ADDITIONAL MATHEMATICS

GCE Ordinary Level

(Syllabus 4018)

CONTENTS

NOTES

Electronic Calculators

1. The use of silent electronic calculators is **expected** in O level Additional Mathematics (4018).

2. More detailed regulations concerning the use of electronic calculators will be issued by the Singapore Examinations and Assessment Board.

Lists of Formulae, etc.

Formulae for O level Additional Mathematics (4018) are printed on the question papers.

Mathematical Instruments

Apart from the usual mathematical instruments, candidates may use flexicurves.

Mathematical Notation

Attention is drawn to the list of mathematical notation at the end of this booklet.

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SYLLABUS AIMS

The course should enable students to:

1. consolidate and extend their elementary mathematical skills, and use these in the context of more advanced techniques;
2. further develop their knowledge of mathematical concepts and principles, and use this knowledge for problem solving;
3. appreciate the interconnectedness of mathematical knowledge;
4. acquire a suitable foundation in mathematics for further study in the subject or in mathematics related subjects;
5. devise mathematical arguments and use and present them precisely and logically;
6. integrate information technology to enhance the mathematical experience;
7. conduct independent and/or cooperative enquiry and experiment, including extended pieces of work of a practical and investigative kind;
8. develop the confidence to apply their mathematical skills and knowledge in appropriate situations;
9. develop creativity and perseverance in the approach to problem solving;
10. derive enjoyment and satisfaction from engaging in mathematical pursuits, and gain an appreciation of the beauty, power and usefulness of mathematics.

ASSESSMENT OBJECTIVES

The examination will test the ability of candidates to:

1. recall and use manipulative technique;
2. interpret and use mathematical data, symbols and terminology;
3. comprehend numerical, algebraic and spatial concepts and relationships;
4. recognise the appropriate mathematical procedure for a given situation;
5. formulate problems into mathematical terms and select and apply appropriate techniques of solution.

EXAMINATION STRUCTURE

There will be two papers, each of 2 hours and each carries 80 marks.

Content for PAPER 1 and PAPER 2 will not be dissected.

Each paper will consist of approximately 10–12 questions of various lengths. There will be no choice of question except that the last question in each paper will consist of two alternatives, only one of which must be answered. The mark allocations for the last question will be in the range of 10–12 marks.

DETAILED SYLLABUS

Knowledge of the content of the Syndicate's Ordinary level Syllabus (or an equivalent Syllabus) is assumed. Ordinary level material, which is not repeated in the syllabus below, will not be tested directly but it may be required in response to questions on other topics.

Proofs of results will not be required unless specifically mentioned in the syllabus.

Candidates will be expected to be familiar with the scientific notation for the expression of compound units e.g. 5 ms^{-1} for 5 metres per second.

THEME OR TOPIC	CURRICULUM OBJECTIVES
	Candidates should be able to:
1 Set language and notation	• use set language and notation, and Venn diagrams to describe sets and represent relationships between sets as follows: $A = \{x : x \text{ is a natural number}\}$ $B = \{(x, y): y = mx + c\}$ $C = \{x : a \leqslant x \leqslant b\}$ $D = \{a, b, c, ...\}$
	• understand and use the following notation: Union of A and B — $A \cup B$ Intersection of A and B — $A \cap B$ Number of elements in set A — n(A) "... is an element of ..." — $\in$ "... is not an element of ..." — $\notin$ Complement of set A — A' The empty set — $\varnothing$ Universal set — $\mathscr{E}$ A is a subset of B — $A \subseteq B$ A is a proper subset of B — $A \subset B$ A is not a subset of B — $A \not\subseteq B$ A is not a proper subset of B — $A \not\subset B$
2 Functions	• understand the terms function, domain, range (image set), one-one function, inverse function and composition of functions;
	• use the notation $f(x) = \sin x$, $f: x \mapsto \lg x$, $(x > 0)$, $f^{-1}(x)$ and $f^2(x)$ [$=f(f(x))$];
	• understand the relationship between $y = f(x)$ and $y = \lvert f(x)\rvert$, where $f(x)$ may be linear, quadratic or trigonometric;
	• explain in words why a given function is a function or why it does not have an inverse;
	• find the inverse of a one-one function and form composite functions;
	• use sketch graphs to show the relationship between a function and its inverse.

