1. $(2-kx)^5 + (3+x)^6$

$= (2)^5 + {}^5C_1(2)^4(-kx) + {}^5C_2(2)^3(-kx)^2 + {}^5C_3(2)^2(-kx)^3 + \cdots$

$+ (3)^6 + {}^6C_1(3)^5(x)^1 + {}^6C_2(3)^4(x)^2 + {}^6C_3(3)^3(x)^3 + \cdots$

$= \cdots + {}^5C_3(2)^2(-kx)^3 + {}^6C_3(3)^3(x^3) + \cdots$

$= \cdots \; -40k^3x^3 + 540x^3 + \cdots$

coefficient of x^3 : $860 = -40k^3 + 540$

$320 = -40k^3$

$k^3 = -8$

$k = -2$ #

2. $\tan(A+B) = 8$

$\dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{8}{1}$

$\dfrac{\tan A + 3}{1 - 3\tan A} = \dfrac{8}{1}$

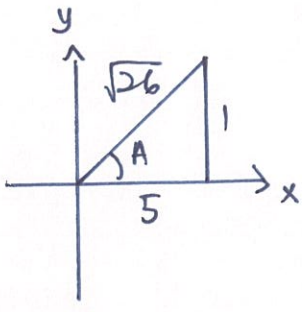
$\tan A + 3 = 8(1 - 3\tan A)$

$\tan A + 3 = 8 - 24\tan A$

$25\tan A = 5$

$\tan A = \dfrac{5}{25}$

$= \dfrac{1}{5}$



$\therefore \sin A = \dfrac{1}{\sqrt{26}} \times \dfrac{\sqrt{26}}{\sqrt{26}}$

$= \dfrac{\sqrt{26}}{26}$ #

3. $y = 2 - \frac{1}{x^2}$ ——(1)

$= 2 - x^{-2}$

$\frac{dy}{dx} = 2x^{-3}$

$\frac{dx}{dt} = 0.12$ & $\frac{dy}{dt} = 0.03$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$0.03 = 2x^{-3} \times 0.12$

$\frac{1}{4} = \frac{2}{x^3}$

$x^3 = 8$

$x = 2$

Sub $x = 2$ into (1),

$y = 2 - \frac{1}{2^2}$

$= 1\frac{3}{4}$ #

4. $\frac{(x+2)^2}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$

$(x+2)^2 = Ax(x-2) + B(x-2) + C(x^2)$

Sub $x=2$, $16 = 4C$

$C = 4$

Sub $x=0$, $4 = B(-2)$

$B = -2$

Sub $x=1$, $3^2 = A(-1) - 2(-1) + 4$

$9 = -A + 6$

$A = -3$

$\therefore \frac{(x+2)^2}{x^2(x-2)} = -\frac{3}{x} - \frac{2}{x^2} + \frac{4}{x-2}$ ⧣

5. (i) See attached graph.

Incorrect reading of $v = 0.263$

(ii) Correct reading of $v = \frac{1}{4.351}$

$= 0.2298$

≈ 0.230 (3 s.f) ⧣

(iii) $\frac{1}{f} = 8.32$ ($\frac{1}{v}$-intercept)

