2012 EM Paper 2. V1.2

1(a) $3x^2 - 48y^2 = 3(x^2 - 16y^2)$
$= 3(x-4y)(x+4y)$ #

(b) $\frac{2x}{5} - \frac{x-1}{15} = \frac{6x-(x-1)}{15}$
$= \frac{6x-x+1}{15}$
$= \frac{5x+1}{15}$ #

(c) $\frac{x}{2y} \div \frac{3x^2y}{4} = \frac{x}{2xy} \times \frac{4^2}{3x^2y}$
$= \frac{2}{3xy^2}$ #

(d) (i) $V = \frac{4\pi}{3}(a^3 - b^3)$.
$= \frac{4\pi}{3}(2.5^3 - 1.9^3)$
$= 36.718$
≈ 36.7 # (Corr to 3 sig fig)

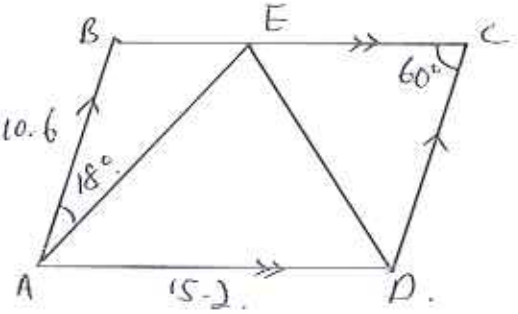
(ii) $V = \frac{4\pi}{3}(a^3 - b^3)$
$3V = 4\pi a^3 - 4\pi b^3$.
$4\pi a^3 = 3V + 4\pi b^3$.
$a^3 = \frac{3V+4\pi b^3}{4\pi}$

$a = \sqrt[3]{\frac{3V+4\pi b^3}{4\pi}}$ #

1(c). $\frac{2x-7}{3x+2} = 4$
$2x - 7 = 12x + 8$.
$10x = -15$.
$x = -\frac{3}{2}$ #

2012 EM Paper 2.

2 (a)(i) $\angle ABC = 180° - \angle BCD$ (Int $\angle$s, AB // DC)
$= 180° - 60°$
$= 120°$ #.



(a)(ii). $\angle DAE + \angle EAB + \angle ABC = 180°$ (Int $\angle$s, BC // AD)
$\therefore \angle DAE = 180° - 120° - 18°$
$= 42°$.

(b) Area of parallelogram ABCD.
= (Area of $\triangle ABD$) x 2.
$= \frac{1}{2}(10.6)(15.2)\sin(18°+42°) \times 2$.
$= 139.53$
≈ 140 cm^2 (Corr to 3 sig figs)

(c). $\angle BEA = \angle DAE = 42°$ (alt $\angle$s, BC // AD)
In $\triangle ABE$, using sine Rule.

$$\frac{AE}{\sin\angle ABE} = \frac{AB}{\sin\angle BEA}$$

$$AE = \frac{10.6}{\sin 42°} \times \sin 120°$$
$$= 13.719$$
$$\approx 13.72 \quad \text{(Corr to 4 sig figs)}.$$

(d) In $\triangle AED$, using cosine rule,
$DE^2 = AE^2 + AD^2 - 2(AE)(AD)\cos\angle EAD$
$= 13.72^2 + 15.2^2 - 2(13.72)(15.2)\cos 42°$
$DE = 10.456$
≈ 10.5 cm (Corr to 3 sig figs)

2013 EM Paper 2.

3 (a)(i) $A(-5, 1) \Rightarrow \overrightarrow{OA} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$.

$B(3, -3) \Rightarrow \overrightarrow{OB} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$.

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ -3 \end{pmatrix} - \begin{pmatrix} -5 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$

(ii) Gradient AB, $m = \frac{-3-1}{3-(-5)} = -\frac{1}{2}$.

$y = mx + c$.

$y = -\frac{1}{2}x + c$.

Given $y = 1$ and $x = -5$.

$1 = -\frac{1}{2}(-5) + c$.

$c = -\frac{3}{2}$.

∴ Equation of line AB:

$y = -\frac{1}{2}x - \frac{3}{2}$ #.

(b). Given line l: $4y - 3x + 8 = 0$.

$4y = 3x - 8$.

$y = \frac{3}{4}x - 2$. ——— (1).

(i)(a). Gradient of line $l = \frac{3}{4}$. #

(b) y-intercept of line $l = -2$ #.

(ii) Given $3x + 2y = 5$. ——— (2).

Sub (1) into (2),

$3x + 2\left(\frac{3}{4}x - 2\right) = 5$.

$3x + \frac{3}{2}x - 4 = 5$

$\frac{9}{2}x = 9$.

