ADDITIONAL MATHEMATICS
GCE Ordinary Level
(Syllabus 4038)

CONTENTS

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AIMS

The syllabus is intended to prepare students adequately for A Level H2 Mathematics and H3 Mathematics, where a strong foundation in algebraic manipulation skills and mathematical reasoning skills are required.

The O Level Additional Mathematics syllabus assumes knowledge of O Level Mathematics.

The general aims of the mathematics syllabuses are to enable students to:

- acquire the necessary mathematical concepts and skills for continuous learning in mathematics and related disciplines, and for applications to the real world
- develop the necessary process skills for the acquisition and application of mathematical concepts and skills
- develop the mathematical thinking and problem solving skills and apply these skills to formulate and solve problems
- recognise and use connections among mathematical ideas, and between mathematics and other disciplines
- develop positive attitudes towards mathematics
- make effective use of a variety of mathematical tools (including information and communication technology tools) in the learning and application of mathematics
- produce imaginative and creative work arising from mathematical ideas
- develop the abilities to reason logically, to communicate mathematically, and to learn cooperatively and independently

ASSESSMENT OBJECTIVES

The assessment will test candidates' abilities to:

AO1 understand and use mathematical concepts and skills in a variety of contexts

AO2 organise and analyse data and information; formulate problems into mathematical terms and select and apply appropriate techniques of solution, including manipulation of algebraic expressions

AO3 solve higher order thinking problems; interpret mathematical results and make inferences; reason and communicate mathematically through writing mathematical explanation, arguments, and proofs

SCHEME OF ASSESSMENT

Paper	Duration	Description	Marks	Weighting
Paper 1	2 hours	There will be 11–13 questions of varying marks and lengths testing more on the fundamental skills and concepts. Candidates are required to answer **all** questions.	80	44%
Paper 2	2 hours 30 minutes	There will be 9–11 questions of varying marks and lengths. Candidates are required to answer **all** questions.	100	56%

NOTES

1. Omission of essential working will result in loss of marks.
2. Some questions may integrate ideas from more than one topic of the syllabus where applicable.
3. Relevant mathematical formulae will be provided for candidates.
4. Scientific calculators are allowed in **both** Paper 1 and Paper 2.
5. Unless stated otherwise within a question, three-figure accuracy will be required for answers. Angles in degrees should be given to one decimal place.
6. SI units will be used in questions involving mass and measures.

   Both the 12-hour and 24-hour clock may be used for quoting times of the day. In the 24-hour clock, for example, 3.15 a.m. will be denoted by 03 15; 3.15 p.m. by 15 15, noon by 12 00 and midnight by 24 00.
7. Candidates are expected to be familiar with the solidus notation for the expression of compound units, e.g. 5 m/s for 5 metres per second.
8. Unless the question requires the answer in terms of π, the calculator value for π or $\pi = 3.142$ should be used.

CONTENT OUTLINE

Knowledge of the content of the O Level Mathematics syllabus is assumed in the syllabus below and will not be tested directly, but it may be required indirectly in response to questions on other topics.

	Topic/Sub-topics	Content
1	**Algebra**	
1.1	Quadratic equations and inequalities	Include: • conditions for a quadratic equation to have: – two real roots – two equal roots – no real roots and related conditions for a given line to: – intersect a given curve – be a tangent to a given curve – not intersect a given curve • solution of quadratic inequalities, and the representation of the solution set on the number line • conditions for ax^2+bx+c to be always positive (or always negative) • relationships between the roots and coefficients of the quadratic equation $ax^2+bx+c=0$
1.2	Indices and surds	Include: • four operations on indices and surds • rationalising the denominator • solving equations involving indices and surds
1.3	Polynomials	Include: • multiplication and division of polynomials • use of remainder and factor theorems • factorisation of polynomials • solving cubic equations
1.4	Simultaneous equations in two unknowns	Include: • solving simultaneous equations with at least one linear equation, by substitution • expressing a pair of linear equations in matrix form and solving the equations by inverse matrix method
1.5	Partial fractions	Include cases where the denominator is no more complicated than: • $(ax+b)(cx+d)$ • $(ax+b)(cx+d)^2$ • $(ax+b)(x^2+c^2)$

