ELEMENTARY MATHEMATICS 4016/01
Paper 1 Suggested Solutions October/November 2008

1. **Topic: Standard Form**

(a) $\frac{4.32}{32.8\times0.593} = 0.2221$

$\approx$ **2.22×10^{-1} (3 sig. fig.)**

(b) $\frac{6.82\times10^{5}}{1.55\times10^{-2}} = \left(\frac{6.82}{1.55}\right)\times 10^{5-(-2)}$

$= \mathbf{4.4 \times 10^{7}}$

$\frac{a_m}{a_n} = a^{m-n}$

Answer (a) 2.22×10^{-1} [1]
(b) 4.4×10^{7} [1]

2. **Topic: Statistics (Bar Chart & Pie Chart)**

(a) Total no. of children $= 6 + 30 + 12$
$= 48$

$\frac{\text{Total no.of children who walked or cycled}}{\text{Total no.of children}} = \frac{30+12}{48}$
$= \mathbf{\frac{7}{8}}$

(b) $\frac{6}{48}\times 360° = \mathbf{45°}$

∠ of sector in pie chart ∝ its given data

Answer (a) $\frac{7}{8}$ [1]
(b) $45°$ [1]

3. **Topic: Indices**

(a) $\left(\frac{2}{x}\right)^{-3} = \left(\frac{x}{2}\right)^{3}$
$= \mathbf{\frac{x^3}{8}}$

$\left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{a}\right)^{c}$

(b) $2^{34} \div 2^{4} = 2^{k}$
$2^{34-4} = 2^{k}$
$\mathbf{k = 30}$

$a^m \div a^n = a^{m-n}$

Answer (a) $\frac{x^3}{8}$ [1]
(b) $k =$ 30 [1]

4. **Topic: Arithmetic**

(a) 3 hours 41 min = $\mathbf{3\frac{41}{60}}$ **hr**

(b) Distance travelled $= 3\frac{41}{60}\times 50$
$= 184.16$
$\approx$ **184 km (nearest km)**

Distance = Speed × Time taken

Answer (a) $3\frac{41}{60}$ hours [1]
(b) 184 km [1]

5. **Topic: Arithmetic**

(a) $\frac{2}{5} \rightarrow \$4.20$

$\frac{3}{5} \rightarrow \frac{\$4.20}{2} \times 3$

$= \mathbf{\$6.30}$

(b) $\$0.99 \rightarrow x$ grams

$\$1 \rightarrow \frac{100x}{99}$ grams

$\$y \rightarrow \mathbf{\frac{100}{99}xy}$ **grams**

Answer (a) $ 6.30 [1]

(b) $\frac{100}{99}xy$ grams [1]

6. **Topic: Mensuration**

Arc $AB = \frac{1}{4}(2)(\pi)(6)$
$= 3\pi$

Circumference of circle = $2\pi r$

Arc $OC = \frac{1}{2}(2)(\pi)(1.5)$
$= 1.5\pi$

Perimeter of the shaded region $= 3\pi + 1.5\pi + 6 + 3$
$= \mathbf{4.5\pi + 9}$

Answer $4.5\pi + 9$ [2]

7. **Topic: Standard Form**

(a) $1.03 \times 10^{10} = 10.3 \times 10^{9}$
$\therefore \boldsymbol{k = 10.3}$

1 billion = 1.0×10^9

(b) Estimated increase $= 1.03 \times 10^{10} - 6.5 \times 10^{9}$
$= 10^{9}[1.03 \times 10 - 6.5]$
$= \mathbf{3.8 \times 10^{9}}$

Answer (a) $k =$ 10.3 [1]

(b) 3.8×10^9 [1]

8. **Topic: Coordinate Geometry**

(a) At y-axis, $x = 0$,
$3(0) + 2y = 8$
$2y = 8$
$y = 4$
$\therefore \boldsymbol{A(0, 4)}$

