

**ELEMENTARY MATHEMATICS****Paper 1 Suggested Solutions****4016/01****October/November 2011****1. Topic: Arithmetic (Approximation & Estimation)**

$$\frac{4.51}{19.6 - 3.91^2} \approx 1.05 \text{ (3 sig. fig.)}$$

Answer 1.05 [2]**2. Topic: Integers**

(a) $32 - (-7) = 39$

(b) $\frac{-3+25-7}{3} = 5$

Answer (a) 39 °C [1](b) 5 °C [1]**3. Topic: Arithmetic (Percentage)**

$120 - 105 = 15^\circ \rightarrow 25$

$$360^\circ \rightarrow \frac{25}{15} \times 360$$

$$= 600$$

∠ of sector in pie chart
∝ its given data

Answer 600 [2]

Difference between angles of
"Mathematics" and "Chemistry" sectors in
pie chart.

4. Topic: Matrices

$$(a) \begin{pmatrix} 59 & 15 & 101 \\ 102 & 3 & 72 \end{pmatrix} \begin{pmatrix} 10 \\ 7 \\ 5 \end{pmatrix} = \begin{pmatrix} 1200 \\ 1401 \end{pmatrix}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax + by + cz \\ dx + ey + fz \end{pmatrix}$$

Answer (a) $\begin{pmatrix} 1200 \\ 1401 \end{pmatrix}$ [1](b) The total amount of money collected for each match, i.e. \$1200 for the first match and \$1401 for the second match [1]**5. Topic: Algebra**

$x \text{ ¢} \rightarrow 1 \text{ litre}$

$1 \text{ ¢} \rightarrow \frac{1}{x} \text{ litre}$

$100 \text{ ¢} \rightarrow \frac{100}{x} \text{ litre}$

$\$y \rightarrow \frac{100y}{x} \text{ litre}$

Answer $\frac{100y}{x}$ litres [2]**6. Topic: Arithmetic (Fractions & Percentages)**

(a) $12 \div \frac{2}{3} = 18$

(b) $\frac{3 \text{ cm}}{6 \times 100 \text{ cm}} \times 100\% = 0.5\%$

$1 \text{ m} = 100 \text{ cm}$

Answer (a) 18 glasses [1](b) 0.5 % [2]



11. Topic: Trigonometry

$$\begin{aligned} \cos p^\circ &= -\cos \widehat{ACD} && \cos(180^\circ - \theta) = -\cos \theta \\ &= -\frac{4}{x} \end{aligned}$$

$$(b) \text{ Area of trapezium } ABCD = 5 \times \text{Area of } \triangle ADC$$

$$\frac{1}{2}(AB + DC) \times AD = 5 \times \frac{1}{2} \times AD \times DC$$

$$\frac{1}{2}(AB + 4) = \frac{5}{2} \times 4$$

$$AB + 4 = 5 \times 4$$

$$AB = 16$$

Area of trapezium
 $= \frac{1}{2} \times \text{sum of // sides} \times \text{height}$

$$\text{Answer (a) } \cos p^\circ = \underline{\underline{-\frac{4}{x}}} \quad [1]$$

$$(b) AB = \underline{\underline{16}} \text{ cm} \quad [2]$$

12. Topic: Approximation & Estimation

$$(b) \text{ Greatest possible mass of 1 cubic centimetre of the glass}$$

$$= \frac{\text{greatest possible mass}}{\text{least possible volume}}$$

$$= \frac{43.49}{14.5}$$

$$\approx 3.00 \text{ g (3 sig. fig.)}$$

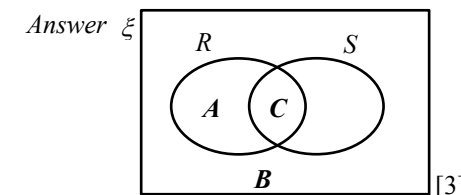
$$\begin{aligned} 42.5 \text{ g} &\leq \text{mass of glass block} < 43.5 \text{ g} \\ 14.5 \text{ cm}^3 &\leq \text{volume of glass block} < 15.5 \text{ cm}^3 \end{aligned}$$

$$\text{Answer (a) } \underline{\underline{42.5}} \text{ g} \quad [1]$$

$$(b) \underline{\underline{3.00}} \text{ g} \quad [2]$$

13. Topic: Set Language & Notation

- $A \in R$ (A has a 90° angle)
 $A \notin S$ (A has two 45° angles $\Rightarrow$ isosceles)
 $B \notin R$ (By Pythagoras Theorem, $7^2 + 3^2 \neq 7^2 \Rightarrow$ not a right-angled triangle)
 $B \notin S$ (B has two equal 7 cm sides)
 $C \in R$ (By Pythagoras Theorem, $3^2 + 4^2 = 5^2 \Rightarrow$ right-angled triangle)
 $C \in S$ (C has three unequal sides)