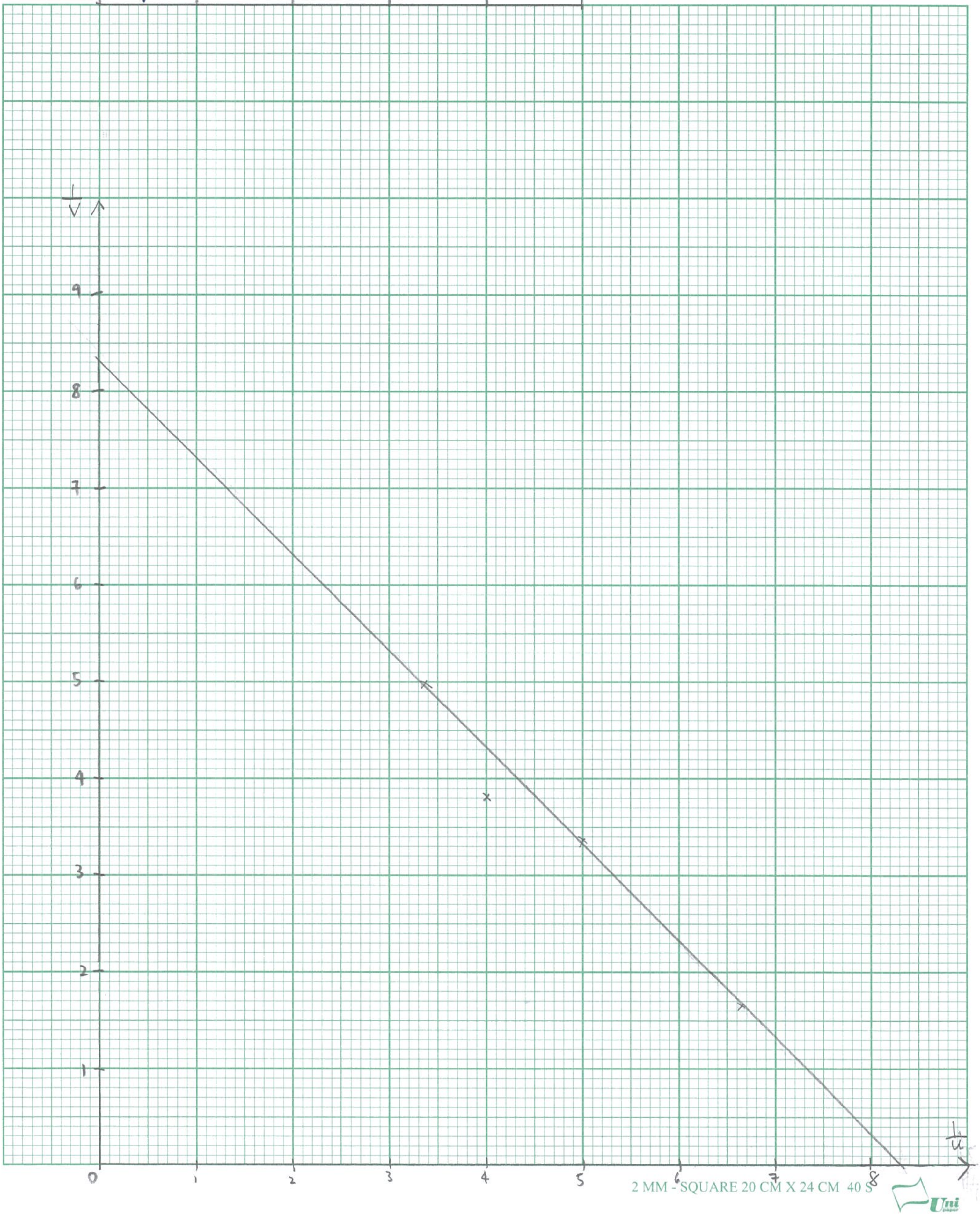
$f = \frac{1}{8.32}$

$= 0.1201$

≈ 0.120 (3 s.f.) ⧣

5.

$\frac{1}{u}$	6.66	5	4	3.33
$\frac{1}{v}$	1.65	3.34	3.80	4.97



6. (i) LHS $= \dfrac{1}{(1+\operatorname{cosec}\theta)(\sec\theta-\tan\theta)}$

$= 1 \div \left[\left(1+\dfrac{1}{\sin\theta}\right)\left(\dfrac{1}{\cos\theta}-\dfrac{\sin\theta}{\cos\theta}\right)\right]$

$= 1 \div \left[\left(\dfrac{\sin\theta+1}{\sin\theta}\right)\left(\dfrac{1-\sin\theta}{\cos\theta}\right)\right]$

$= 1 \div \left[\dfrac{1-\sin^2\theta}{\sin\theta\cos\theta}\right]$

$= 1 \div \dfrac{\cos^2\theta}{\sin\theta\cos\theta}$

$= 1 \div \dfrac{\cos\theta}{\sin\theta}$

$= 1 \times \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

$=$ RHS (shown) #

(ii) $\dfrac{1}{(1+\operatorname{cosec}\theta)(\sec\theta-\tan\theta)} = 3\cot\theta$

$\tan\theta = \dfrac{3}{\tan\theta}$

$\tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3}$

Basic $\angle = \dfrac{\pi}{3}$

$\theta = \dfrac{\pi}{3},\ \pi-\dfrac{\pi}{3},\ \pi+\dfrac{\pi}{3},\ 2\pi-\dfrac{\pi}{3}$

$= 1.0471,\ \underset{(rej)}{2.0943},\ \underset{(rej)}{4.1887},\ \underset{(rej)}{5.2359}$

≈ 1.05 radians (3 s.f.) #

7. (i) Sub $x=h$ into $y=2x$,

$y=2h$

$\therefore A(h, 2h)$ #

Sub $x=h$ into $y=\frac{1}{2}x$,

$y=\frac{1}{2}h$

$\therefore C(h, \frac{1}{2}h)$ #

Given AB is parallel to x-axis,

$\Rightarrow B(x, 2h)$

Gradient of $BC = 2$

$$\frac{2h-\frac{1}{2}h}{x-h} = \frac{2}{1}$$

$$2x-2h = \frac{3}{2}h$$

$$2x = \frac{7}{2}h$$

$$x = \frac{7h}{4}$$

$\therefore B(\frac{7h}{4}, 2h)$ #

(ii) Given $h=4$, $A(4,8)$, $B(7,8)$, $C(4,2)$.

