ELEMENTARY MATHEMATICS 4016/02
Paper 2 Suggested Solutions October/November 2009

1. **Topics: Arithmetic (Time, Speed and Percentages)**

(a) (i) $1\frac{1}{2}$ hours $= 1\frac{1}{2} \times 60$

$= 90$ minutes

Time he spends warming up $= \frac{90}{12} \times 3$

$=$ **22.5 minutes**

(ii) % of the $1\frac{1}{2}$ hours at the sports centre he spends running

$= \frac{7}{2+3+7} \times 100\%$

$= \mathbf{58\frac{1}{3}}\%$

(b) (i) Speed $= \frac{3000 \text{ m}}{9\frac{1}{2} \text{ minutes}}$

Speed = $\frac{\text{Distance travelled}}{\text{Time taken}}$

$= \frac{3}{\frac{9\frac{1}{2}}{60}}$ km/h

$= \mathbf{18\frac{18}{19}}$ **km/h**

(ii) His best time in 2009 $= 90\% \times 9\frac{1}{2}$ minutes

$= 8.55$ minutes

$=$ **8 minutes 33 seconds**

100% → 2008 time
90 % → 2009 time

(iii) His best time in 2007 $= \frac{9.5}{95} \times 100$

$=$ **10 minutes 0 seconds**

100% → 2007 time
95 % → 2008 time

2. **Topic: Algebra (Factorisation, Formulae)**

(a) (i) $25 - p^2 = 5^2 - p^2$

$= \mathbf{(5 + p)(5 - p)}$

$a^2 - b^2 = (a + b)(a - b)$

(ii) $\frac{25-p^2}{15+3p} = \frac{(5+p)(5-p)}{3(5+p)}$

$= \mathbf{\frac{5-p}{3}}$

Using answer from part (i)

(b) $\frac{3}{(x+2)^2} - \frac{4}{x+2} = \frac{3-4(x+2)}{(x+2)^2}$

$= \frac{3-4x-8}{(x+2)^2}$

$= \mathbf{\frac{-4x-5}{(x+2)^2}}$

(c) $v^2 = u^2 - 2gh$

(i) When $u = 30$, $g = 9.8$ and $h = 24$,

$v^2 = 30^2 - 2(9.8)(24)$

$= 900 - 470.4$

$v^2 = 429.6$

$v = \pm\sqrt{429.6}$

$= \pm 20.726$

$\approx$ **$\pm$ 20.7 (3 sig. fig.)**

(ii) $v^2 = u^2 - 2gh$

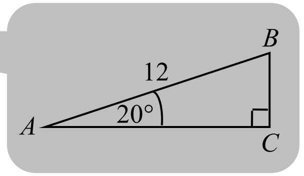
$u^2 = v^2 + 2gh$

$\mathbf{u = \pm\sqrt{v^2 + 2gh}}$

3. **Topic: Trigonometry (Trigonometrical Ratios, Pythagoras' Theorem)**

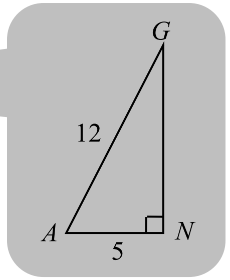
(a)

$$\begin{aligned} \sin 20° &= \frac{BC}{12} \\ BC &= 12 \sin 20° \\ &= 4.1042 \\ \therefore BD &= 6 + 4.1042 \\ &= 10.1042 \\ &\approx \mathbf{10.1\ m\ (3\ sig.\ fig.)} \end{aligned}$$



(b) (i)

$$\begin{aligned} (AG)^2 &= (AN)^2 + (GN)^2 \\ 12^2 &= 5^2 + (GN)^2 \ (\because AN = EF) \\ (GN)^2 &= 12^2 - 5^2 \\ GN &= \sqrt{119} \text{ or } -\sqrt{119} \text{ (rejected)} \\ &= 10.908 \\ &\approx \mathbf{10.9\ m\ (3\ sig.\ fig.)} \end{aligned}$$