$x = 2$.

Put $x = 2$ into (1).

$y = \frac{3}{4}(2) - 2$.

$= -\frac{1}{2}$.

∴ Coordinate of C is $(2, -\frac{1}{2})$ #.

2012 EM Paper 2.

4 (a). Given £1 = $2.10
€1 = £0.93.

(i) Amt received = 200 × 2.10
= $420.00

(ii) 1st $500 converted to pounds = £ $\frac{500}{2.10}$.
Then converted to euros = € $\frac{500}{2.10} \times \frac{1}{0.93}$.
= € 256.02.
≈ € 256 #. (cor to nearest euros)

(b). 107% —— $295
100% —— $295 ÷ 1.07 = $275.70 (cor to nearest cents)

(c) (i). Given 1 light year = 9.46×10^{15} m
8.6 light year = $9.46 \times 10^{15} \times 8.6$.
= 81.356×10^{15}
= 8.1356×10^{13} km (exact).

(ii) Time taken = $3.97 \times 10^{13} \div 60{,}000$
= $3.97 \times 10^{13} \div 6 \times 10^{4}$
= $\frac{3.97}{6} \times 10^{9}$ hrs.

Assume 1 yr = 365 days
= 365 × 24 hrs
= 8760 hrs.

∴ Time taken in years = $\frac{3.97}{6} \times 10^{9} \div 8760$
= 75532.7 yrs
≈ 75500 yrs (Cor to 3 sig figs)

5 (a)(i) $\angle ADE = \angle ABE$ ($\angle$s in the same segment)
$= 60°$

$\therefore \angle BAE = 90°$ ($\angle$ in semi-circle, BE is diameter).
$\angle AEB = 180° - 90° - 60°$ (Sum of $\angle$s in $\triangle AEB$)
$= 30°$

In Quad ABCD.
$\angle BAD = 180° - \angle BCD$ ($\angle$s in Opp Segment)
$= 180° - 110°$
$= 70°$

(ii)(a) $\angle BFD = \angle ABE + \angle BAD$ (ext $\angle$ = sum of 2 Int opp $\angle$s).
$= 60° + 70°$
$= 130°$.

(b). $\angle ADB = \angle AEB = 30°$ ($\angle$s in the same segment)

In $\triangle BFD$,
$\angle DBF = 180° - 130° - 30°$ (Sum of $\angle$s in $\triangle$)
$= 20°$.

$\angle DBF + \angle DBC + \angle DCB = 180°$ (Int $\angle$s, $EB // DC$)
$\angle DBC = 180° - 110° - 20°$
$= 50°$ #.

(b). Volume of the solid = Area of cross-section $\times$ ht.
$= \left(\frac{\theta}{360} \times \pi r^2\right) \times h$.
$= \frac{76}{360} \times \pi (6)^2 \times 4$
$= 95.50$ (corr to 3 sig fig).
$\approx 95.5 cm^3$

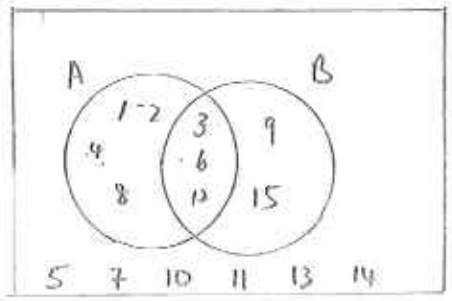
2012 EM P2.

6 (a) $\xi = \{$ integers $x : 1 \leq x \leq 15\}$.

$A = \{$ factors of 24$\} = \{1, 2, 3, 4, 6, 8, 12\}$

$B = \{$ multiples of 3$\} = \{3, 6, 9, 12, 15\}$.

(i)



(ii) $n(A \cap B) = 3$.

(iii) $A' \cup B = \{3, 5, 6, 7, 9, 10, 11, 13, 14, 12, 15\}$

(b)(i) $M = 5L = 5\begin{pmatrix} 25 & 10 \\ 20 & 30 \end{pmatrix} = \begin{pmatrix} 125 & 50 \\ 100 & 150 \end{pmatrix}$

(ii) $N = \begin{pmatrix} 3 \\ 2.5 \end{pmatrix}$.

(iii) $MN = \begin{pmatrix} 125 & 50 \\ 100 & 150 \end{pmatrix}\begin{pmatrix} 3 \\ 2.5 \end{pmatrix}$.

$= \begin{pmatrix} 125 \times 3 + 50 \times 2.5 \\ 100 \times 3 + 150 \times 2.5 \end{pmatrix}$

$= \begin{pmatrix} 500 \\ 675 \end{pmatrix}$

(iv) The matrix P represents the total fees collected in a week from boys ($500) and girls ($675).

