2012 H2 Maths Paper 2. V1.1

1. (a) Given $\frac{d^2y}{dx^2} = 16 - 9x^2$.

$$\frac{dy}{dx} = \int (16 - 9x^2)\, dx$$
$$= 16x - 3x^3 + C_1 \quad , \text{ where } C_1 \text{ is a const.}$$

$$y = \int (16x - 3x^3 + C_1)\, dx$$
$$= 8x^2 - \frac{3}{4}x^4 + C_1 x + C_2 \quad , \text{ where } C_2 \text{ is a const.}$$ #

(b). Given $\frac{du}{dt} = 16 - 9u^2$.

$$\int \frac{1}{(4-3u)(4+3u)}\, du = \int dt.$$

$$\int \left[\frac{1}{8(4-3u)} + \frac{1}{8(4+3u)}\right] du = \int dt.$$

$$-\frac{1}{24}\ln|4-3u| + \frac{1}{24}\ln|4+3u| = t + d_1 \quad , \text{ where } d_1 \text{ is a const.}$$

$$\frac{1}{24}\ln\left|\frac{4+3u}{4-3u}\right| = t + d_1$$

Given $u = 1$ when $t = 0$.

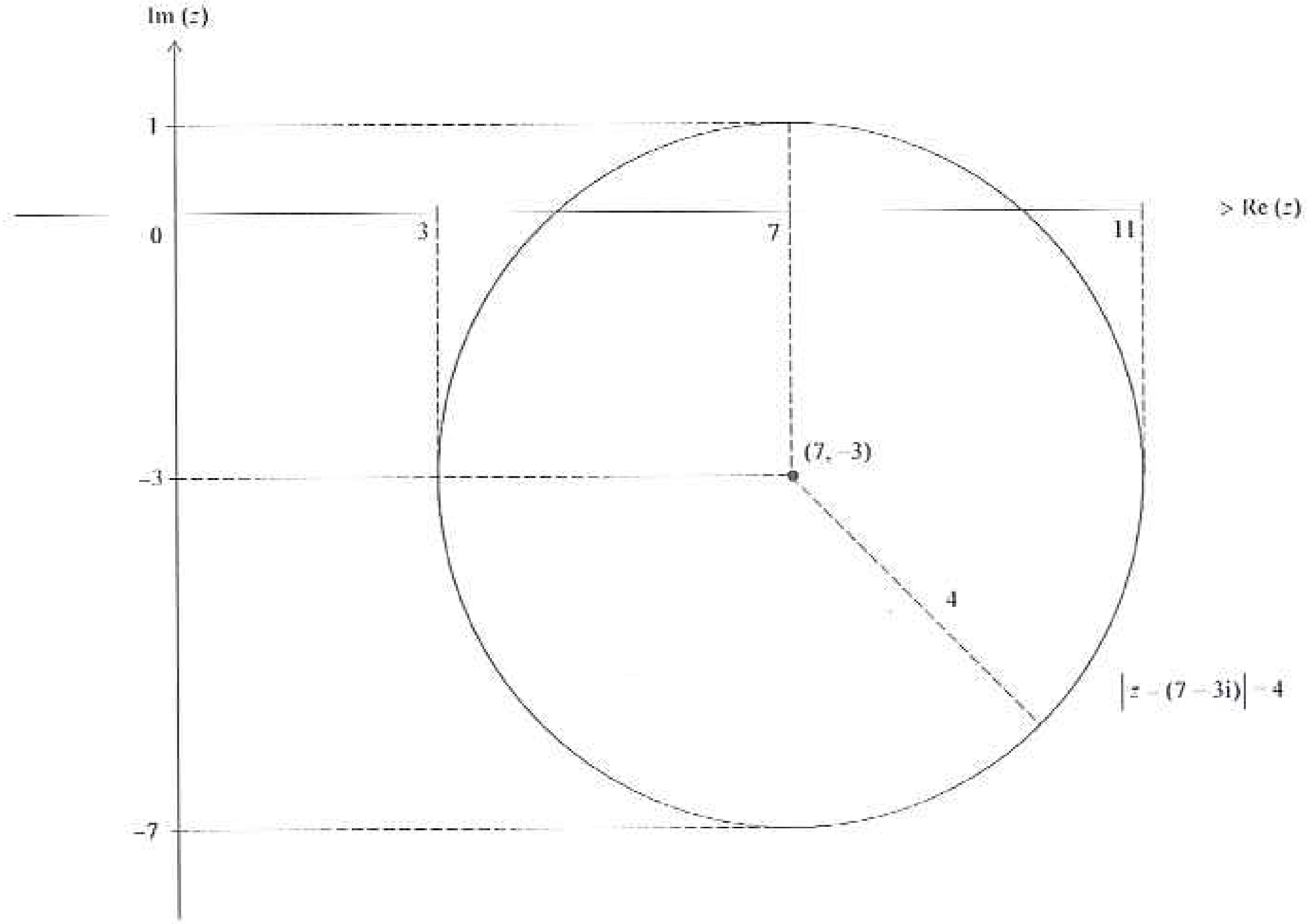
$$\frac{1}{24}\ln 7 = d_1.$$

$$\frac{1}{24}\ln\left|\frac{4+3u}{4-3u}\right| = t + \frac{1}{24}\ln 7.$$

$$t = \frac{1}{24}\ln\left|\frac{4+3u}{7(4-3u)}\right|$$ #

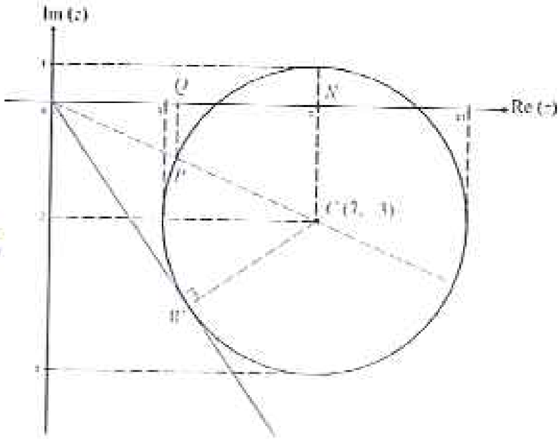
2012 H2 Maths Paper 2

2.(i)



2012 H2 Maths Paper 2.

