ELEMENTARY MATHEMATICS 4017/02
Paper 2 Suggested Solutions October/November 2008

1. **Topics: Algebraic Manipulation, Solutions to Quadratic Equations**

(a) $\frac{7p^2-28}{p^2+2p} = \frac{7(p^2-4)}{p(p+2)}$

$= \frac{7(p+2)(p-2)}{p(p+2)}$

$= \boldsymbol{\frac{7(p-2)}{p}}$

(b) $1 - \frac{3f-g}{f+2g} = \frac{f+2g-(3f-g)}{f+2g}$

$= \frac{f+2g-3f+g}{f+2g}$

$= \boldsymbol{\frac{3g-2f}{f+2g}}$

Completing the Square:

$$x^2+bx+c = x^2+bx+\left(\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c$$
$$= \left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c$$

(c) (i) $x^2+11x-15 = x^2+11x+\left(\frac{11}{2}\right)^2-\left(\frac{11}{2}\right)^2-15$

$= (x+\frac{11}{2})^2-45.25$

$= \boldsymbol{(x+5.5)^2-45.25}$

(ii) $x^2+11x-15=0$

Sub $a=1$, $b=11$, $c=-15$ into $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$,

$$x = \frac{-11\pm\sqrt{11^2-4(1)(-15)}}{2(1)}$$
$$= \frac{-11\pm\sqrt{120-60}}{2}$$
$$= \frac{-11\pm\sqrt{181}}{2}$$
$$= \mathbf{1.23, -12.23}$$

2. **Topic: Trigonometry**

(a) $\cos 49° = \frac{AD}{9.4}$

$AD \approx$ **6.17 m (3 sig. fig.)**

(b) $\angle PAB = 90° - 49° - 32° = 9°$

$\sin 9° = \frac{PB}{12.1}$

$PB \approx$ **1.89 m (3 sig. fig.)**

(c) Area of $\Delta APQ = \frac{1}{2}(9.4)(12.1)\sin 32°$

$\approx$ **30.1 m² (3 sig. fig.)**

(d) Using cosine rule, $PQ^2 = 9.4^2 + 12.1^2 - 2(9.4)(12.1)\cos 32°$

$PQ \approx$ **6.47 m (3 sig. fig.)**

3. **Topic: Arithmetic (Application of Mathematics in Practical Situations)**

(a) Total cost of making 25 000 souvenirs = 25000 × \$0.90
= **\$22500**

(b) Cost of materials per souvenir = \$ $[\frac{0.9}{15} \times 5]$ = **\$0.30**

Cost of wages per souvenir = \$ $[\frac{0.9}{15} \times 4]$ = **\$0.24**

(c) Total no. of hours spent = 7 × 5 = 35 hours
⇒ Salary for 35 hours = \$630 (Given)
Salary per hour = \$630 ÷ 35 = \$18
∴ no. of souvenirs John made in one hour = $\frac{18}{0.24}$ = **75**

(d) From (b): Original cost of materials = \$0.30
⇒ Increase in cost of materials = \$0.30 × 50% = \$0.15
Original wages = \$0.24
⇒ Increase in wages = \$0.24 × 10% = \$0.024

% increase in cost of making a souvenir = $\frac{\$0.15 + \$0.024}{\$0.90} \times 100\%$
≈ **19.3% (3 sig. fig.)**

(e) 125% of cost price = \$2.00
⇒ Cost price = $\frac{\$2.00}{\frac{125}{100}} = \mathbf{\$1.60}$