(v) Original fees $= 500 + 675 = \$1175$.

After increase, fees collected $= \$1175 \times 1.05$

$\approx \$1234$ (1 dec to nearest dollar)

2012 EM Paper 2.

7 (a) when $x = 6$.

$$y = \frac{6^3}{20} + \frac{10}{6} - 5 = 7.47 \text{ (cor to 2 dec pl)}.$$

(b) refer to graph.

(c) Given $\frac{x^3}{20} + \frac{10}{x} = 5$.

$$\frac{x^3}{20} + \frac{10}{x} - 5 = 0.$$

$$\therefore y = 0.$$

Solution is where curve cuts the x-axes:

$\therefore x = 2.26$ # and $x = 3.45$ #

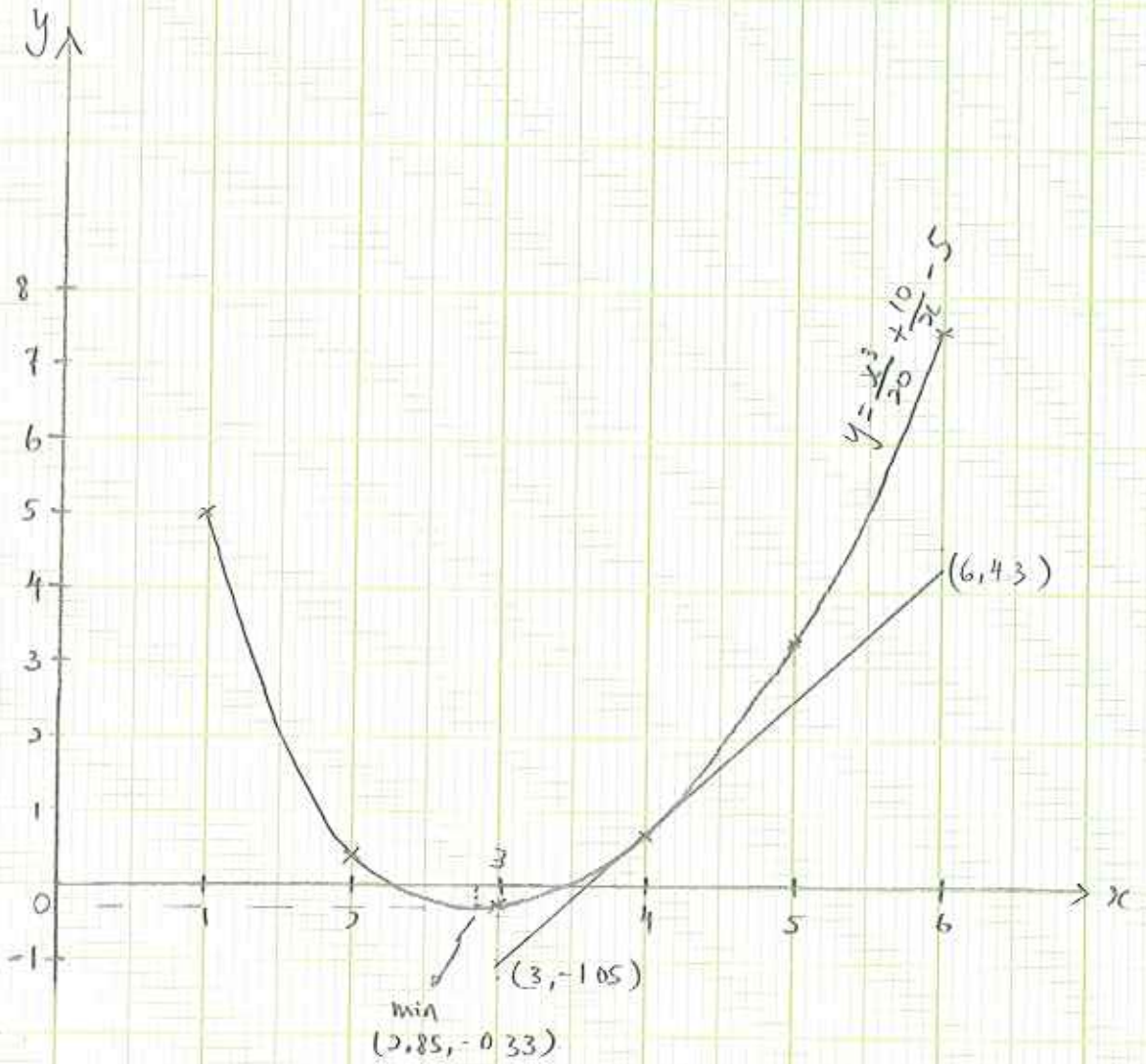
(d) Gradient of the curve at $(4, 0.70) = \frac{4.3 - (-1.05)}{6 - 3}$.