14. Topic: Geometry

- (a) (i) From $\triangle MDC$, $M\widehat{DC} = 180^\circ - D\widehat{MC} - D\widehat{CM}$ (sum of interior $\angle$ s in $\triangle$)
 $= 180^\circ - 28^\circ - x^\circ$
 $= 152^\circ - x^\circ$
 (ii) From $\triangle BNC$, $N\widehat{BC} = 180^\circ - B\widehat{NC} - B\widehat{CN}$ (sum of interior $\angle$ s in $\triangle$)
 $= 180^\circ - 22^\circ - x^\circ$
 $= 158^\circ - x^\circ$

Alternative Method:

[using answer from (a)(i)]

$$\begin{aligned} N\widehat{BC} &= 180^\circ - M\widehat{DC} \text{ (opposite } \angle \text{s in cyclic quad)} \\ &= 180^\circ - (152^\circ - x^\circ) \\ &= 28^\circ + x^\circ \end{aligned}$$



(b) $N\hat{B}C + M\hat{D}C = 180^\circ$ (opposite $\angle$ s in cyclic quad)

$$152^\circ - x^\circ + 158^\circ - x^\circ = 180^\circ$$

$$2x^\circ = 130^\circ$$

$$x = 65$$

Alternative Method:

From $\triangle BNC$,

$$B\hat{N}C + N\hat{B}C + B\hat{C}N = 180^\circ \text{ (sum of interior } \angle\text{s in } \triangle)$$

$$22^\circ + 28^\circ + x^\circ + x^\circ = 180^\circ$$

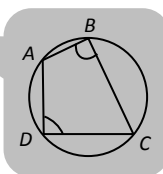
$$2x^\circ = 130^\circ$$

$$x = 65$$

Answer (a)(i) $M\hat{D}C = \frac{152^\circ - x^\circ}{2}$ [1]

(ii) $N\hat{B}C = \frac{158^\circ - x^\circ \text{ or } 28^\circ + x^\circ}{2}$ [1]

(b) $x = 65$ [1]



15. Topic: Mensuration

(a) Area of the minor sector $AOB = \frac{1}{2}(4)^2(2.5)$ Area of sector $= \frac{1}{2}r^2\theta$
 $= 20$

(b) (i) Reflex angle $AOB = 2\pi - 2.5$

(ii) Length of arc $ACB = 4(2\pi - 2.5)$ Length of arc $= r\theta$
 $= 8\pi - 10$

Answer (a) $\frac{20}{1} \text{ cm}^2$ [1]

(b)(i) $\frac{(2\pi - 2.5)}{1} \text{ radians}$ [1]

(ii) $\frac{(8\pi - 10)}{1} \text{ cm}$ [1]

16. Topic: Algebra

(a) $3(a - 4) = 1 - (3 - a)$

$$3a - 12 = 1 - 3 + a$$

$$2a = 10$$

$$a = 5$$

(b) $y = 3x - 7$ (1)

$$x = y + 4$$
 (2)

Sub (2) into (1),

$$y = 3(y + 4) - 7$$

$$y = 3y + 12 - 7$$

$$2y = -5$$

$$y = -2.5$$

Sub $y = -2.5$ into (2),

$$x = -2.5 + 4$$

$$= 1.5$$

Answer (a) $a = 5$ [2]

(b) $x = 1.5$

$y = -2.5$ [2]



17. Topic: Geometry

- (a) (i) $\widehat{BCD} = \widehat{FDF}$ (corresponding $\angle$ s, $FD \parallel BC$)
 $= 44^\circ$
- (ii) $\widehat{BDF} = \widehat{CBD} = 56^\circ$ (alternate $\angle$ s, $FD \parallel BC$)
 $\widehat{ABD} = \widehat{BDC} = 180^\circ - 44^\circ - 56^\circ$ (adjacent $\angle$ s on straight line)
 $= 80^\circ$
- (iii) $\widehat{AFD} = 360^\circ - \widehat{BAF} - \widehat{ABD} - \widehat{BDF}$ (sum of interior $\angle$ s in quadrilateral)
 $= 360^\circ - 101^\circ - 80^\circ - 56^\circ$
 $= 123^\circ$