4. **Topic: Number Patterns**

(a) $u_5 = 2^4 + 9 =$ **25 (shown)**

(b) $u_6 = 2^5 + 11 =$ **43**

(c) $\boldsymbol{u_n = 2^{n-1} + 2n - 1}$

(d) $u_{20} = 2^{19} + 2(20) - 1 =$ **524327**

(e) (i) L.H.S.: $2^{n-1} - 2^{n-2} = \frac{2^n}{2} - \frac{2^n}{2^2}$

$= 2^n\,(\frac{1}{2} - \frac{1}{4})$

$= 2^n\,(\frac{1}{4})$

$= 2^n\,(2^{-2})$

$= \boldsymbol{2^{n-2}}$ **= R.H.S. (Shown)**

(ii) $u_n - u_{n-1} = [2^{n-1} + 2n - 1] - [2^{n-2} + 2(n-1) - 1]$

$= [2^{n-1} + 2n - 1] - [2^{n-2} + 2n - 3]$

$= 2^{n-1} + 2n - 1 - 2^{n-2} - 2n + 3$

$= 2^{n-1} - 2^{n-2} + 2$

$= \boldsymbol{2^{n-2} + 2}$

Sub $2^{n-1} - 2^{n-2} = 2^{n-2}$ as proven in (e)(i).

5. **Topic: Algebraic Representation & Formulae**

(a) Cost of each apple $= \frac{\$12}{m} = \boldsymbol{\left(\frac{1200}{m}\right)}$**¢**

$12 = 1200¢
→ question requires this to be expressed in cents.

(b) No. of remaining apples $= (m - 3)$

Selling price of each apple $= \left(\frac{1200}{m} + 2\right)$¢ ………. (1)

∴ Total sum received from the sale of the apples

$= \boldsymbol{(m - 3)\left(\frac{1200}{m} + 2\right)}$

(c) $(m - 3)\left(\frac{1200}{m} + 2\right) - 1200 = 96$

$(m - 3)\left(\frac{1200 + 2m}{m}\right) = 1296$

$(m - 3)(1200 + 2m) = 1296m$

$1200m + 2m^2 - 3600 - 6m = 1296m$

$2m^2 - 102m - 3600 = 0$

$\boldsymbol{m^2 - 51m - 1800 = 0}$ **(Shown)**

Total cost = $12 = 1200¢ (given)
Total sales = $(m-3)\left(\frac{1200}{m} + 2\right)$ from (b)
Profit = total sales – total cost

(d) $m^2 - 51m - 1800 = 0$

$m = \frac{51 \pm \sqrt{(-51)^2 - 4(1)(-1800)}}{2}$

$= \frac{51 \pm \sqrt{9801}}{2}$

= 75 or –24

(e) $m = 75$ ($m = -24$ rejected ∵ number of apples cannot be negative)

Sub $m = 75$ into (1): Selling price $= \left(\frac{1200}{75} + 2\right)$¢

= 18¢ = $0.18

6. **Topics: Congruence & Similarity, Angles & Triangles**

(a) $\angle LAD = \angle LCB$ (angles in same segment)
$\angle LDA = \angle LBC$ (angles in same segment)
$\angle ALD = \angle CLB = 90°$ (vertically opposite angles)

∴ **ΔLAD and ΔLCB are similar (AAA) (Shown)**

(b) (i) $\angle CNO = 90°$ ($ON \perp BC$ ∵ N is midpt of BC of isosceles ΔOBC)

(ii) $\angle DCB = \angle DAB = 58°$ (angles in same segment)

$\angle CON = 180° - \angle CNO - (\angle DCO + \angle DCB)$ (sum of ∠s in a triangle)

$= 180° - 90° - (18° + 58°)$

$= \boldsymbol{14°}$

$\angle CNO = 90°$ from (b)(i)

(iii) $\angle CBA = 180° - \angle CLB - \angle DCB$ (sum of ∠s in a triangle)

$= 180° - 90° - 58°$

$= \boldsymbol{32°}$

$\angle DCB = 58°$ from (b)(ii)

(iv) $\angle CDO = \angle DCO = 18°$ (base ∠s of isosceles ΔDCO)

$\angle ADC = \angle CBA$ (angles in same segment)

$= 32°$

$\angle CBA = 32°$ from (b)(iii)

$\angle ADO = \angle ADC - \angle CDO$

$= 32° - 18°$

$= \boldsymbol{14°}$

7. **Topics: Geometrical Properties of Circles, Trigonometry**

(a) (i) $\tan \angle AOC = \frac{AC}{OC}$ ($OC \perp AC \because OC$ is perpendicular bisector of chord AB)

$= \frac{40}{50}$

$\angle AOC = 38.66°$

$\therefore \angle AOB = 2 \times \angle AOC$

$= 2 \times 38.66°$

$\approx 77.32°$

≈ 77.3° (3 sig. fig.)