(ii)

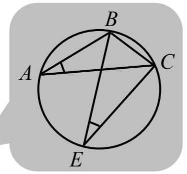
$$\begin{aligned} \cos \angle GAN &= \frac{5}{12} \\ \angle GAN &= \cos^{-1}\left(\frac{5}{12}\right) \\ &= 65.375° \end{aligned}$$

$\therefore$ The angle through which the jib has rotated $= 65.375° - 20°$

$= 45.375°$

$\approx$ **45.4° (1 d.p.)**

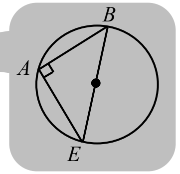
4. **Topic: Geometry (Geometrical Properties of Circles & Similarity)**



(a) (i) $\angle BEC = \angle BAC$ (angle in the same segment)

$= \mathbf{33°}$

(ii) $\angle BAE = 90°$ (Δ in semi circle)

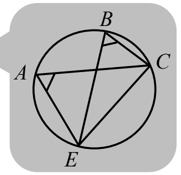


$$\begin{aligned} \angle CAE &= \angle BAE - \angle BAC \\ &= 90° - 33° \\ &= 57° \end{aligned}$$

$\angle BAC = 33°$ [from (a)(i)]

$\therefore \angle EBC = \angle CAE$

$= \mathbf{57°}$ (angle in the same segment)

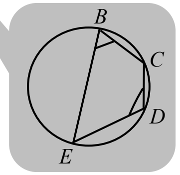


(iii) $\angle CDE + \angle EBC = 180°$ (opp. $\angle$s of cyclic quad)

$\therefore \angle CDE = 180° - 57°$

$= \mathbf{123°}$

$\angle EBC = \angle CAE = 57°$ [from (a)(ii)]



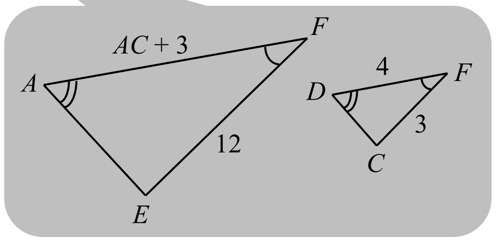
(b) (i) $\angle AFE = \angle DFC$ (Common $\angle$)

$\angle FAE = \angle FDC = 57°$

$\therefore \Delta FAE$ and ΔFDC are similar

(c) Since ΔFAE and ΔFDC are similar,

$$\begin{aligned} \frac{FA}{FD} &= \frac{FE}{FC} \\ \frac{AC+3}{4} &= \frac{8+4}{3} \\ 3(AC+3) &= 4(12) \\ 3AC &= 39 \\ \mathbf{AC} &= \mathbf{13\ cm} \end{aligned}$$



5. **Topics: Set Language and Notation, Matrices**

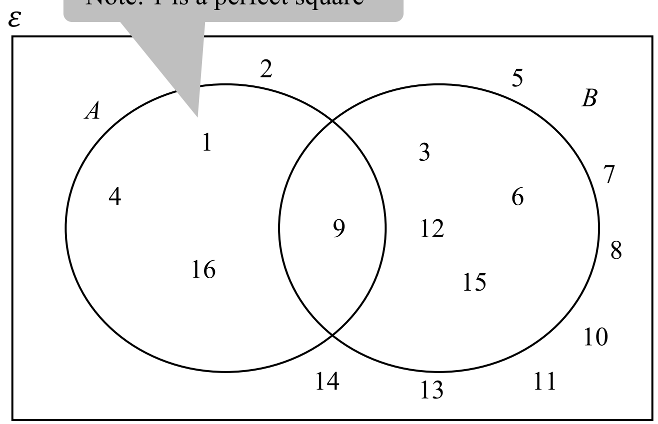
(a) ε = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16}

A = {1, 4, 9, 16}

B = {3, 6, 9, 12, 15}

(i)

Note: 1 is a perfect square



(ii) A' = {2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15}

Integers outside of A → non-perfect squares

$A' \cap B$ = {3, 6, 12, 15}

Integers in both A' and B → non-perfect squares and divisible by 3

(iii) $A \cup B$ = {1, 3, 4, 6, 9, 12, 15, 16}

n($A \cup B$) = 8

All perfect squares and all integers divisible by 3.