Answer (a)(i) $\widehat{BCD} = \underline{\hspace{2cm} 44^\circ \hspace{2cm}}$ [1]
 (ii) $\widehat{ABD} = \underline{\hspace{2cm} 80^\circ \hspace{2cm}}$ [1]
 (iii) $\widehat{AFD} = \underline{\hspace{2cm} 123^\circ \hspace{2cm}}$ [1]
 (b) AF is not parallel to BD because $\widehat{BAF} + \widehat{ABD} \neq 180^\circ$,
 since these two interior angles have to be supplementary in
 order for AF to be parallel to BD . [1]

18. Topic: Indices

- (a) (i) $7^{12-3} = 7^9$ $a^m \div a^n = a^{m-n}$
- (ii) $\frac{1}{49} = \frac{1}{7^2}$
 $= 7^{-2}$ $\frac{1}{a^m} = a^{-m}$
- (iii) $\sqrt[4]{7} = 7^{\frac{1}{4}}$ $\sqrt[n]{a^m} = a^{\frac{m}{n}}$
- (b) $3^{-2+k} = 3^0$
 $-2 + k = 0$ $a^0 = 1$
 $k = 2$

Answer (a)(i) $\underline{\hspace{2cm} 7^9 \hspace{2cm}}$ [1]
 (ii) $\underline{\hspace{2cm} 7^{-2} \hspace{2cm}}$ [1]
 (iii) $\underline{\hspace{2cm} 7^{\frac{1}{4}} \hspace{2cm}}$ [1]
 (b) $k = \underline{\hspace{2cm} 2 \hspace{2cm}}$ [1]





19. Topic: Coordinate Geometry

(a) $x = 8 \dots\dots\dots (1)$

$$y = 6 - x \dots\dots\dots (2)$$

Sub (1) into (2),

$$y = 6 - 8$$

$$= -2$$

$$\therefore A (8, -2)$$

$x = 8$

$$2y = 3x + 2 \dots\dots\dots (3)$$

Sub (1) into (3),

$$2y = 3(8) + 2$$

$$y = 13$$

$$\therefore B(8, 13)$$

(b) $y = 6 - x$

$$\therefore m = -1$$

(c) $k = \frac{13+(-2)}{2}$

= 5.5

Equation of straight line with gradient m and y-intercept c :

$$y = mx + c$$

k is the y -coordinate of the midpoint of AB i.e. $\frac{y_A + y_B}{2}$

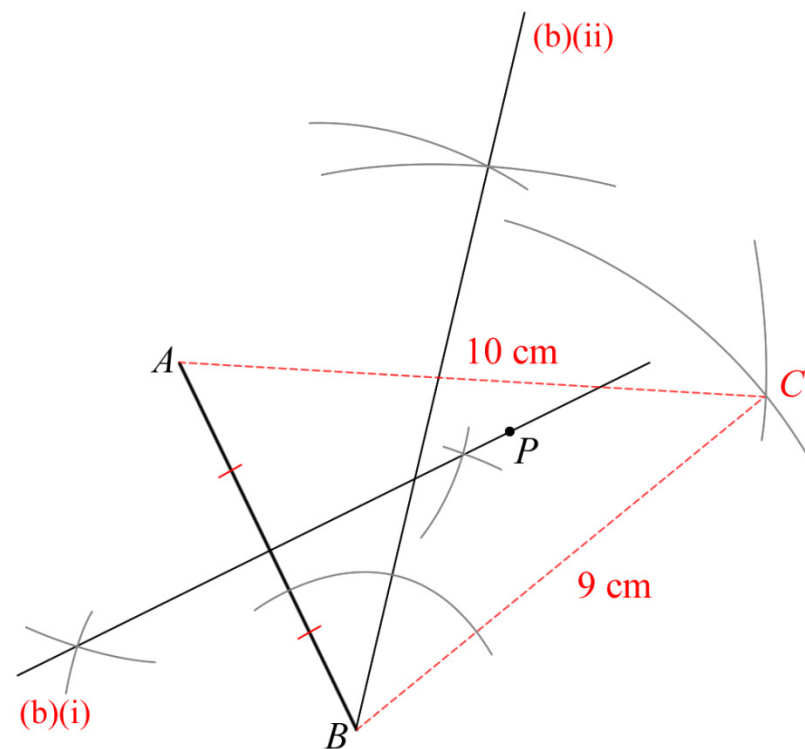
Answer (a) A is $(\underline{8}, \underline{-2})$ [1]

$$B \text{ is } \left(\begin{array}{c|c} 8 & 13 \end{array} \right) \quad [1]$$

$$(b) \quad \begin{array}{cc} & -1 \\ \hline & [1] \end{array}$$

(c) $k = 5.5$ [1]