(ii) Using Pythagoras' theorem for ΔOAC,

AO (length of radius of sector OAB) $= \sqrt{40^2 + 50^2}$ cm

$= \sqrt{4100}$ cm

Area of window = Area of sector OAB – Area of ΔOAB

$= \frac{1}{2} \times AO^2 \times \angle AOB - \frac{1}{2} \times AB \times OC$

$= \frac{1}{2}(\sqrt{4100})^2\left(\frac{77.32°}{180°} \times \pi\right) - \frac{1}{2}(80)(50)$

≈ 766 cm² (3 sig. fig.)

$\angle AOB$ must to be converted to radians: $\frac{\theta}{180°} \times \pi$ radians

(b) (i) Using cosine rule for ΔDEX,

$EX^2 = DE^2 + DX^2 - 2(DE)(DX)\cos\angle EDX$

$= 80^2 + 80^2 - 2(80)(80)\cos 38°$

$\therefore EX \approx 52.09$ cm

≈ 52.1 cm (3 sig. fig.)

(ii) Using Pythagoras' theorem,

$DF = DY = \sqrt{200^2 + 80^2}$

$= \sqrt{46400}$ cm

Using cosine rule for ΔFDY,

$FY^2 = DF^2 + DY^2 - 2(DF)(DY)\cos\angle FDY$

$\cos \angle FDY = \frac{(\sqrt{46400})^2 + (\sqrt{46400})^2 - (52.09)^2}{2\sqrt{46400}\sqrt{46400}}$

$= 0.97076$

$\therefore \angle FDY \approx$ **13.9° (3 sig. fig.)**

$FY = EX \approx 52.09$ cm from (b)(i)

8. **Topic: Mensuration**

(a) (i) Using Pythagoras' theorem,

Slant height, $s = \sqrt{\left(\frac{0.8}{2}\right)^2 + 2^2}$ cm

Radius of pencil $= \frac{0.8}{2}$ cm

$= \sqrt{0.4^2 + 2^2}$ cm

≈ 2.0396 cm

≈ 2.04 cm (3 sig. fig.)

(ii) Total surface area

= Area of cone + area of cylinder + area of circular base

$= \pi(0.4)(2.0396) + 2\pi(0.4)(16) + \pi(0.4)^2$

≈ 43.3 cm² (3 sig. fig.)

Sub $s = 2.0396$ from (a)(i) into area of cone $= \pi rs$

(b) Volume of pencil

= Volume of cone + volume of cylinder

$= \frac{1}{3}\pi(0.4)^2(2) + \pi(0.4)^2(16)$

≈ 8.378 cm³

≈ 8.38 cm³ (3 sig. fig.)

(c) (i) Width of box = 6 × pencil diameter = 6 × 0.8 cm = 4.8 cm

Height of box = 2 × pencil diameter = 2 × 0.8 cm = 1.6 cm

Length of box = 1 × pencil length = (16+2) cm = 18 cm

$\therefore$ volume of box = 4.8 cm × 1.6 cm × 18 cm

=138.24 cm³ (Shown)

(ii) Volume of box not occupied by the pencils

= Volume of box – total volume of 12 pencils in box

= 138.24 cm³ – 12 × 8.378 cm³

= 37.704 cm³

Volume of each pencil ≈ 8.378 cm³ from (b)

$\therefore$ % of the volume not occupied by the pencils

$= \frac{37.704}{138.24} \times 100\%$

≈ 27.3 %

9. **Topics: Vectors in Two Dimensions, Probability**

(a) (i) $BC = \sqrt{(5-7)^2 + (4-9)^2}$ units
$= \sqrt{29}$ units
$\approx$ **5.39 units (3 sig. fig.)**