3 Quadratic functions	• find the maximum or minimum value of the quadratic function f : $x \mapsto ax^2 + bx + c$ by any method; • use the maximum or minimum value of f(x) to sketch the graph or determine the range for a given domain; • know the conditions for f(x) = 0 to have (i) two real roots, (ii) two equal roots, (iii) no real roots; and the related conditions for a given line to (i) intersect a given curve, (ii) be a tangent to a given curve, (iii) not intersect a given curve; • solve quadratic equations for real roots and find the solution set for quadratic inequalities.
4 Indices and surds	• perform simple operations with indices and with surds, including rationalising the denominator.
5 Factors of polynomials	• know and use the remainder and factor theorems; • find factors of polynomials; • solve cubic equations.
6 Simultaneous equations	• solve simultaneous equations in two unknowns with at least one linear equation.
7 Logarithmic and exponential functions	• know simple properties and graphs of the logarithmic and exponential functions including ln$x$ and e^x (series expansions are not required); • know and use the laws of logarithms (including change of base of logarithms); • solve equations of the form $a^x = b$.
8 Straight line graphs	• interpret the equation of a straight line graph in the form $y = mx + c$; • transform given relationships, including $y = ax^n$ and $y = Ab^x$, to straight line form and hence determine unknown constants by calculating the gradient or intercept of the transformed graph; • solve questions involving mid-point and length of a line; • know and use the condition for two lines to be parallel or perpendicular.
9 Circular measure	• solve problems involving the arc length and sector area of a circle, including knowledge and use of radian measure.

10 Trigonometry

- know the six trigonometric functions of angles of any magnitude (sine, cosine, tangent, secant, cosecant, cotangent);
- understand amplitude and periodicity and the relationship between graphs of e.g. $\sin x$ and $\sin 2x$;
- draw and use the graphs of $y = a\sin(bx) + c$, $y = a\cos(bx) + c$, $y = a\tan(bx) + c$, where a, b are positive integers and c is an integer;
- use the relationships $\dfrac{\sin A}{\cos A} = \tan A$, $\dfrac{\cos A}{\sin A} = \cot A$, $\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$, $\operatorname{cosec}^2 A = 1 + \cot^2 A$, and solve simple trigonometric equations involving the six trigonometric functions and the above relationships (not including general solution of trigonometric equations);
- prove simple trigonometric identities**.**

11 Permutations and combinations

- recognise and distinguish between a permutation case and a combination case;
- know and use the notation $n!$, (with $0! = 1$), and the expressions for permutations and combinations of n items taken r at a time;
- answer simple problems on arrangement and selection (cases with repetition of objects, or with objects arranged in a circle or involving both permutations and combinations, are excluded).

12 Binomial expansions

- use the Binomial Theorem for expansion of $(a + b)^n$ for positive integral n;
- use the general term $\binom{n}{r} a^{n-r} b^r$, $0 \leqslant r \leqslant n$

  (knowledge of the greatest term and properties of the coefficients is not required).

13 Vectors in 2 dimensions

- use vectors in any form, e.g. $\begin{pmatrix} a \\ b \end{pmatrix}$, $\overrightarrow{AB}$, $\mathbf{p}$, $a\mathbf{i} + b\mathbf{j}$;
- know and use position vectors and unit vectors;
- find the magnitude of a vector. Add and subtract vectors and multiply vectors by scalars;
- compose and resolve velocities;
- use relative velocity including solving problems on interception (but not closest approach).

14 Matrices

- display information in the form of a matrix of any order and interpret the data in a given matrix;
- solve problems involving the calculation of the sum and product (where appropriate) of two matrices and interpret the results;
- calculate the product of a scalar quantity and a matrix;
- use the algebra of 2 x 2 matrices (including the zero and identity matrix);
- calculate the determinant and inverse of a non-singular 2 x 2 matrix and solve simultaneous linear equations.

15 Differentiation and integration

- understand the idea of a derived function;
- use the notations $f'(x)$, $f''(x)$, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}\left[= \frac{d}{dx}\left(\frac{dy}{dx}\right)\right]$;
- use the derivatives of the standard functions x^n (for any rational n), $\sin x$, $\cos x$, $\tan x$, e^x, $\ln x$, together with constant multiples, sums and composite functions of these;
- differentiate products and quotients of functions;
- apply differentiation to gradients, tangents and normals, stationary points, connected rates of change, small increments and approximations and practical maxima and minima problems;
- discriminate between maxima and minima by any method;
- understand integration as the reverse process of differentiation;
- integrate sums of terms in powers of x excluding $\frac{1}{x}$;
- integrate functions of the form $(ax + b)^n$ (excluding $n = -1$), e^{ax+b}, $\sin(ax + b)$, $\cos(ax + b)$;
- evaluate definite integrals and apply integration to the evaluation of plane areas;
- apply differentiation and integration to kinematics problems that involve displacement, velocity and acceleration of a particle moving in a straight line with variable or constant acceleration, and the use of x-t and v-t graphs.