(b) (i) $\mathbf{D} = \begin{pmatrix} \mathbf{25} & \mathbf{15} \\ \mathbf{10} & \mathbf{30} \end{pmatrix}$

(ii) $$\begin{aligned} \mathbf{E} &= 5\mathbf{C} + \mathbf{D} \\ &= 5\begin{pmatrix} 10 & 30 \\ 20 & 10 \end{pmatrix} + \begin{pmatrix} 25 & 15 \\ 10 & 30 \end{pmatrix} \\ &= \begin{pmatrix} 50 & 150 \\ 100 & 50 \end{pmatrix} + \begin{pmatrix} 25 & 15 \\ 10 & 30 \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{75} & \mathbf{165} \\ \mathbf{110} & \mathbf{80} \end{pmatrix} \end{aligned}$$

(iii) Total number of adults and children carried by bus from Monday to Saturday.

(iv) (a) $$\begin{aligned} \mathbf{F} &= \mathbf{C}\begin{pmatrix} 25 \\ 15 \end{pmatrix} \\ &= \begin{pmatrix} 10 & 30 \\ 20 & 10 \end{pmatrix}\begin{pmatrix} 25 \\ 15 \end{pmatrix} \\ &= \begin{pmatrix} 700 \\ 650 \end{pmatrix} \end{aligned}$$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$

(b) **F** represents the total fare in cents collected from both adults and children on a weekday morning and weekday afternoon respectively.

(c) $$\begin{aligned} \mathbf{G} &= \tfrac{1}{100}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 700 \\ 650 \end{pmatrix} \\ &= \tfrac{1}{100}(1350) \\ &= \mathbf{(13.50)} \end{aligned}$$

(d) **G** represents the total fare amount (in dollars) collected on a weekday.

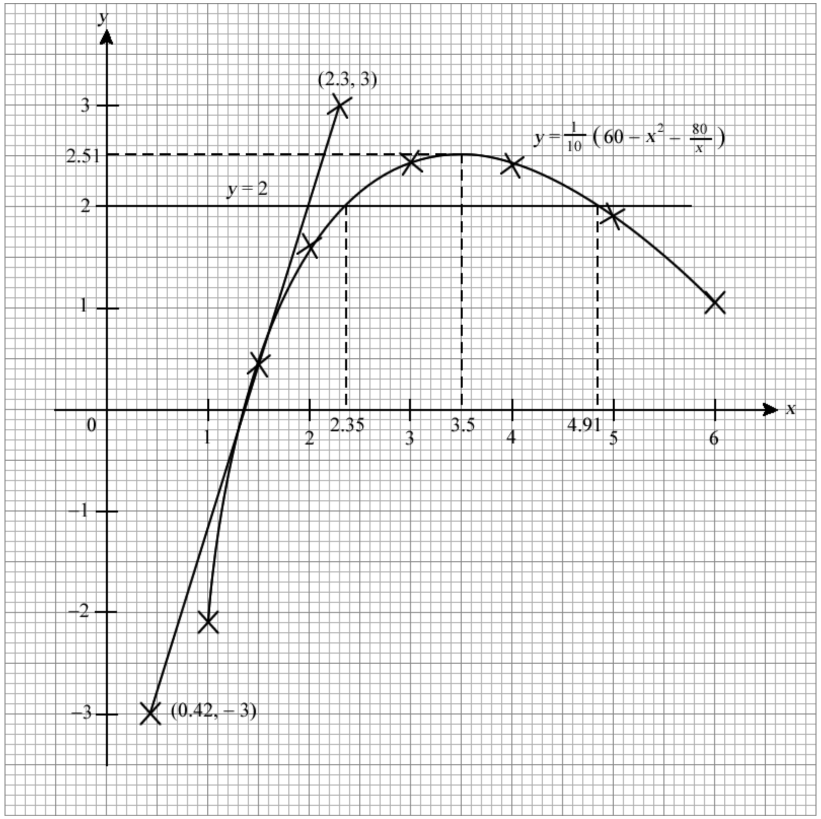
6. **Topic: Graphical Solution of Equations**