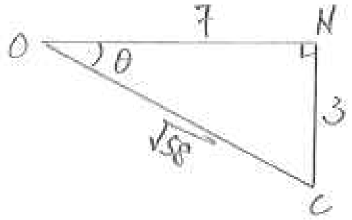
2. (ii)(a) $OC = \sqrt{7^2 + 3^2} = \sqrt{58}$.



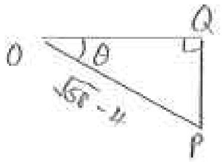
Given $|z|$ is as small as possible,
Then $|z| = OP_1$
$= OC - 4$
$= \sqrt{58} - 4$ units.

(ii)(b) In ΔOCN, we have

$\cos\theta = \frac{7}{\sqrt{58}}$
$\sin\theta = \frac{3}{\sqrt{58}}$.



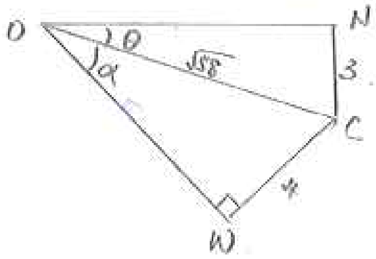
In ΔOPQ, we have
$PQ = OP \sin\theta$
$= (\sqrt{58} - 4)\left(\frac{3}{\sqrt{58}}\right)$
$= \left(3 - \frac{12}{\sqrt{58}}\right) = 3 - \frac{6\sqrt{58}}{29}$



$OQ = OP \cos\theta$
$= (\sqrt{58} - 4)\left(\frac{7}{\sqrt{58}}\right)$
$= \left(7 - \frac{28}{\sqrt{58}}\right) = 7 - \frac{14\sqrt{58}}{29}$

$\therefore$ At point P, $z = \left(7 - \frac{14\sqrt{58}}{29}\right) - \left(3 - \frac{6\sqrt{58}}{29}\right)i$ #.

(iii). In ΔOCN, we have
$\sin\theta = \frac{3}{\sqrt{58}}$
$\therefore \theta = \sin^{-1}\left(\frac{3}{\sqrt{58}}\right)$.



In ΔOCW, we have
$\sin\alpha = \frac{4}{\sqrt{58}}$
$\alpha = \sin^{-1}\left(\frac{4}{\sqrt{58}}\right)$