$\therefore$ Area of the trapezium OABC

$$= \frac{1}{2}\begin{vmatrix} 0 & 4 & 7 & 4 & 0 \\ 0 & 8 & 8 & 2 & 0 \end{vmatrix}$$

$$= \frac{1}{2}|0+32+14+0-0-56-32-0|$$

$$= \frac{1}{2}|-42| = 21 \text{ units}^2$$ #

8. $f'(x) = \sin 4x - \cos 2x$

$f(x) = \int (\sin 4x - \cos 2x)\, dx$

$= -\frac{\cos 4x}{4} - \frac{\sin 2x}{2} + c$

when $x = \frac{\pi}{2}$, $f(x) = 0$,

$0 = -\frac{\cos 2\pi}{4} - \frac{\sin \pi}{2} + c$

$c = \frac{1}{4}$

$\therefore f(x) = -\frac{\cos 4x}{4} - \frac{\sin 2x}{2} + \frac{1}{4}$

LHS $= f''(x) = 4\cos 4x + 2\sin 2x$

$\therefore f''(x) + 4f(x)$

$= 4\cos 4x + 2\sin 2x + 4\left[-\frac{\cos 4x}{4} - \frac{\sin 2x}{2} + \frac{1}{4}\right]$

$= 4\cos 4x + 2\sin 2x - \cos 4x - 2\sin 2x + 1$

$= 3\cos 4x + 1$

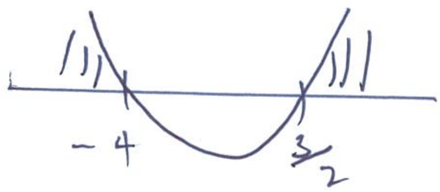
$=$ RHS (shown) #

9. $y = ax^2 + 5x + a - 5$

(i) Given $a = 2$, $2x^2 + 5x + 2 - 5 > 9$

$$2x^2 + 5x - 12 > 0$$

$$(2x - 3)(x + 4) > 0$$



$\therefore$ Set of values of $x = \{x : x < -4 \cup x > \frac{3}{2}\}$ #

(ii) Given $a = 4$, $y = 4x^2 + 5x - 1$ —— (1)

$y = x - 2$ —— (2)

(1) = (2), $4x^2 + 5x - 1 = x - 2$

$$4x^2 + 4x + 1 = 0$$

$$a = 4,\ b = 4,\ c = 1$$

$$b^2 - 4ac = 4^2 - 4(4)(1)$$
$$= 0$$

Since $b^2 - 4ac = 0$, $\therefore$ the line $y = x - 2$ is a tangent to the curve. (shown) #

(iii) $y = ax^2 + 5x + a - 5$ —— (3)

$y = x - 2$ —— (4)

(3) = (4) $ax^2 + 5x + a - 5 = x - 2$

$$ax^2 + 4x + a - 3 = 0$$

$$a = a,\ b = 4,\ c = a - 3$$

$$b^2 - 4ac = 0$$

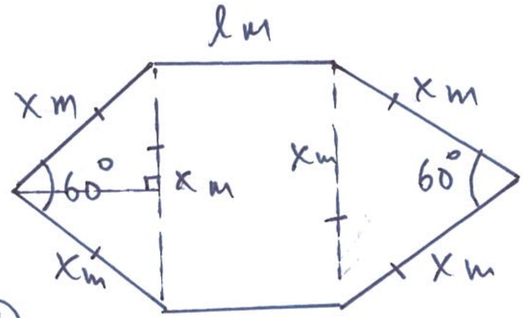
$$16 - 4(a)(a - 3) = 0$$

$$16 - 4a^2 + 12a = 0$$

$$a^2 - 3a - 4 = 0$$

$$(a + 1)(a - 4) = 0$$

$a = -1$ # or $a = 4$ (rejected)



10.(i) $x + x + x + x + l + l = 130$

$4x + 2l = 130$

$2l = 130 - 4x$

$l = 65 - 2x$ —— ①

(ii) Area of the plot $= 2\left[\frac{1}{2}(x)^2 \sin 60^\circ\right] + xl$

$= x^2 \frac{\sqrt{3}}{2} + xl$

Sub ①,

$= x^2 \frac{\sqrt{3}}{2} + x(65 - 2x)$

$= \frac{\sqrt{3}}{2}x^2 + 65x - 2x^2$

$= \frac{\sqrt{3}x^2 + 130x - 4x^2}{2}$

$= \frac{130x - (4-\sqrt{3})x^2}{2}$ m^2 (shown)

#

(ii) $A = \frac{130x - (4-\sqrt{3})x^2}{2}$

$\frac{dA}{dx} = \frac{1}{2}\left[130 - 2(4-\sqrt{3})x\right] = 0$

$2(4-\sqrt{3})x = 130$

$x = \frac{130}{2(4-\sqrt{3})}$

$= 28.660$

≈ 28.7 (3 s.f) #

(iii) $\frac{d^2A}{dx^2} = \frac{1}{2}\left[-2(4-\sqrt{3})\right]$

$= \sqrt{3} - 4 < 0$

$\therefore$ Area is maximum.