(a) $y = \frac{1}{10}\left(60 - x^2 - \frac{80}{x}\right)$

When $x = 6$,

$$p = \frac{1}{10}\left(60 - 6^2 - \frac{80}{6}\right)$$
$$= \mathbf{1.07\ (2\ d.p.)}$$

(b)



(c) Values of x where $\frac{1}{10}\left(60 - x^2 - \frac{80}{x}\right) = 2$ is obtained from x-coordinate of intersection points of the curve $y = \frac{1}{10}\left(60 - x^2 - \frac{80}{x}\right)$ and the line $y = 2$.

From the graph, $\mathbf{x = 2.35, 4.91}$

Check:
$\frac{1}{10}\left(60 - x^2 - \frac{80}{x}\right) = 2$
$x^3 - 40x + 80 = 0$
$x = -7.15$, **2.31, 4.85**

(d) From the graph, gradient of tangent $= \frac{3-(-3)}{2.3-0.42}$

$= \mathbf{3.19\ (3\ sig.\ fig.)}$

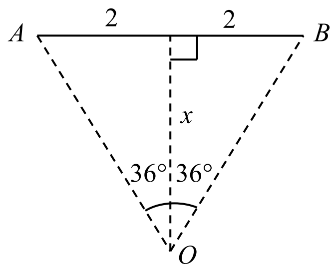
AMaths students:
Check: $\frac{dy}{dx} = \frac{1}{10}\left(-2x + \frac{80}{x^2}\right)$
Sub $x = 1.5 \Rightarrow \frac{dy}{dx} = 3.26$

(e) Largest value of y = **2.51**, when $x = 3.5$

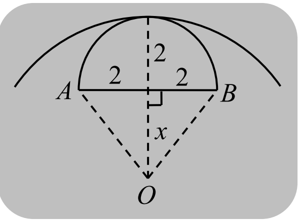
7. **Topics: Trigonometry, Mensuration**

(a) $\angle AOB = \frac{360°}{5}$
$= 72°$

Let the length of the perpendicular from O to AB be x.



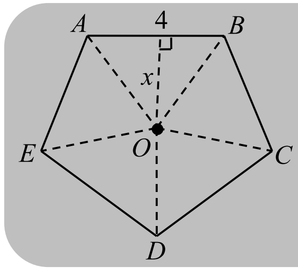
$$\tan 36° = \frac{2}{x}$$
$$x = \frac{2}{\tan 36°}$$
$$= 2.7527$$
$$\approx \mathbf{2.75\ cm\ (3\ sig.\ fig.)}$$



(b) Radius of outer circle $= x + 2$
$= 2.7527 + 2$
$= 4.7527$
$\approx$ 4.75 cm (3 sig. fig.) **(Shown)**

(c) Total length of wire needed to make the ornament
$= 5(4) + 5\left(\frac{1}{2}\right)(2\pi)(2) + 2\pi(4.7527)$
$= 81.278$
≈ 81.3 cm (3 sig. fig.)

(5 × sides of pentagon)
+ (5 × semicircles)
+ Circumference of outer circle



Area of 5 triangles of height x cm and base 4 cm

(d) Area enclosed by the wire pentagon $= 5\left(\frac{1}{2}\right)(2.7527)(4)$
$= 27.527$
$\approx$ **27.5 cm^2 (3 sig. fig.)**

(e) Area of the shaded region on the diagram
$= [\pi(4.7527)^2 - 27.527 - 5\left(\frac{1}{2}\right)(\pi)(2)^2] \div 5$
$= (12.019) \div 5$
≈ 2.40 cm^2 (3 sig. fig.)