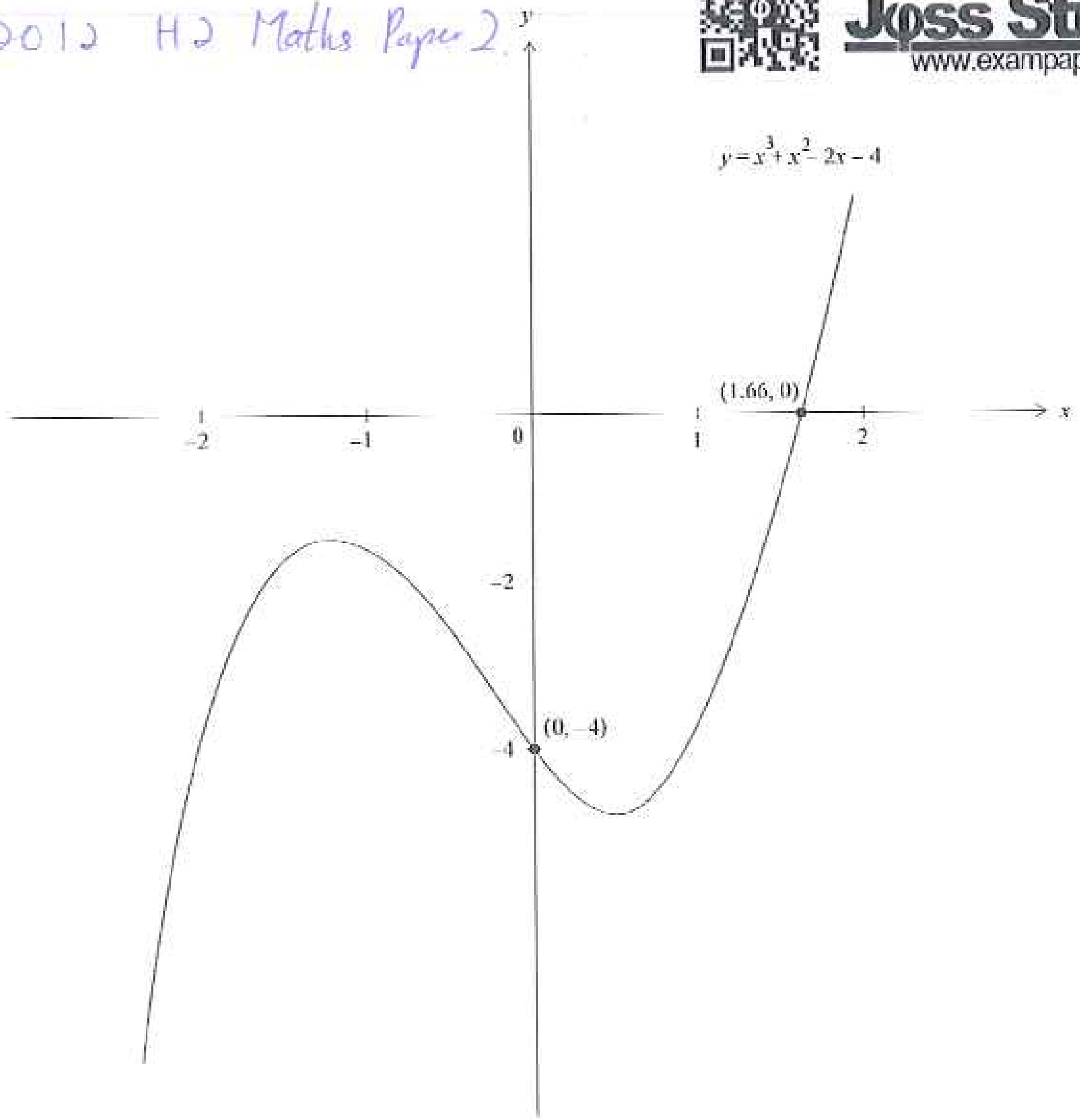
Given $-\pi < \arg z \leq \pi$

$\therefore$ largest value of $\arg z = -\left[\sin^{-1}\left(\frac{3}{\sqrt{58}}\right) + \sin^{-1}\left(\frac{4}{\sqrt{58}}\right)\right]$
$= -0.95787$