20. Topic: Geometrical Construction





21. Topic: Graphs of Quadratic Equations

(a) When $y = 0$,

$$-(x-3)(x+1) = 0$$

$$x-3 = 0 \text{ or } x+1 = 0$$

$$x = 3 \quad x = -1$$

x-intercepts

When $x = 0$,

$$y = -(0-3)(0+1)$$

$$= 3$$

y-intercept

(b) line of symmetry: $x = \frac{-1+3}{2}$

$$= 1$$

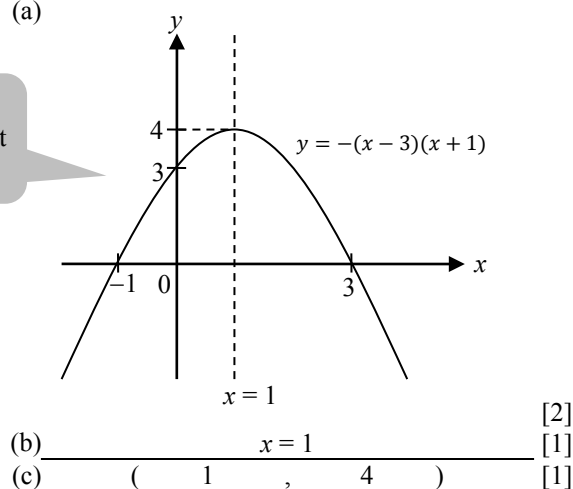
Line of symmetry of quadratic graph passes through the max/min point.

(c) Sub $x = 1$ into y ,

$$y = -(1-3)(1+1) = 4$$

 $\therefore$ Maximum point (1, 4)

Answer (a)

Coefficient of $x^2 < 0$
 $\Rightarrow$ graph has max point with shape $\cap$ 

22. Topic: Standard Form

$$(a) \quad 1.34 \times 10^{-7} \text{ seconds} = 1.34 \times 10^2 \times 10^{-9} \text{ seconds} \\ = \mathbf{134 \text{ nanoseconds}}$$

$$(b) \quad (i) \quad 6.1 \times 10^7 = 4.8 \times 10^6 \\ = 10^6(6.1 \times 10 - 4.8) \\ = 56.2 \times 10^6 \\ \approx \mathbf{5.6 \times 10^7}$$

$$1 \text{ nano} = 1.0 \times 10^{-9}$$

When quantities are multiplied or divided, the no. of sig. fig. is the answer equal to the no. of sig. fig. in the quantity with the smallest no. of sig. fig.

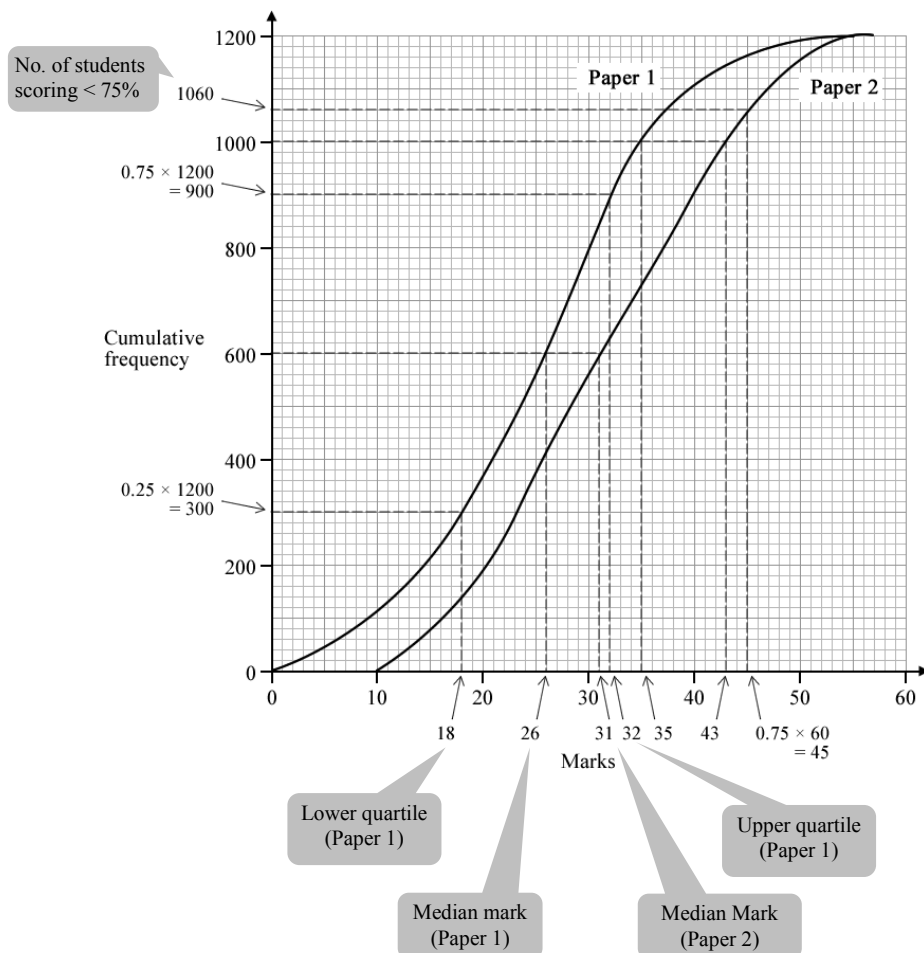
$$(ii) \quad 273\% \rightarrow 6.1 \times 10^7 \\ 100\% \rightarrow \frac{6.1 \times 10^7}{273} \times 100 \\ = 0.02234 \times 10^9 \\ \approx \mathbf{2.2 \times 10^7}$$