Sub $B(5, 4)$, $C(7, 9)$ into $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(ii) Gradient of AB, $m = \frac{4-1}{5-(-3)} = \mathbf{\frac{3}{8}}$

Sub $A(-3, 1)$, $B(5, 4)$ into $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

(iii) Sub $m = \frac{3}{8}$ into $y = mx + c$:

$y = \frac{3}{8}x + c$ ………. (1)

Sub $A(-3, 1)$ into (1): $1 = \frac{3}{8}(-3) + c$

$c = \mathbf{\frac{17}{8}}$

$\therefore \boldsymbol{y = \frac{3}{8}x + \frac{17}{8}}$

(iv) Mid-point of $AC = \left(\frac{-3+7}{2}, \frac{1+9}{2}\right)$

$\therefore$ ***E*= (2, 5)**

Sub $A(-3, 1)$, $C(7, 9)$ into $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

(v) Let the coordinates of $D = (x, y)$

Since $ABCD$ are the vertices of a parallelogram,
$\Rightarrow$ Mid-point of BD = Mid-point of $AC = E$

Hence, mid-point of BD: $\left(\frac{x+5}{2}, \frac{y+4}{2}\right) = (2, 5)$

Coordinates of E from (a)(iv)

$\Rightarrow \quad \frac{x+5}{2} = 2 \qquad \frac{y+4}{2} = 5$
$x + 5 = 4 \qquad y = 10 - 4$
$x = -1 \qquad y = 6$

$\Rightarrow D = (-1, 6) \Rightarrow$ Position vector $\overrightarrow{OD} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$

Position vector $\overrightarrow{OE} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$\overrightarrow{ED} = \overrightarrow{OD} - \overrightarrow{OE}$
$= \begin{pmatrix} -1 \\ 6 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix}$
$= \mathbf{\begin{pmatrix} -3 \\ 1 \end{pmatrix}}$

(b) (i)

	1	**2**	**3**	**4**	**5**	**6**
1		1,2	1,3♦	1,4	1,5	1,6♦
2	2,1		2,3♦	2,4♠	2,5	2,6♠♣♦
3	3,1♦	3,2♦		3,4♦	3,5♣♦	3,6♦
4	4,1	4,2♠	4,3♦		4,5	4,6♠♦
5	5,1	5,2	5,3♣♦	5,4		5,6♦
6	6,1♦	6,2♠♣♦	6,3♦	6,4♠♦	6,5♦	

Without replacement $\Rightarrow$ (1, 1), (2, 2) … (6, 6) outcomes are impossible.

♠ - both balls even
♣ - sum is 8
♦ - at least one is multiple of 3

From the possibility diagram, total no. of possible outcomes = 30

(ii) (a) P (both have even number) $= \frac{6}{30}$
$= \mathbf{\frac{1}{5}}$

$\dfrac{\text{\# of ♠}}{\text{Total \# of outcomes}}$
Check: $\frac{1}{6} \times \frac{1}{5} \times 6$

(b) P (sum of numbers drawn is 8) $= \frac{4}{30}$
$= \mathbf{\frac{2}{15}}$

$\dfrac{\text{\# of ♣}}{\text{Total \# of outcomes}}$
Check: $\frac{1}{6} \times \frac{1}{5} \times 4$

(c) P (product is 7) $= \frac{0}{30} = 0$

(d) P (at least one of the no. drawn is a multiple of 3) $= \frac{8}{30}$
$= \mathbf{\frac{3}{5}}$

$\dfrac{\text{\# of ♦}}{\text{Total \# of outcomes}}$
Check: $\frac{1}{6} \times \frac{1}{5} \times 18$

10. **Topic: Graphical Solution of Equations**

(a) Sub $x = 4$ into $y = \frac{1}{5}x(12 - x^2)$: $p = \frac{1}{5}(4)(12 - 4^2) = \mathbf{-3.2}$

(b) See graph.