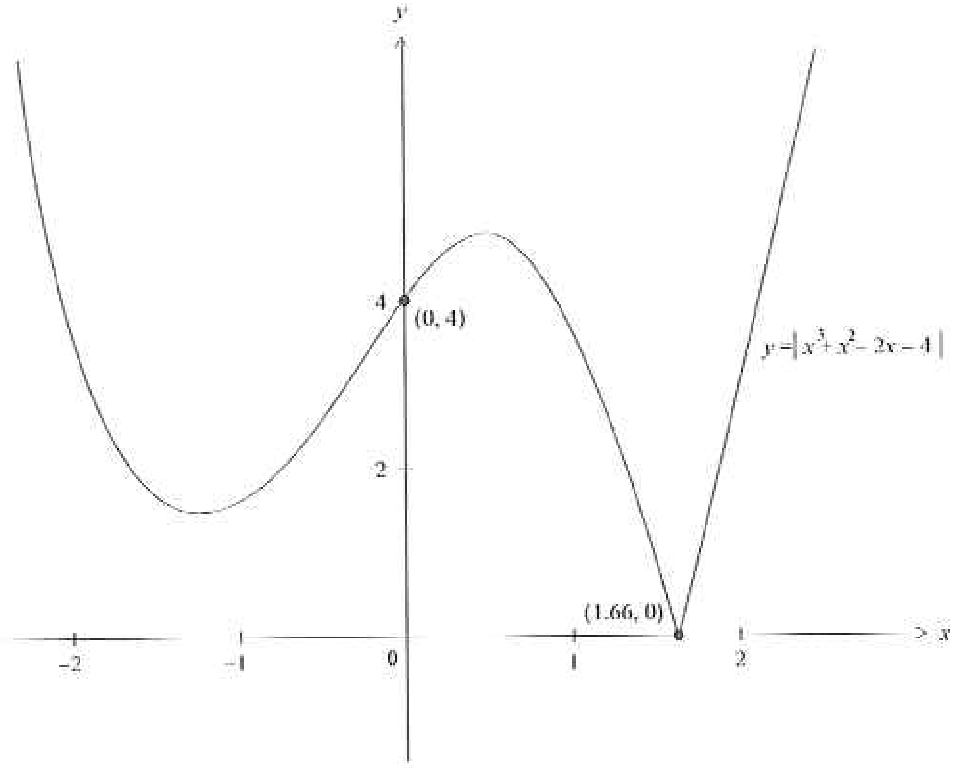
≈ -0.9579 (cor to 4 sig fig) #

2012 H2 Maths Paper 2.

3 (i)



(iv)



2012 H2 Maths Paper 2.

3 (ii) Given that $f(x) = x^3 + x^2 - 2x - 4$.

Given $f(x) = 4$,
From GC, $x = 2$.

$$x^3 + x^2 - 2x - 4 = 4.$$
$$x^3 + x^2 - 2x - 8 = 0.$$
$$(x-2)(x^2 + 3x + 4) = 0.$$

$\therefore x - 2 = 0$ or $x^2 + 3x + 4 = 0$.

$x = 2$.

$$x = \frac{-3 \pm \sqrt{9-16}}{2} = \frac{-3 \pm \sqrt{-7}}{2}.$$

(reject as $\sqrt{-7}$ has no solution.)

$\therefore x = 2$ is the only solution for $f(x)$ #

(iii) Given $(x+3)^3 + (x+3)^2 - 2(x+3) - 4 = 4$.
Let $p = x+3$, we have

$$p^3 + p^2 - 2p - 4 = 4$$
$\therefore p = 2$ (result from part (ii)).
$\therefore x + 3 = 2$.
$x = -1$. #

(iv) Refer to attached drawing.

(v) For $|f(x)| = 4$, we have.

$x^3 + x^2 - 2x - 4 = 4$ or $x^3 + x^2 - 2x - 4 = -4$.

$x^3 + x^2 - 2x - 8 = 0$. $\quad x^3 + x^2 - 2x = 0$.

$x = 2$ is the solution from part (ii) $\quad x(x^2 + x - 2) = 0$.

$x(x+2)(x-1) = 0$.

$x = 0, -2$ or 1.

$\therefore$ Roots of $|f(x)| = 4$ are $-2, 0, 1, 2$ #

2012 H2 Maths Paper 2.

4.

	Deposit
1-Jan-2001	\$100.
1-Feb-2001	\$110.
1-Mar-2001	\$120
⋮	

(i) In n months.., sum of money in account be S_n.

$$S_n = 100 + 110 + 120 + \cdots$$
$$= \frac{n}{2}\left[2(100) + (n-1)10\right].$$

For $S_n > 5000$.

$$\frac{n}{2}\left[200 + (n-1)10\right] > 5000.$$
$$10n^2 + 190n > 10000$$
$$n^2 + 19n - 1000 > 0.$$
$$n < -42.51 \quad \text{or} \quad n > 23.518$$

$\therefore$ No of month to exceed \$5000 = 24 month.
The date will be 1-Dec-2002. #

2012 H2 Maths Paper 2

4(ii)

	Beg	Interest	End
$n=1$	100	0.005	$100(1.005)$
$n=2$	$100(1.005)+100$	0.005	$[100(1.005)+100]\times 1.005$ $= 100(1.005)^2 + 100(1.005)$
$n=3$	$100(1.005)^2+100(1.005)$	0.005	$100(1.005)^3+100(1.005)^2+100(1.005)$
$\vdots$			