$$\text{Answer (a) } k = \frac{134}{5.6 \times 10^7} \quad [1] \\ (b)(i) \quad \frac{5.6 \times 10^7}{2.2 \times 10^7} \quad [2] \\ (ii) \quad \frac{2.2 \times 10^7}{5.6 \times 10^7} \quad [2]$$





23. Topic: Statistics (Cumulative Frequency)



(a) From graph,

Lower quartile = 18

Upper quartile = 32

 Interquartile range = $32 - 18$
 = **14**

 Interquartile range
 = Upper quartile – Lower quartile
(b) No. of students gaining $\geq 75\%$

= total no. of students – no. of students gaining < 75%

= $1200 - 1060$ = **140**

Answer (a) 14 marks [2]

(b) 140 [2]

(c) 43 [1]

(d) Paper 1 was more difficult because its median mark of 26 is lower than the median mark of 31 for paper 2. [1]



**24. Topic: Kinematics**

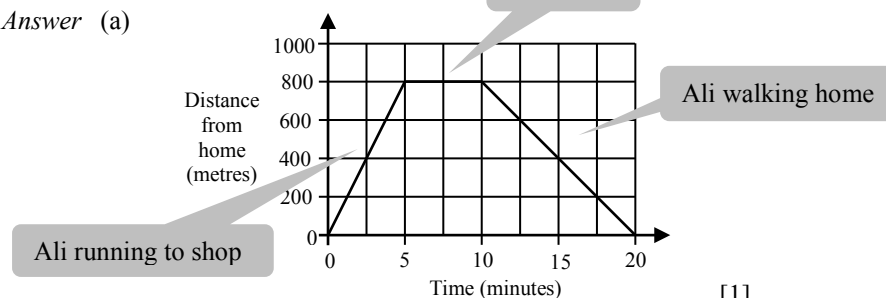
(b) Ali's speed as he runs to the shop = $\frac{0.8 \text{ km}}{\frac{5}{60} \text{ hr}}$ Speed = $\frac{\text{Distance}}{\text{Time taken}}$

$$= 9.6 \text{ km/h}$$

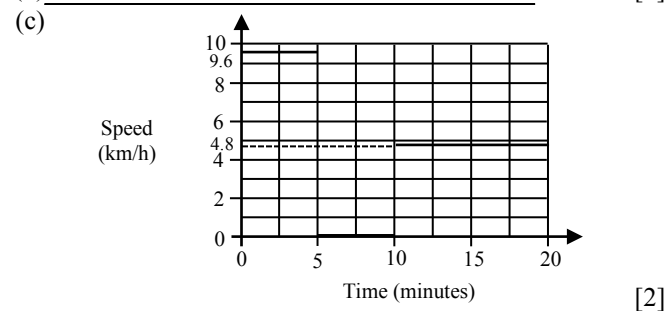
(c) Ali's speed as he walks home = $\frac{0.8 \text{ km}}{\frac{10}{60} \text{ hr}}$

$$= 4.8 \text{ km/h}$$

Answer (a)



(b) 9.6 km/h [1]



[2]