(c) Plot $y = 1$ for the range $-3 \le x \le 4$.

From graph,

$y = \frac{1}{5}x(12 - x^2)$ intersects $y = 1$ at $x = 0.42, 3.23$

$\therefore$ Solution of $\frac{1}{5}x(12 - x^2) = 1$: $\mathbf{x = 0.42, 3.23}$

Check: $x^3 - 12x + 5 = 0$

$\Rightarrow x = \mathbf{3.23}, -3.66, \mathbf{0.42}$

(d) From graph, gradient of tangent at (3, 1.8) $= \frac{4-(-0.5)}{2.25-3.75}$

$= \mathbf{-3}$

AMaths students:

Check: $\frac{dy}{dx} = \frac{12}{5} - \frac{3x^2}{5}$

Sub $x = 3 \Rightarrow \frac{dy}{dx} = -3$

(e) Since $2x + y = 2$ is linear, sub the values of $x = -1$ and $x = 3$ to obtain the y-values of the two points:

x	-1	3
y	4	-4

Join up these two points to get the graph of $2x + y = 2$.

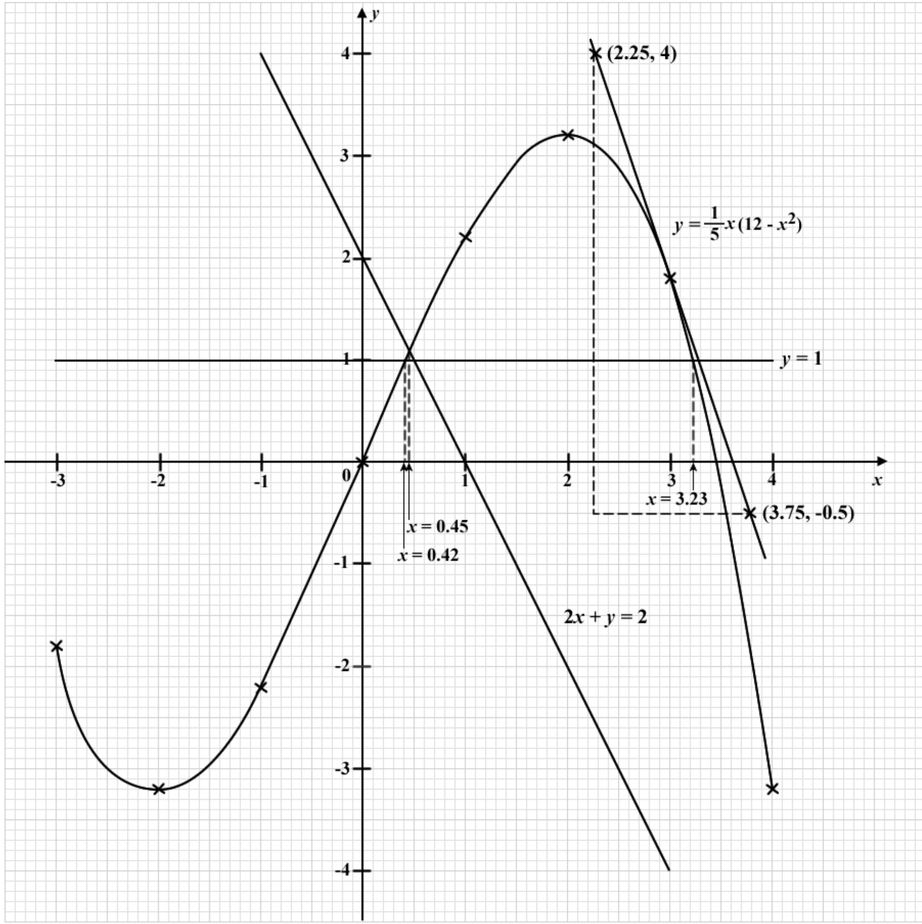
(f) (i) From graph, x-coordinate = **0.45**

(ii) $\frac{1}{5}x(12 - x^2) = 2 - 2x$

$x^3 - 22x + 10 = 0$ ………. (1)

Comparing coefficients of (1) with $x^3 + Ax^2 + Bx + C = 0$,

$\mathbf{A = 0, B = -22, C = 10}$



11. **Topic: Data Analysis (Statistics, Cumulative Frequency Distribution)**

(a) Mean mark $= \frac{\Sigma fx}{\Sigma f}$

$= \frac{30(5) + 70(15) + 25(140) + 35(110) + 45(50)}{400}$

$=$ **27**

Use the mid-value of each interval for x.