[Area of outer circle
– Area of pentagon
– Area of 5 semicircles]
÷ 5

8. **Topic: Trigonometry (Cosine Rule, Sine Rule, Bearings, Angle of Elevation)**

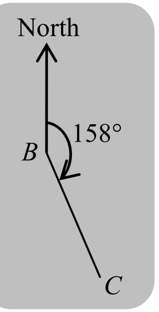
(a) (i) $(BC)^2 = (AB)^2 + (AC)^2 - 2(AB)(AC)\cos\angle BAC$
$= 550^2 + 645^2 - 2(550)(645)\cos 38°$
$BC = \sqrt{159431.3703}$
$= 399.28$
$\approx$ **399 m (3 sig. fig.)**

Cosine Rule:
$c^2 = a^2 + b^2 - 2ab\cos C$

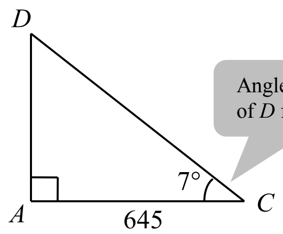
(ii) $\frac{\sin\angle ACB}{AB} = \frac{\sin\angle BAC}{BC}$
$\frac{\sin\angle ACB}{550} = \frac{\sin 38°}{399.28}$
$\sin\angle ACB = 0.84806$
$\angle ACB = 58.00°$
$\approx$ **58.0° (1 d.p.)**

Sine Rule:
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

(iii) $\angle ABC = 180° - 38° - 58.0°$
$= 84°$
Bearing of C from B $= 360° - (180° - 62°) - 84°$
$=$ **158°**



(b) (i) $\tan 7° = \frac{AD}{645}$
$AD = 79.196$
$\approx$ **79.2 m (3 sig. fig.)**



Angle of elevation of D from C

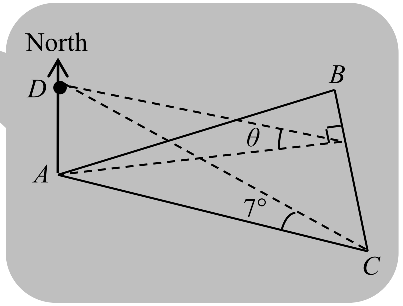
(ii) Let x be the shortest distance from A to BC.

Max ∠ of elevation occurs at min distance from A to BC.

$$\frac{1}{2}(x)(BC) = \text{Area of } \Delta ABC$$
$$\frac{1}{2}(x)(399.28) = \frac{1}{2}(550)(645)\sin 38°$$
$$x = 546.999$$

Let θ be the greatest possible angle of elevation of D from a point on BC.

$$\tan\theta = \frac{79.196}{546.999}$$
$$\theta = 8.238$$
$$\approx \mathbf{8.2° \ (1\ d.p.)}$$



9. **Topics: Pythagoras' Theorem, Solution to Quadratic Equation**

(a) $AT = TP = x$

$SB = SQ = 7$

$$\therefore ST = 17 - AT - SB$$
$$= 17 - x - 7$$
$$= (10 - x) \text{ cm } \textbf{(Shown)}$$

(b) (i) $PQ = \mathbf{(x + 7)\ cm}$

(ii) $QR = \mathbf{(7 - x)\ cm}$

(c) By Pythagoras' Theorem,

$$(PQ)^2 = (PR)^2 + (QR)^2$$
$$(x+7)^2 = (ST)^2 + (7-x)^2$$
$$x^2 + 14x + 49 = (10-x)^2 + 49 - 14x + x^2$$
$$x^2 + 14x + 49 = 100 - 20x + x^2 + 49 - 14x + x^2$$
$$x^2 - 48x + 100 = 0 \ \textbf{(Shown)}$$