Coordinates of *y*-intercept

(b) $3x + 2y = 8$
$2y = -3x + 8$
$y = -\frac{3}{2}x + 4$

Equation of straight line with gradient *m* and y-intercept *c*:
$y = mx + c$

Answer (a) (0 , 4) [1]

(b) $-\frac{3}{2}$ [1]

9. **Topics: Pythagoras Theorem, Trigonometry**

(a) If $\angle ABC$ is a right angle, by Pythagoras' Theorem,
$AC^2 = AB^2 + BC^2$
$AC^2 = 17^2 = 289$ cm^2
$AB^2 + BC^2 = 15^2 + 8^2 = 225 + 64 = 289$ cm^2 $= AC^2$
$\therefore \angle ABC$ is a right angle

(b) (i) Area of $\Delta DAC = \frac{1}{2}(5)(8)$
$= \mathbf{20\ cm^2}$

Area of $\Delta = \frac{1}{2} \times$ base $\times$ height

(ii) $\cos \angle DAC = \cos(180° - \angle BAC)$
$= -\cos \angle BAC$
$= -\frac{\mathbf{15}}{\mathbf{17}}$

$-\cos\theta = \cos(180° - \theta)$

Answer (a) $AC^2 = AB^2 + BC^2$ (Pythagoras' theorem)
$\therefore \angle ABC$ is a right angle [1]

Answer (b)(i) 20 cm^2 [1]
(ii) $\cos \angle DAC =$ $-\frac{\mathbf{15}}{\mathbf{17}}$ [1]

10. **Topic: Matrices**

(a) $\begin{pmatrix} 14 & 5 & 1 \\ 15 & 0 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} \mathbf{27} \\ \mathbf{25} \end{pmatrix}$

$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax + by + cz \\ dx + ey + fz \end{pmatrix}$

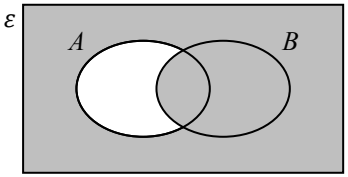
(b) **Sandy's score is 27 marks and Roger'score is 25 marks.**

Answer (a) $\begin{pmatrix} 27 \\ 25 \end{pmatrix}$ [2]

Answer (b) Sandy's score is 27 marks and Roger's score is 25 marks [1]

11. **Topic: Set Language and Notation**

(a)



(b) $\varepsilon = \{2, 3, 4, 5, 6, 7\ldots\}$
$P = \{2, 3, 5, 7\ldots\}$
$S = \{4, 9, 16\ \ldots\}$
$T = \{2, 12, 22\ \ldots\}$

(i) $P \cap T = \{2\}$
$n(P \cap T) = \mathbf{1}$

(ii) $S \cap T = \{\ \}$
$\therefore n(S \cap T) = \mathbf{0}$

$n(A)$ denotes the number of elements in set A

Answer (b)(i) 1 [1]
(b) 0 [1]

12. **Topics: Algebra (Algebraic Representation, Formulae)**

(a) $A = \frac{1}{2}(3x + 5x)(h) + (h + 2)(5x)$
$= 4xh + 5xh + 10x$
$= 9xh + 10x$ **(Shown)**

Area of trapezium ($\frac{1}{2}$ × sum of // sides × height) + Area of rectangle (length × breadth)

(b) $A = 9xh + 10x$
$A = x(9h + 10)$
$\boldsymbol{x = \frac{A}{(9h+10)}}$

Answer (a) $A = \frac{1}{2}(3x + 5x)(h) + (h + 2)(5x)$
$= 4xh + 5xh + 10x$
$= 9xh + 10x$ [1]

Answer (b) $x = \frac{A}{(9h+10)}$ [2]

13. **Topics: Direct & Inverse Variation, Percentages**

Let s be the speed of the car.
$D = ks^2$

Direct variation: $y \propto x \Rightarrow y = kx$

When $s = p$, $D = 6$
$6 = kp^2$
$k = \frac{6}{p^2}$
$\therefore D = \frac{6s^2}{p^2}$

(a) When the speed is increased by 300%,
Initial speed = p
Increase in speed = 300% × p
= $3p$
New speed = initial speed + increase in speed
= $p + 3p$
= $4p$

(b) When $s = 4p$,
$D_1 = \frac{6(4p)^2}{p^2}$
= **96**

% increase = $\frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$

(c) Percentage increase = $\frac{96-6}{6} \times 100\%$
= **1500 %**

Answer (a) $4p$ m/s [1]
(b) 96 m [1]
(c) 1500 % [1]

14. **Topic: Statistics (Stem-and-Leaf Diagram, Box-and-Whisker Diagram)**

(a) (i) **22**

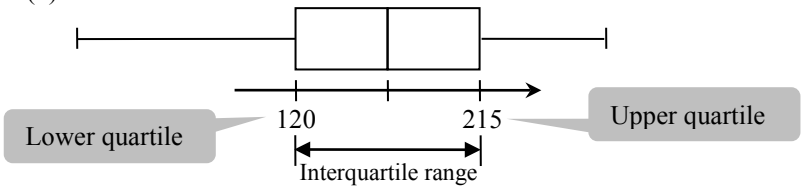
Modal score = score that appears most frequently

(ii) Median score = average of 8th & 9th terms
= $\frac{20+21}{2}$
= **20.5**

Total number of terms = 16 (even)
⇒ median term = average of 2 middle terms

Answer (a)(i) 22 [1]
(ii) 20.5 [1]

(b)



Interquartile range = upper quartile – lower quartile
= 215 − 120
= **95**

Answer (b) 95 g [1]

15. **Topic: Algebra (Factorisation, Expansion)**

(a) $2x^3 - 13x^2 + 6x = x(2x^2 - 13x + 6)$
$= \boldsymbol{x(2x - 1)(x - 6)}$

Answer (a) $x(2x - 1)(x - 6)$ [2]

(b) $9a^2 + 1 - [9a^2 - 6a + 1] = 9a^2 + 1 - 9a^2 + 6a - 1$
$= \boldsymbol{6a}$

Answer (b) $6a$ [2]

16. **Topic: Angle Properties of Polygons**

(a) $\frac{(n-2)180°}{n} = 165°$
$180n - 360 = 165n$
$180n - 165n = 360$
$15n = 360$
$\boldsymbol{n = 24}$

Each interior ∠ of a regular n-sided polygon
$= \frac{(n-2)\times 180°}{n}$

(b) Let x be the remaining interior ∠.
$x + 6(125) = (7 - 2)180$
$\boldsymbol{x = 150°}$

Answer (a) 24 [2]
(b) 150° [2]

17. **Topic: Factors and Multiples**

(a) (i) $1800 = \mathbf{2^3 \times 3^2 \times 5^2}$

(ii) $k = 3 \times 5$
$= \mathbf{15}$

(iii) $42 = 2 \times 3 \times 7$
HCF $= 2 \times 3$
$= \mathbf{6}$

(b) $12 = 2^2 \times 3$
$27 = 3^3$
$90 = 2 \times 3^2 \times 5$
LCM $= 2^2 \times 3^3 \times 5$
$= 540$
540 seconds = 9 min
They next flash together at **09 09**.

Question asked for the time, NOT time elapsed!

Answer (a)(i) $2^3 \times 3^2 \times 5^2$ [1]
(ii) $k =$ 15 [1]
(iii) 6 [1]
(b) 09 09 [2]

18. **Topics: Linear Inequalities and Simultaneous Equations**

(a) (i) $2 - 2x > 9$
$-2x > 9 - 2$
$-2x > 7$
$\boldsymbol{x < -3.5}$

Inequality sign reversed when multiplying both sides by –1

(ii) **The greatest integer is −4.**

Answer(a)(ii) x < -3.5 [1]
(ii) -4 [1]

(b) $x - 2y = 8$
$x = 2y + 8$ ..... (1)
$3x = 19 + 4y$ ..... (2)

Sub (1) into (2),

$3(2y + 8) = 19 + 4y$
$6y + 24 = 19 + 4y$
$2y = -5$
$y = \mathbf{-2.5}$

Sub $y = -2.5$ into (1),

$x = 2(-2.5) + 8$
$= \mathbf{3}$

Answer (b) $x =$ 3
$y =$ -2.5 [3]

19. **Topic: Areas & Volumes of Similar Figures**

Let h_1, A_1, V_1 be the height, surface area and volume, respectively, of the smaller jug.
Let h_2, A_2, V_2 be the height, surface area and volume, respectively, of the larger jug.

(a) $\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2$
$\frac{h_1}{h_2} = \sqrt{\frac{45}{125}}$
$= \mathbf{\frac{3}{5}}$

(b) $\frac{A_1}{A_2} = \frac{45}{125}$
$\frac{63}{A_2} = \frac{45}{125}$
$A_2 = \mathbf{175\ cm^2}$

(c) $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$
$\frac{V_1}{2.5} = \left(\frac{3}{5}\right)^3$

Using ratio of $\frac{h_1}{h_2}$ obtained in (a)

$V_1 = 0.54\ l$
$= \mathbf{540\ cm^3}$

1 litre = 1000 cm^3

Answer (a) 3 : 5 [1]
(b) 175 cm^2 [2]
(c) 540 cm^3 [2]

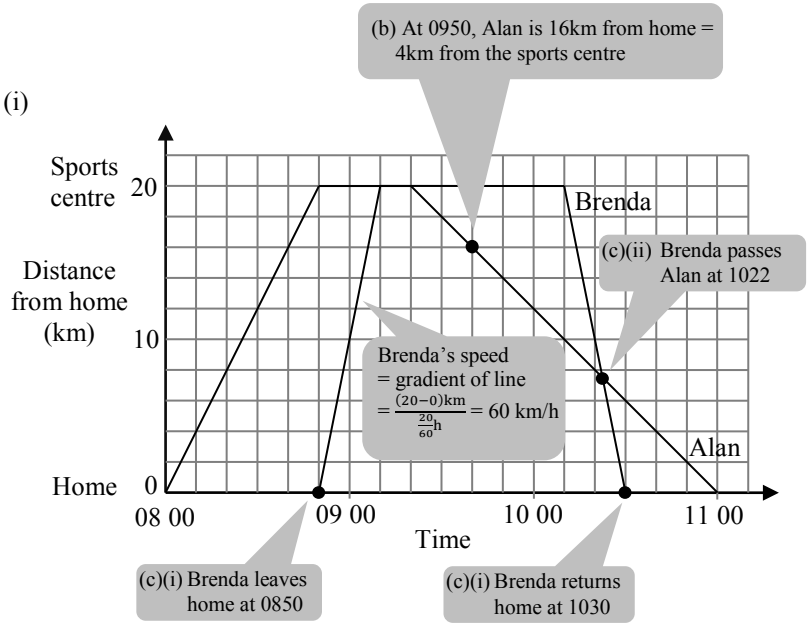
20. **Topic: Graphs in Practical Situations**

(a) Speed = $20 \div \frac{50}{60}$
= **24 km/h**

(b) **He was 4 km from the sports centre.**

Answer (a) 24 km/h [1]
(b) 4 km [1]

(c) (i)



(ii) Brenda passed Alan at 1022.

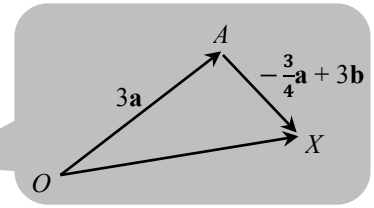
Answer(c)(ii) 1022 [1]

21. **Topic: Vectors in Two Dimensions**

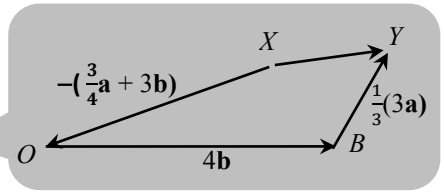
(a) (i) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
$= -\overrightarrow{OA} + \overrightarrow{OB}$
$= -3\mathbf{a} + 4\mathbf{b}$

(ii) $\overrightarrow{AX} = \frac{3}{4}\overrightarrow{AB}$
$= \frac{3}{4}(-3\mathbf{a} + 4\mathbf{b})$
$= -\frac{9}{4}\mathbf{a} + 3\mathbf{b}$

(iii) $\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$
$= 3\mathbf{a} - \frac{9}{4}\mathbf{a} + 3\mathbf{b}$
$= \frac{3}{4}\mathbf{a} + 3\mathbf{b}$



(iv) $\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OY}$
$= -\overrightarrow{OX} + \overrightarrow{OB} + \overrightarrow{BY}$
$= -\overrightarrow{OX} + \overrightarrow{OB} + \frac{1}{3}\overrightarrow{OA}$
$= -\left(\frac{3}{4}\mathbf{a} + 3\mathbf{b}\right) + 4\mathbf{b} + \frac{1}{3}(3\mathbf{a})$
$= \frac{1}{4}\mathbf{a} + \mathbf{b}$



(b) $\overrightarrow{OX} = 3\left(\frac{1}{4}\mathbf{a} + \mathbf{b}\right)$
$= 3\overrightarrow{XY}$

$\overrightarrow{AB} = k\overrightarrow{BC}$
$\Rightarrow$ *A*, *B*, *C* are collinear (straight line)

$\therefore$ *O*, *X* and *Y* lie in a straight line

Answer(a)(i) $\overrightarrow{AB} = -3\mathbf{a} + 4\mathbf{b}$ [1]
(ii) $\overrightarrow{AX} = -\frac{9}{4}\mathbf{a} + 3\mathbf{b}$ [1]
(iii) $\overrightarrow{OX} = \frac{3}{4}\mathbf{a} + 3\mathbf{b}$ [1]
(iv) $\overrightarrow{XY} = \frac{1}{4}\mathbf{a} + \mathbf{b}$ [1]

22. **Topic: Graphs**

(a) (i) x-intercepts: When $y = 0$, $x(3 - x) = 0$
$x = 0$ or $3 - x = 0$
$x = 3$

y-intercept: When $x = 0$, $y = 0$

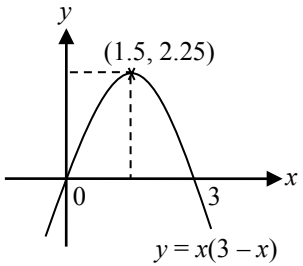
$y = x(3 - x)$: coefficient of x^2 is -1
$\Rightarrow$ graph takes the shape of $\cap$ (max point).

At max point:

$x = \frac{3-0}{2} = 1.5$

Line of symmetry of quadratic graph passes through max/min point

When $x = 1.5$, $y = 1.5(3 - 1.5)$
$= 2.25$



(ii) Line of symmetry:
$x = \frac{3-0}{2}$
$= \mathbf{1.5}$

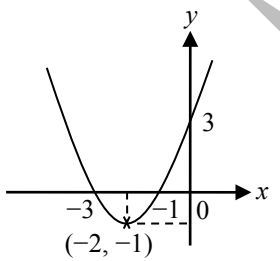
Answer (ii) $x = 1.5$ [1]

(b) (i) x-intercepts: When $y = 0$,
$(x + 2)^2 - 1 = 0$
$(x + 2)^2 = 1$
$x + 2 = 1$ or $x + 2 = -1$
$x = -1$ $x = -3$

y-intercept: When $x = 0$, $y = 2^2 - 1$
$= 3$

$y = (x + 2)^2 - 1$: coefficient of x^2 is 1
$\Rightarrow$ graph takes the shape of $\cup$ (min point).

Min point = $(-2, -1)$



$y = (x - h)^2 + k$
$\Rightarrow$ Line of symmetry $x = h$
$\Rightarrow$ min/max point = (h, k)

(ii) Coordinates of min pt = **(−2, −1)**

Answer (ii) $(-2, -1)$ [1]

23. **Topics: Bearings and Geometrical Constructions**

(a) Bearing of C from A = **116°**

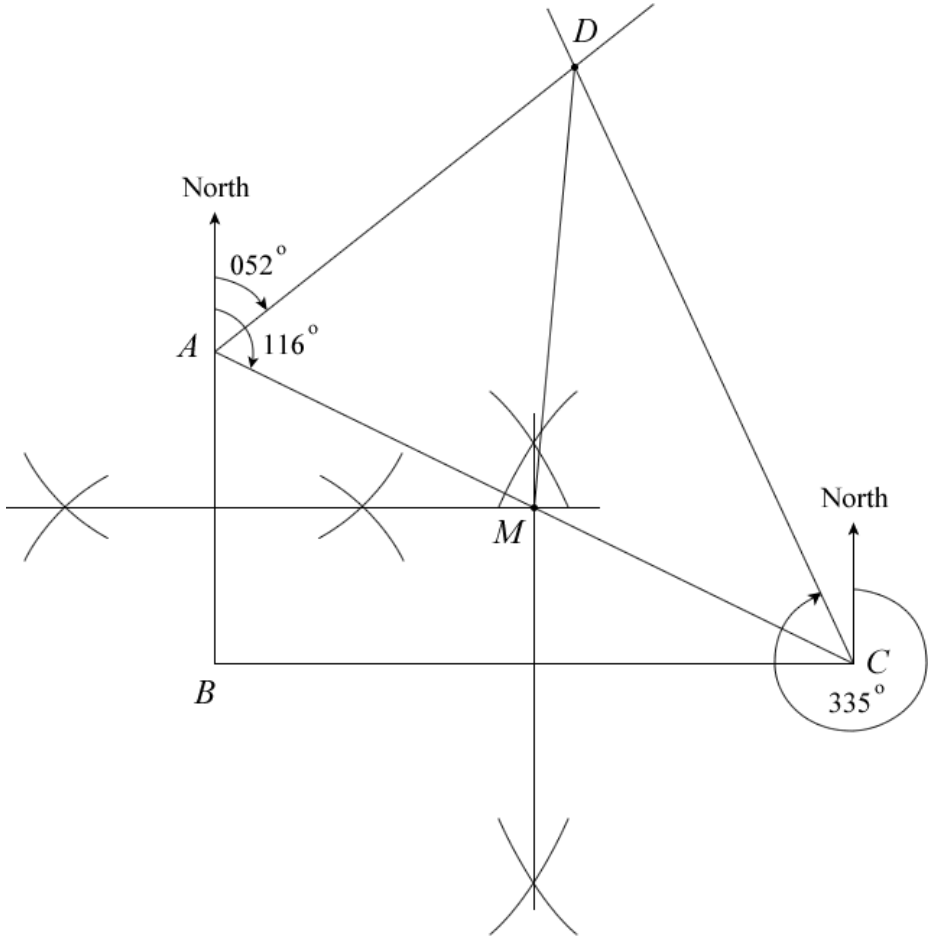
Answer (a) 116° [1]

(d) Given AB = 60 km

Scale: 6 cm : 60 km
1 cm : 10 km
DM = 8.35 cm
⇒ **83.5 km**

Measured on actual question paper.

Answer (b) and (c) [4]



Answer (d) 83.5 km [1]