(b)

Mark (x)	$x = 0$	$x \le 10$	$x \le 20$	$x \le 30$	$x \le 40$	$x \le 50$
No. of people	0	30	100	240	350	400

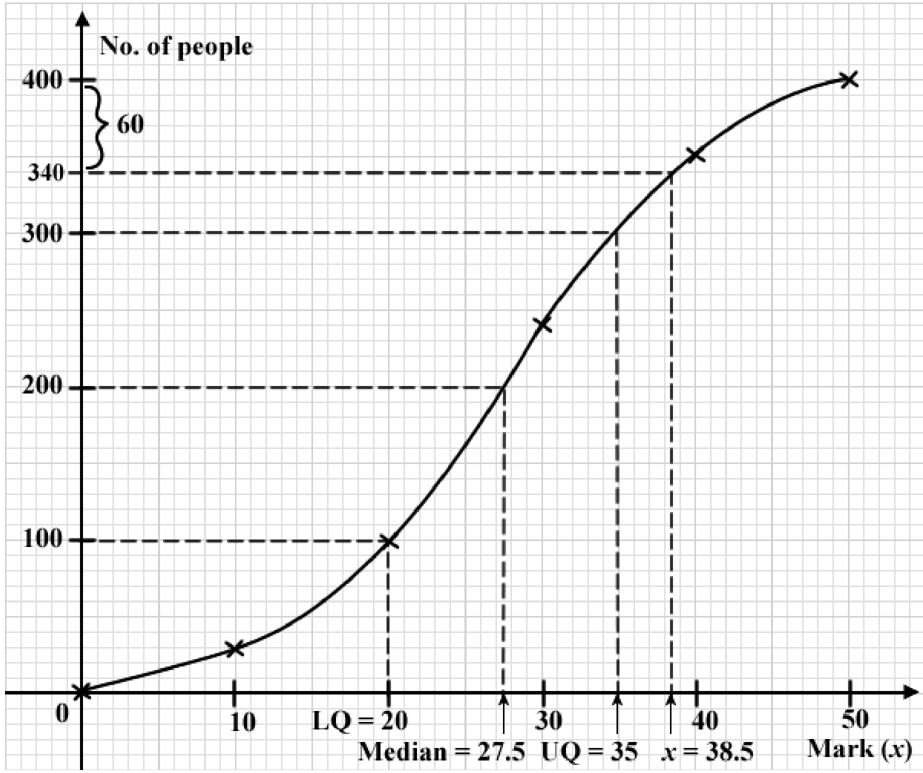
freq. ($x \le 10$) + freq. ($10 < x \le 20$) = 30 + 70

freq. ($x \le 20$) + freq. ($20 < x \le 30$) = 100 + 140

freq. ($x \le 30$) + freq. ($30 < x \le 40$) = 240 + 110

freq. ($x \le 40$) + freq. ($40 < x \le 50$) = 350 + 50

(c)



(d) From the curve in (c):

(i) Median = **27.5**

50% of 400 people scored below this mark

(ii) Upper Quartile (UQ) = **35**

75% of 400 people scored below this mark

(iii) Lower Quartile (LQ) = 20

Interquartile Range = UQ – LQ
= 35 – 20
= **15**

(e) 15% of people $= \frac{14}{100} \times 400 = 60$

15% scored *above* this mark = 85% scored *below* this mark

From the curve in (c),

Least mark needed to go on to the second round = **38.5**