	Topic/Sub-topics	Content
1.6	Binomial expansions	Include: • use of the Binomial Theorem for positive integer n • use of the notations $n!$ and $\binom{n}{r}$ • use of the general term $\binom{n}{r}a^{n-r}b^r$, $0 < r \leqslant n$ Exclude: • proof of the theorem • knowledge of the greatest term and properties of the coefficients
1.7	Exponential, logarithmic and modulus functions	Include: • functions a^x, e^x, $\log_a x$, $\ln x$ and their graphs • laws of logarithms • equivalence of $y = a^x$ and $x = \log_a y$ • change of base of logarithms • function $\lvert x \rvert$ and graph of $\lvert f(x) \rvert$, where $f(x)$ is linear, quadratic or trigonometric • solving simple equations involving exponential, logarithmic and modulus functions
2	**Geometry and Trigonometry**	
2.1	Trigonometric functions, identities and equations	Include: • six trigonometric functions for angles of any magnitude (in degrees or radians) • principal values of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ • exact values of the trigonometric functions for special angles (30°, 45°, 60°) or $(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3})$ • amplitude, periodicity and symmetries related to the sine and cosine functions • graphs of $y = a\sin(bx) + c$, $y = a\sin\left(\frac{x}{b}\right) + c$, $y = a\cos(bx) + c$, $y = a\cos\left(\frac{x}{b}\right) + c$ and $y = a\tan(bx)$, where a and b are positive integers and c is an integer • use of the following – $\frac{\sin A}{\cos A} = \tan A$, $\frac{\cos A}{\sin A} = \cot A$, $\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$, $\operatorname{cosec}^2 A = 1 + \cot^2 A$ – the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ – the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ – the formulae for $\sin A \pm \sin B$ and $\cos A \pm \cos B$ – the expression for $a\cos\theta + b\sin\theta$ in the form $R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$

	Topic/Sub-topics	Content
		• simplification of trigonometric expressions • solution of simple trigonometric equations in a given interval • proofs of simple trigonometric identities Exclude general solution of trigonometric equations
2.2	Coordinate geometry in two dimensions	Include: • condition for two lines to be parallel or perpendicular • mid-point of line segment • finding the area of rectilinear figure given its vertices • graphs of equations – $y = ax^n$, where n is a simple rational number – $y^2 = kx$ • coordinate geometry of the circle with the equation $(x-a)^2+(y-b)^2=r^2$ and $x^2+y^2+2gx+2fy+c=0$ • transformation of given relationships, including $y = ax^n$ and $y = kb^x$, to linear form to determine the unknown constants from the straight line graph Exclude: • finding the equation of the circle passing through three given points • intersection of two circles
2.3	Proofs in plane geometry	Include: • symmetry and angle properties of triangles, special quadrilaterals and circles♦ • mid-point theorem and intercept theorem for triangles • tangent-chord theorem (alternate segment theorem), intersecting chords theorem and tangent-secant theorem for circles • use of above properties and theorems
3	**Calculus**	
3.1	Differentiation and integration	Include: • derivative of $\mathrm{f}(x)$ as the gradient of the tangent to the graph of $y = \mathrm{f}(x)$ at a point • derivative as rate of change • use of standard notations $\mathrm{f}'(x)$, $\mathrm{f}''(x)$, $\frac{\mathrm{d}y}{\mathrm{d}x}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ $[=\frac{\mathrm{d}}{\mathrm{d}x}(\frac{\mathrm{d}y}{\mathrm{d}x})]$ • derivatives of x^n, for any rational n, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$, together with constant multiples, sums and differences • derivatives of composite functions • derivatives of products and quotients of functions • increasing and decreasing functions • stationary points (maximum and minimum turning points and stationary points of inflexion)

♦ These are properties learnt in O Level Mathematics.