MATHEMATICAL NOTATION

The list which follows summarizes the notation used in the Syndicate's Mathematics examinations. Although primarily directed towards A level, the list also applies, where relevant, to examinations at all other levels, i.e. O level, AO level and N level.

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	universal set
A'	the complement of the set A
$\mathbb{N}$	the set of positive integers, {1, 2, 3, …}
$\mathbb{Z}$	the set of integers, {0, ±1, ±2, ±3, …}
$\mathbb{Z}^+$	the set of positive integers, {1, 2, 3, …}
$\mathbb{Z}_n$	the set of integers modulo n, {0, 1, 2, …, n – 1}
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geqslant 0\}$
$\mathbb{R}^n$	the real n tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\nsubseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$
yRx	y is related to x by the relation R

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$; $\ll$	is less than; is much less than
$\leqslant$; $\ngtr$	is less than or equal to; is not greater than
$>$; $\gg$	is greater than; is much greater than
$\geqslant$; $\nless$	is greater than or equal to; is not less than
∞	infinity

3. *Operations*

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\dfrac{a}{b}$, a/b	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + ... + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$ (0! = 1)
$\dbinom{n}{r}$	the binomial coefficient $\dfrac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leqslant r \leqslant n$ $\dfrac{n(n-1)...(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. *Functions*

f	function f
$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f}: A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim\limits_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
Δx ; δx	an increment of x
$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, …, $\mathrm{f}^{(n)}(x)$	the first, second, … nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y \, \mathrm{d}x$	indefinite integral of y with respect to x
$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x for values of x between a and b
$\dfrac{\partial y}{\partial x}$	the partial derivative of y with respect to x
$\dot{x}$, $\ddot{x}$, …	the first, second, …derivatives of x with respect to time

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

6. *Circular Functions and Relations*

sin, cos, tan, cosec, sec, cot	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ $\operatorname{cosec}^{-1}$, $\sec^{-1}$, $\cot^{-1}$	the inverse circular functions

7. *Complex Numbers*

i	square root of -1
z	a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta),\ r \in \mathbb{R}_0^+$ $= r\mathrm{e}^{\mathrm{i}\theta},\ r \in \mathbb{R}_0^+$
$\mathrm{Re}\ z$	the real part of z, $\mathrm{Re}\ (x + \mathrm{i}y) = x$
$\mathrm{Im}\ z$	the imaginary part of z, $\mathrm{Im}\ (x + \mathrm{i}y) = y$
$\lvert z \rvert$	the modulus of z, $\lvert x + \mathrm{i}y \rvert = \sqrt{(x^2 + y^2)}$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta) \rvert = r$
$\arg z$	the argument of z, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta$, $-\pi < \theta \leqslant \pi$
z^*	the complex conjugate of z, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$

8. *Matrices*

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
det $\mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

9. *Vectors*

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$\lvert \mathbf{a} \rvert$	the magnitude of $\mathbf{a}$
$\lvert \overrightarrow{AB} \rvert$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a.b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10. *Probability and Statistics*

A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A, the event 'not A'
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
X, Y, R, etc.	random variables
x, y, r, etc.	value of the random variables X, Y, R, etc.
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations, $x_1, x_2 \ldots$ occur

$\mathrm{p}(x)$	the value of the probability function $\mathrm{P}(X = x)$ of the discrete random variable X
$p_1,\ p_2 \cdots$	probabilities of the values $x_1,\ x_2, \ldots$ of the discrete random variable X
$\mathrm{f}(x), \mathrm{g}(x)\ldots$	the value of the probability density function of the continuous random variable X
$\mathrm{F}(x), \mathrm{G}(x)\ldots$	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of the random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{B}(n, p)$	binominal distribution, parameters n and p
$\mathrm{N}(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\overline{x}$	sample mean
s^2	unbiased estimate of population variance from a sample, $s^2 = \dfrac{1}{n-1}\sum(x - \overline{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $\mathrm{N}\ (0, 1)$
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample
$\mathrm{Cov}(X, Y)$	covariance of X and Y