≈ 1.78 # (cor to 3 sig fig)

(e) Minimum pt $= (2.85, -0.33)$.

2012 EM Paper 2

Q7



2011 EM Paper 2.

8 (a). Time taken for the small pump to empty the full tank $= \frac{2000}{x}$ mins

(b). Time taken for the big pump to empty the full tank $= \frac{2000}{x+100}$ min

(c) $\frac{2000}{x} - \frac{2000}{x+100} = 1$.

$100(x+100) - 100x = x(x+100)$

$100x + 10000 - 100x = x^2 + 100x$.

$\therefore x^2 + 100x - 10000 = 0$.

(d). $x^2 + 100x - 10000 = 0$.

$x = \frac{-100 \pm \sqrt{100^2 - 4(-10000)(1)}}{2(1)}$.

$x = \frac{-100 - \sqrt{50000}}{2}$ or $\frac{-100 + \sqrt{50000}}{2}$

$= -161.803$ or 61.803.

$\therefore x = 61.8$ # (corr to 1 dec pl)

or $x = -161.8$

(e). Take $x = 61.8$

Time taken to empty the full tank using large pump $= \frac{2000}{61.803 + 100}$

$= 12.36$ mins.

$= 12$ min 21.64 sec.

≈ 12 min 22 sec #

(corr. to min and sec).

2011 EM Paper 2.

9 (a). $\angle BOC = \frac{360^\circ}{6} = 60^\circ$.

$\therefore$ $\triangle OBC$ is equilateral $\triangle$ since $OB = OC = OD = OE = OF = OA$.

Area of $\triangle OBC = \frac{1}{2}(OB)(OC)\sin\angle BOC$
$= \frac{1}{2}(4)(4)\sin 60^\circ$
$= 8\sin 60^\circ$.

$\therefore$ Area of the hexagon ABCDEF $= 6 \times 8\sin 60^\circ$
$= 41.569$
$\approx 41.6\ cm^2$ (corr to 3 sig fig)

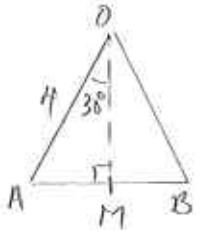
9(b) Area of $\triangle BXC = \frac{1}{2}(4) \times 10) = 20\ cm^2$.

$\therefore$ Total Surface area $= 20 \times 6 + 41.569$
$= 120 + 41.569$
$= 161.569$
$\approx 162\ cm^2$ (Corr to 3 sig fig)

9(c). In $\triangle OAM$

$\cos 30^\circ = \frac{OM}{4}$

$OM = 4\cos 30^\circ$.



In $\triangle XOM$, using phythagoras thm.

$XO^2 + OM^2 = 10^2$.

$XO^2 = 10^2 - (4\cos 30^\circ)^2$

$XO^2 = 88$

$XO = \sqrt{88}$ cm.

≈ 9.38 cm (Corr to 3 sig fig)

(d) Vol of the pyramid $= \frac{1}{3}$ base area $\times$ ht.

$= \frac{1}{3}(41.569)\cdot\sqrt{88}$

$= 129.98$

$\approx 130\ cm^3$ (Corr to 3 sig fig)

2011 EM Paper 2:

9 (c). $\frac{V_B}{V_S} = \left(\frac{9}{4}\right)^3$

Let V_B be bigger pyramid.
V_S be smaller pyramid.

$V_B = \left(\frac{9}{4}\right)^3 \times 129.98$
$= 1480.598$
$\approx 1480\ cm^3$ # (Corr to 3sig fig)

2012 EM Paper 2.

10 (a) (i) (a) median = 37.4 marks.

(b) Upper Quartile = 41 marks
Lower Quartile = 24 marks
Interquartile = 41 − 24 = 17 marks

(ii) No. of students below pass marks 35 = 34

∴ No of students who pass the test = 80 − 34
= 46

(iii) The group of students did better in physics with a higher median mark of 37.3, with a smaller spread of physics marks as indicated by their smaller Interquartile range of 17.

(b) (i) (a) P (right handed male) = $\frac{6}{16} = \frac{3}{8}$.

(b) P (a female) = $\frac{7}{16}$

(ii) (a) P (both are males) = $\frac{9}{16} \times \frac{8}{15} = \frac{3}{10}$.

(b) P (neither of them is a right handed female)
= $\frac{11}{16} \times \frac{10}{15}$
= $\frac{11}{24}$