$\Rightarrow$ This value of x gives the gardener the largest area possible. #

11. (i) $y = x\ln x$ —— ①

$\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x$

$= 1 + \ln x$

Given the tangent at p is parallel to $y = 2x - 3$,

$\Rightarrow \quad 1 + \ln x = 2$

$\ln x = 1$

$x = e^1$

Sub $x = e$ into ①,

$y = e\ln e$

$= e$

$\therefore\ p(e, e)$ #

(ii) Gradient of normal at $p = -\frac{1}{2}$

Equation of normal at $p \Rightarrow\ y - e = -\frac{1}{2}(x - e)$

$y = -\frac{1}{2}x + \frac{3}{2}e$ —— ②

$y = 2x - 3$ —— ③

② = ③, $-\frac{1}{2}x + \frac{3}{2}e = 2x - 3$

$\frac{5}{2}x = \frac{3}{2}e + 3$

$5x = 3e + 6$

$x = \frac{3}{5}(e + 2)$

$\Rightarrow \quad k = \frac{3}{5}$ #

12. (i) For $y=(2x-3)^2-4$, the minimum value of y is -4 and it occurs when $x=\frac{3}{2}$.

Since coefficient of $x^2 = 4 > 0$, the turning pt $\left(\frac{3}{2}, -4\right)$ is a minimum point.

$\Rightarrow$ The lowest point on the curve has coordinates $\left(\frac{3}{2}, -4\right)$ #

(ii) When the curve intersects the x-axis, $y=0$.

$$0=(2x-3)^2-4$$

$$(2x-3)^2=4$$

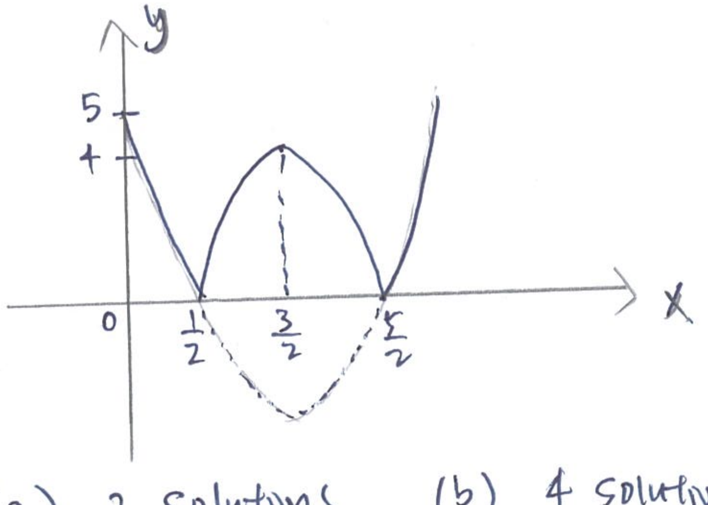
$2x-3=2$ or $2x-3=-2$

$2x=5$ $\qquad$ $2x=1$

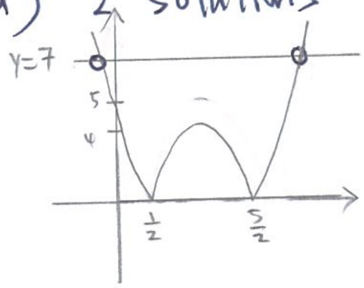
$x=\frac{5}{2}$ $\qquad$ $x=\frac{1}{2}$

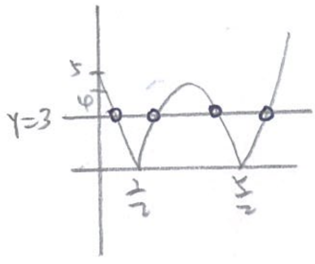
$\Rightarrow$ Coordinates on x-axis $= \left(\frac{1}{2}, 0\right), \left(\frac{5}{2}, 0\right)$ #

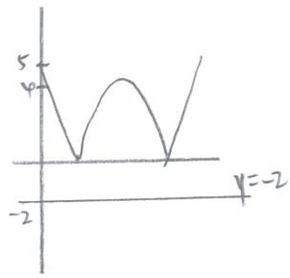
(iii)



(iv) (a) 2 solutions $\qquad$ (b) 4 solutions $\qquad$ (c) No solution.







* For illustrative purpose only.