	Topic/Sub-topics	Content
		• use of second derivative test to discriminate between maxima and minima • applying differentiation to gradients, tangents and normals, connected rates of change and maxima and minima problems • integration as the reverse of differentiation • integration of x^n for any rational n, sin x, cos x, $\sec^2 x$ and e^x, together with constant multiples, sums and differences • integration of $(ax + b)^n$ for any rational n, $\sin(ax + b)$, $\cos(ax + b)$ and $e^{(ax + b)}$ • definite integral as area under a curve • evaluation of definite integrals • finding the area of a region bounded by a curve and lines parallel to the coordinate axes • finding areas of regions below the x-axis • application of differentiation and integration to problems involving displacement, velocity and acceleration of a particle moving in a straight line with variable or constant acceleration Exclude: • differentiation of functions defined implicitly and parametrically • finding the area of a region between a curve and an oblique line, or between two curves • use of formulae for motion with constant acceleration

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$$

$$\sin A - \sin B = 2\cos\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$$

$$\cos A + \cos B = 2\cos\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$$

$$\cos A - \cos B = -2\sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

MATHEMATICAL NOTATION

The list which follows summarises the notation used in Cambridge's Mathematics examinations. Although primarily directed towards A Level, the list also applies, where relevant, to examinations at all other levels.

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	universal set
A'	the complement of the set A
$\mathbb{Z}$	the set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geqslant 0\}$
$\mathbb{R}^n$	the real n tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\nsubseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\propto$	is proportional to
$<$	is less than
$\leqslant$; $\ngtr$	is less than or equal to; is not greater than
$>$	is greater than
$\geqslant$; $\nless$	is greater than or equal to; is not less than
∞	infinity

3. *Operations*

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\frac{a}{b}$, a/b	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + ... + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, $(0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!\,(n-r)!}$, for $n,\ r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leqslant r \leqslant n$ $\frac{n(n-1)...(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. *Functions*

f	function f
$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f}: A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim\limits_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
Δx ; δx	an increment of x
$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x), \mathrm{f}''(x), \ldots, \mathrm{f}^{(n)}(x)$	the first, second, … nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y \,\mathrm{d}x$	indefinite integral of y with respect to x
$\int_a^b y \,\mathrm{d}x$	the definite integral of y with respect to x for values of x between a and b
$\dot{x}, \ddot{x}, \ldots$	the first, second, …derivatives of x with respect to time

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

6. *Circular Functions and Relations*

sin, cos, tan, cosec, sec, cot	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$	the inverse circular functions

7. *Complex Numbers*

i	square root of -1
z	a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta),\ r \in \mathbb{R}_0^+$ $= r\mathrm{e}^{\mathrm{i}\theta},\ r \in \mathbb{R}_0^+$
$\operatorname{Re} z$	the real part of z, $\operatorname{Re}(x + \mathrm{i}y) = x$
$\operatorname{Im} z$	the imaginary part of z, $\operatorname{Im}(x + \mathrm{i}y) = y$
$\lvert z \rvert$	the modulus of z, $\lvert x + \mathrm{i}y \rvert = \sqrt{(x^2 + y^2)}$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta) \rvert = r$
$\arg z$	the argument of z, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta$, $-\pi < \theta \leqslant \pi$
z^*	the complex conjugate of z, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$

8. *Matrices*

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
det $\mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

9. *Vectors*

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$\lvert \mathbf{a} \rvert$	the magnitude of $\mathbf{a}$
$\lvert \overrightarrow{AB} \rvert$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a}.\mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10. *Probability and Statistics*

A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A, the event 'not A'
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
X, Y, R, etc.	random variables
x, y, r, etc.	value of the random variables X, Y, R, etc.
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations, $x_1, x_2 \ldots$ occur
$\mathrm{p}(x)$	the value of the probability function $\mathrm{P}(X = x)$ of the discrete random variable X
$p_1, p_2 \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
$\mathrm{f}(x)$, $\mathrm{g}(x) \ldots$	the value of the probability density function of the continuous random variable X
$\mathrm{F}(x)$, $\mathrm{G}(x) \ldots$	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of the random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{B}(n, p)$	binominal distribution, parameters n and p
$\mathrm{Po}(\mu)$	Poisson distribution, mean μ
$\mathrm{N}(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$	sample mean
s^2	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1}\Sigma(x - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $\mathrm{N}(0, 1)$
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample