1. (i) $T = 20 + Ae^{-kt}$

When $t=0$, $T=80$,

$80 = 20 + Ae^{0}$

$A = 60$ (shown) #

(ii) $T = 20 + 60e^{-kt}$

When $t=1$ and $T=65°C$,

$65 = 20 + 60e^{-k}$

$60e^{-k} = 45$

$e^{-k} = \frac{3}{4}$

$-k \ln e = \ln\frac{3}{4}$

$k = 0.2876$

≈ 0.288 (3 s.f.) #

(iii) $20 + 60e^{-0.2876t} < 40$

$60e^{-0.2876t} < 20$

$e^{-0.2876t} < \frac{1}{3}$

$-0.2876t \ln e < \ln\frac{1}{3}$

$t > 3.8199$

$\therefore$ It is safe to give the food 4 minutes after removal from the microwave. #

2. $f(x) = 2x^3 - 3x^2 - 11x + 6$

(i) When $f(x)$ is divided by $x-2$,

remainder, $R = f(2)$

$= 2(2)^3 - 3(2)^2 - 11(2) + 6$

$= -12$ #

(ii) $f(-2) = 2(-2)^3 - 3(-2)^2 - 11(-2) + 6$

$= 0$

By factor theorem, $(x+2)$ is a factor of $f(x)$.

(shown) #

$f(x) = (x+2)(2x^2 + bx + 3)$

Comparing coeff. of x: $-11 = 3 + 2b$

$2b = -14$

$b = -7$

$\therefore f(x) = (x+2)(2x^2 - 7x + 3)$

$= (x+2)(2x-1)(x-3)$

$\Rightarrow (x+2)(2x-1)(x-3) = 0$

$x + 2 = 0$ or $2x - 1 = 0$ or $x - 3 = 0$

$x = -2$ $\quad x = \frac{1}{2}$ $\quad x = 3$

#

3. (i) Length of the rectangle

$$= \frac{13-\sqrt{48}}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$$

$$= \frac{(13-4\sqrt{3})(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})}$$

$$= \frac{39+13\sqrt{3}-12\sqrt{3}-4(3)}{9-3}$$

$$= \frac{27+\sqrt{3}}{6}$$

$$= \frac{27}{6}+\frac{\sqrt{3}}{6}$$

$$= \frac{9}{2}+\frac{1}{6}\sqrt{3} \ \#$$

(ii) $(2\sqrt{3}+c)^2 = 13-\sqrt{48}$

$4(3)+4\sqrt{3}c+c^2 = 13-\sqrt{48}$

$12+c^2+4\sqrt{3}c = 13-4\sqrt{3}$

Comparing coefficients of $\sqrt{3}$,

$\Rightarrow 4c = -4$

$c = -1 \ \#$

4. $2x^2+5x+4=0$, roots α and β.

$a=2,\ b=5,\ c=4$

$\alpha+\beta=-\frac{5}{2} \qquad \alpha\beta=2$

(i) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\left(-\frac{5}{2}\right)^2-2(2)$

$=2\frac{1}{4}$

(ii) $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$=\left(-\frac{5}{2}\right)\left[\alpha^2+\beta^2-\alpha\beta\right]$

$=\left(-\frac{5}{2}\right)\left[\frac{9}{4}-2\right]$

$=-\frac{5}{8}$

$\alpha^3\beta^3=(\alpha\beta)^3$

$=(2)^3$

$=8$

An equation with roots α^3 and β^3 is

$x^2-\left(-\frac{5}{8}\right)x+8=0$

$8x^2+5x+64=0$ #

5(a) $2\log_2 x - \log_2 (x-4) = 3$

$\log_2 x^2 - \log_2 (x-4) = 3$

$\log_2 \frac{x^2}{x-4} = 3$

$\frac{x^2}{x-4} = 2^3$

$x^2 = 8(x-4)$

$x^2 - 8x + 32 = 0$

$a=1,\ b=-8,\ c=32$

$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(32)}}{2(1)}$

$= \frac{8 \pm \sqrt{-64}}{2}$ (N.A)

$\therefore$ There are no real solutions. ✱

(b) $\frac{(\log_x y)^2}{\log_y x} + 8 = 0$

$(\log_x y)^2 \log_x y + 8 = 0$

let $A = \log_x y$, $A^3 + 8 = 0$

$(A+2)(A^2 - 2A + 4) = 0$

$A + 2 = 0$ or $A^2 - 2A + 4 = 0$

$A = -2$ (NA) ✱

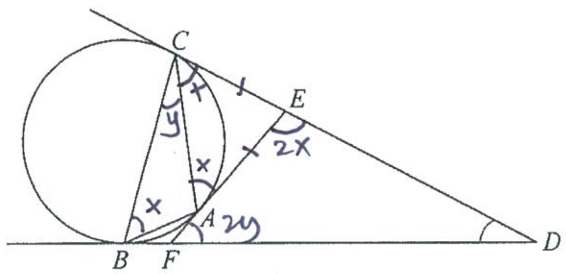
$\log_x y = -2$

$y = x^{-2} = \frac{1}{x^2}$ ✱

6 (i) let $\hat{ACE} = x$.

$\hat{CAE} = \hat{ACE} = x^\circ$

[$\triangle CEA$ is an isos. $\triangle$,
$\because$ CE and EA are tangents from E, then CE = EA]



$\hat{ABC} = \hat{ACE}$

$= x^\circ$ (Tangent - Chord Theorem)

$\Rightarrow \hat{DEF} = \hat{ACE} + \hat{CAE}$ (Ext $\angle$ = Sum of Int. $\angle$s)

$= 2x$

$= 2 \times \hat{ABC}$ (shown) #

(ii) $\hat{DFE} = 2 \times \hat{ACB}$

(iii) let $\hat{ACB} = y$

$\hat{DFE} = 2y$

$\hat{BAC} = 180^\circ - x - y$

$2 \times \hat{BAC} = 2[180^\circ - x - y]$

$= 360^\circ - 2x - 2y$

$= 180^\circ + 180^\circ - 2x - 2y$

$= 180^\circ + \hat{EDF}$ (shown) #

7. (i) $y = 2 - (3-x)^4$ —(1)

Sub (p, q) into (1),

$q = 2 - (3-p)^4$ —(2)

$\frac{dy}{dx} = -4(3-x)^3(-1)$

$= 4(3-x)^3$ —(3)

Sub $x = p$ and $\frac{dy}{dx} = 0$ into (3),

$0 = 4(3-p)^3$

$3 - p = 0$

$p = 3$

Sub $p = 3$ into (2),

$q = 2 - (3-3)^4$

$= 2$

$\therefore p = 3$ and $q = 2$ #

(ii)

x	2.9	3	3.1
$\frac{dy}{dx}$	+	0	−
	/	—	\

[$p = 3$]

(a). For values of x less than 3, y is an increasing function.

(b) For values of x greater than 3, y is a decreasing function.

7 (iii) The stationary point is a maximum point.

(iv) $\frac{dy}{dx} = 4(3-x)^3$

$\frac{d^2y}{dx^2} = -12(3-x)^2$

when $x=3$, $\frac{d^2y}{dx^2} = -12(3-3)^2$

$= 0$ #

8. (i) $a = -e^{-0.1t}$

$v = \int a\, dt$

$= \int -e^{-0.1t}\, dt$

$= \frac{-e^{-0.1t}}{-0.1} + c$

when $t=0$, $v=8$, $8 = 10e^0 + c$

$c = -2$

$\therefore v = 10e^{-0.1t} - 2$

when $v=0$, $10e^{-0.1t} - 2 = 0$

$10e^{-0.1t} = 2$

$e^{-0.1t} = \frac{1}{5}$

$\ln e^{-0.1t} = \ln\frac{1}{5}$

$-0.1t = \ln 1 - \ln 5$

$0.1t = \ln 5$

$t = 10\ln 5$ # (shown)

8(ii) $s = \int v \, dt$

$= \int (10e^{-0.1t} - 2) \, dt$

$= -100e^{-0.1t} - 2t + c$

When $t=0$, $s=0$,

$0 = -100e^{0} - 0 + c$

$c = 100$

$\therefore s = -100e^{-0.1t} - 2t + 100$

When $t = 10\ln 5$, $OP = -100e^{-0.1(10\ln 5)} - 2(10\ln 5) + 100$

$= -100e^{-\ln 5} - 20\ln 5 + 100$

$= -20 - 20\ln 5 + 100$

$= 80 - 20\ln 5$

$= 47.8$ m (3 s.f) #

(iii) $s = -100e^{-0.1t} - 2t + 100$

When $t = 49$, $s = -100e^{-0.1(49)} - 2(49) + 100$

$= 1.255$ m

When $t = 50$, $s = -100e^{-0.1(50)} - 2(50) + 100$

$= -0.6737$ m

Since the particle's displacement from O is positive when $t = 49$ s and negative when $t = 50$ s, this shows that it has passed O again at some instant during the fiftieth second. #

9. (i) $3\cos 2A + \sin A - 2 = 0$

$3(1 - 2\sin^2 A) + \sin A - 2 = 0$

$3 - 6\sin^2 A + \sin A - 2 = 0$

$6\sin^2 A - \sin A - 1 = 0$

$(2\sin A - 1)(3\sin A + 1) = 0$

$2\sin A - 1 = 0$ or $3\sin A + 1 = 0$

$\sin A = \frac{1}{2}$; $\sin A = -\frac{1}{3}$

$\alpha = 30°$; $\alpha = 19.471°$

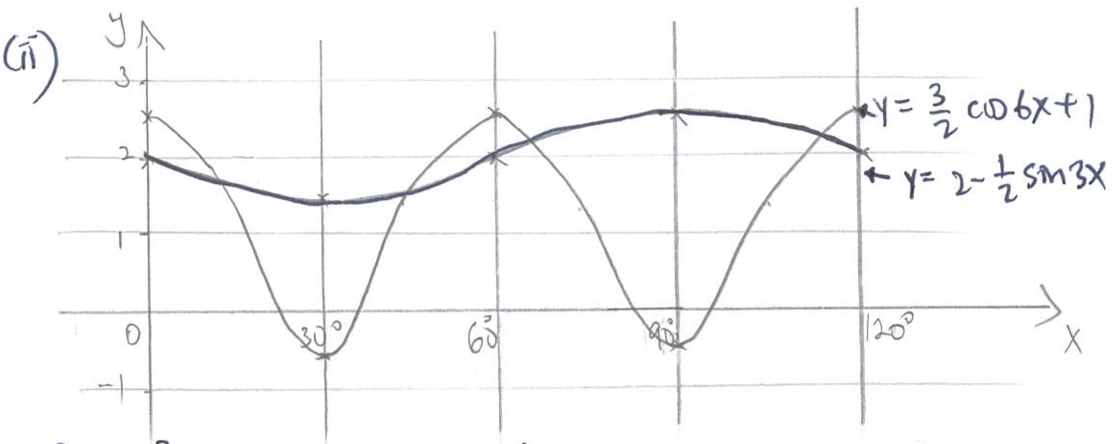
$A = 30°, 150°$; $A = 180° + 19.471°, 360° - 19.471°$

$= 199.471, 340.528°$

$\approx 199.5°, 340.5°$ (1 d.p)

$\therefore A = 30°, 150°, 199.5°, 340.5°$ #

(ii)



(iii) $\frac{3}{2}\cos 6x + 1 = 2 - \frac{1}{2}\sin 3x$

$3\cos 6x + 2 = 4 - \sin 3x$

$3\cos 6x + \sin 3x - 2 = 0$ [refer to 9(i) answer]

let $A = 3x$,

$3x = 30°, 150°$ & $3x = 199.471°, 340.528°$

$x = 10°, 50°$; $x = 66.49°, 113.52°$

$\approx 66.5°, 113.5°$ (1 d.p)

10. $x^2 + y^2 + 4x - 6y = 36$

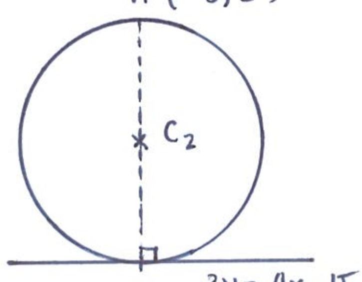
(i) $x^2 + 4x + y^2 - 6y = 36$

$(x+2)^2 - 2^2 + (y-3)^2 - 3^2 = 36$

$(x+2)^2 + (y-3)^2 = 49$

$(x+2)^2 + (y-3)^2 = 7^2$

$\Rightarrow$ radius $= 7$ units and centre of $C_1 = (-2, 3)$ #



(ii) Gradient of the tangent to C_2 at B
$= \frac{4}{3}$

Gradient of diameter AB $= -\frac{3}{4}$

Equation of diameter AB $\Rightarrow y - 5 = -\frac{3}{4}(x+5)$

$\Rightarrow \quad y = -\frac{3}{4}x + \frac{5}{4}$ # —①

$3y = 4x - 15$ —②

Sub ① into ②, $3\left(-\frac{3}{4}x + \frac{5}{4}\right) = 4x - 15$

$-\frac{9}{4}x + \frac{15}{4} = 4x - 15$

$6\frac{1}{4}x = 18\frac{3}{4}$

$x = 3$

Sub $x = 3$ into ①, $y = -\frac{3}{4}(3) + \frac{5}{4}$

$= -1$

$\therefore$ B $(3, -1)$ #

10.(iii) C_2 = midpt of AB

$= \left(\frac{3-5}{2}, \frac{-1+5}{2}\right)$

$= (-1, 2)$ #

radius of $C_2 = \sqrt{(-1+5)^2 + (2-5)^2}$

$= 5$ units #

(iv) let (4,6) be represented by the point D.

$DC_1 = \sqrt{(4+2)^2 + (6-3)^2}$

$= \sqrt{45}$ units

$DC_2 = \sqrt{(4+1)^2 + (6-2)^2}$

$= \sqrt{41}$ units

$DC_2 = \sqrt{41}$ > radius of circle C_2

$\Rightarrow$ (4,6) does not lie in Circle C_2.

radius of circle C_2 $< DC_1 = \sqrt{45}$ < radius of circle C_1

$\Rightarrow$ (4,6) only lies in Circle C_1 but not Circle C_2. (shown) #

11(a) $\frac{d}{dx}\left[\frac{x}{\sqrt{2x-1}}\right] = \frac{\sqrt{2x-1}(1) - x(\frac{1}{2})(2x-1)^{-\frac{1}{2}}(2)}{(\sqrt{2x-1})^2}$

$= \left[\frac{\sqrt{2x-1}}{1} - \frac{x}{\sqrt{2x-1}}\right] \times \frac{1}{2x-1}$

$= \frac{(2x-1) - x}{\sqrt{2x-1}} \times \frac{1}{2x-1}$

$= \frac{x-1}{(2x-1)^{\frac{1}{2}}(2x-1)}$

$= \frac{x-1}{(2x-1)^{\frac{3}{2}}}$

$= \frac{x-1}{\sqrt{(2x-1)^3}}$ (shown)

(b) $y = \frac{8(x-1)}{\sqrt{(2x-1)^3}}$ — (1)

when $y=0$, $8(x-1)=0$
$x=1$

$\Rightarrow A(1,0)$.

Equation of line AB $\Rightarrow y = 1(x-1)$
$y = x-1$ — (2)

(1)=(2), $x-1 = \frac{8(x-1)}{\sqrt{(2x-1)^3}}$

$(x-1)\sqrt{(2x-1)^3} = 8(x-1)$

$(x-1)\sqrt{(2x-1)^3} - 8(x-1) = 0$

$(x-1)\left[\sqrt{(2x-1)^3} - 8\right] = 0$

$x=1$ (rej). or $\sqrt{(2x-1)^3} = 8$

$(2x-1) = 8^{\frac{2}{3}}$

$2x-1 = 4$

$2x = 5$

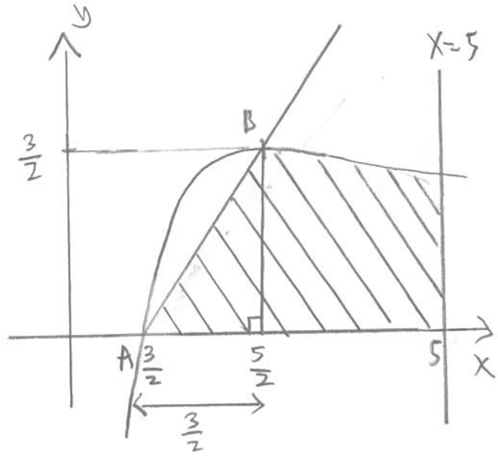
$x = \frac{5}{2}$

11 (b) (i) (Cont') Sub $x = \frac{5}{2}$ into (1),

$$y = \frac{8(\frac{5}{2}-1)}{\sqrt{(5-1)^3}}$$

$$= \frac{3}{2} \ \# \text{ (verified)}$$

(ii) Area of shaded region



$$= \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{3}{2}\right) + \int_{\frac{5}{2}}^{5} \frac{8(x-1)}{\sqrt{(2x-1)^3}}\,dx$$

$$= \frac{9}{8} + 8\int_{\frac{5}{2}}^{5} \frac{x-1}{\sqrt{(2x-1)^3}}\,dx$$

$$= \frac{9}{8} + 8\left[\frac{x}{\sqrt{2x-1}}\right]_{\frac{5}{2}}^{5}$$

$$= \frac{9}{8} + 8\left[\frac{5}{\sqrt{9}} - \frac{\frac{5}{2}}{\sqrt{4}}\right]$$

$$= \frac{9}{8} + 8\left[\frac{5}{3} - \frac{5}{4}\right]$$

$$= \frac{9}{8} + \frac{10}{3}$$

$$= 4\frac{11}{24} \text{ units}^2 \ \#$$