(d) $a = 1$, $b = -48$, $c = 100$

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{48 \pm \sqrt{(-48)^2 - 4(1)(100)}}{2(1)}$$
$$= \frac{48 \pm \sqrt{1904}}{2}$$
$$= 45.817 \quad \text{or} \quad 2.182$$
$$\approx \mathbf{45.82} \quad\quad \mathbf{2.18\ (2\ d.p.)}$$

(e) When $x = 2.182$, (45.82 rejected)

$\because x < 14$ in the diagram

$$\text{Diameter of smaller cylinder} = 2(2.182)$$
$$= 4.364 \text{ cm}$$
$$= 43.64 \text{ mm}$$
$$\approx \mathbf{44\ mm\ (nearest\ millimeter)}$$

1 cm = 10 mm

10. **Topics: Statistics, Simple Probability**

(a) (i)

Scores	4	5	6	7	8	10
Frequency	2	5	11	8	3	1

(ii) (a) Mean score $= \frac{\Sigma fx}{\Sigma f}$

$= \frac{4(2)+5(5)+6(11)+7(8)+8(3)+10(1)}{30}$

$= \frac{189}{30}$

$=$ **6.3**

(b) Standard deviation $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$

$= \sqrt{\frac{2(4)^2+5(5)^2+11(6)^2+8(7)^2+3(8)^2+1(10)^2}{30} - (6.3)^2}$

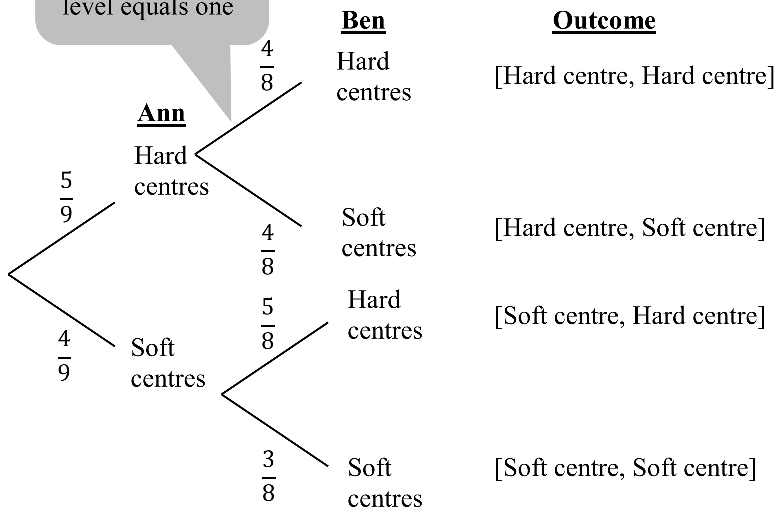
$= 1.2423$

$\frac{\Sigma fx}{\Sigma f} = 6.3$ from (ii)(a)

$\approx$ **1.24 (3 sig. fig.)**

(b) (i)

Check: Sum of probabilities of branches at each level equals one



(ii) (a) P(Ann and Ben both choose a chocolate with hard centre)

$= \frac{5}{9} \times \frac{4}{8}$

P(Ann gets hard centre) AND P(Ben gets hard centre)

$= \mathbf{\frac{5}{18}}$

(b) P(Ben chooses a chocolate with soft centre)

$= \frac{5}{9}\left(\frac{4}{8}\right) + \frac{4}{9}\left(\frac{3}{8}\right)$

P(Ann gets hard centre AND Ben gets soft centre) OR P(Ann gets soft centre AND Ben gets soft centre)

$= \mathbf{\frac{4}{9}}$

(c) P(One of them chooses a chocolate with hard centre and the other chooses one with soft centre)

$= \frac{5}{9}\left(\frac{4}{8}\right) + \frac{4}{9}\left(\frac{5}{8}\right)$

P(Ann gets hard centre AND Ben gets soft centre) OR P(Ann gets soft centre AND Ben gets hard centre)

$= \mathbf{\frac{5}{9}}$