$\therefore$ Sum of Mr B's account on the last day of the nth month, X_n

$$= 100(1.005)^n + 100(1.005)^{n-1} + \ldots + 100(1.005)^3 + 100(1.005)^2 + 100(1.005)$$

$$= 100(1.005)\left[1.005^{n-1} + 1.005^{n-2} + \ldots + (1.005)^2 + (1.005)^1 + 1\right].$$

$$= 100(1.005)\frac{1.005^n - 1}{1.005 - 1}$$

$$= 20100(1.005^n - 1) \#$$

$$X_n > 5000.$$

$$20100(1.005^n - 1) > 5000.$$

$$1.005^n > \frac{5000}{20100} + 1.$$

$$n > \lg\left(\frac{50}{201}+1\right) \div \lg(1.005)$$

$$n > 44.54$$

Minimum months for Mr B's account to exceed \$5000 = 45 mths.

where the 45th month falls on Sept-2004 #.

2012 H2 Maths Paper 2.

4 (iii). Given Mr B wanted the value of his account to be \$5000 on 2-Dec 2003.

$= \$4900 + \$100.$

Where \$4900 is the money he earns with interest till End Nov-2003. ($n = 35$).

\$100 is the money he deposit on 1st Day of Dec.

Then. $100\left(1+\frac{r}{100}\right)\left[\frac{\left(1+\frac{r}{100}\right)^{35}-1}{1+\frac{r}{100}-1}\right] = 4900.$

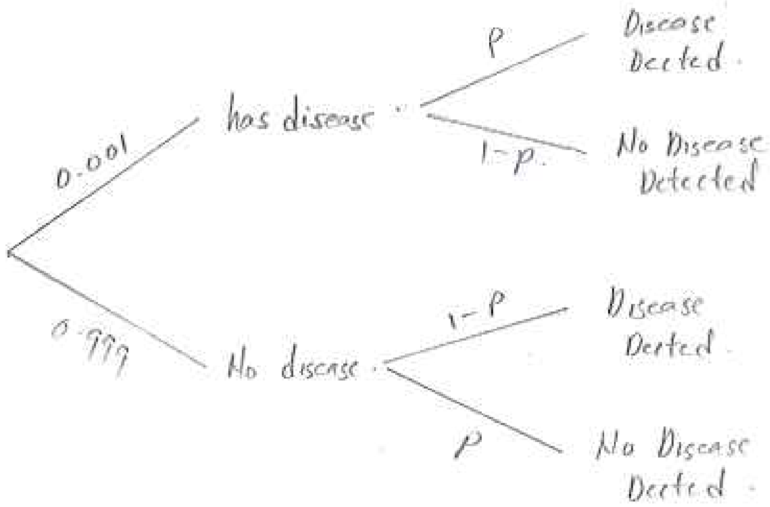
$$\left(1+\frac{r}{100}\right)^{36} - \left(1+\frac{r}{100}\right) = \frac{49}{100}r.$$

$$\left(1+\frac{r}{100}\right)^{36} = 1 + \frac{1}{2}r.$$

Using G.C, we have $r = 1.79576$
$\approx 1.80\%$ (corr to 3 sig fig.)

2012 H2 Maths Paper 2.

5.



(i)(a) Given $p = 0.995$.

$$P(\text{result is positive}) = 0.001 \times p + 0.999(1-p)$$
$$= 0.001 \times 0.995 + 0.999 \times 0.005$$
$$= 5.99 \times 10^{-3}$$

(b) Let event A : the patient has the disease.
event B : the result of the test is +ve.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.001 \times (0.995)}{5.99 \times 10^{-3}} = 0.1661$$
$$\approx 0.166 \text{ (corr to 3 sig fig)}$$

(ii) Given P(patient has the disease given that result of test is positive) = 0.75.

$$\frac{0.001 \times p}{0.001 \times p + 0.999(1-p)} = 0.75$$
$$\frac{p}{p + 999(1-p)} = 0.75$$

$$p = 0.75(999 - 998p)$$
$$p = \frac{0.75 \times 999}{1 + 0.75 \times 998}$$
$$= 0.9996664$$
$$\approx 0.999666 \text{ (corr to 6 dec pl)}$$

2012 H2 Maths Paper 2.

6 (i). $H_0 : \mu = 14.0$ cm.
$H_1 : \mu \neq 14.0$ cm.

(ii). $\bar{X} \sim N(14, \frac{3.8^2}{20})$

For H_0 not to be rejected at the 5% significance level.

$$Z_{2.5\%} < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < Z_{97.5\%}$$

$$-1.96 < \frac{\bar{x}-14}{3.8/\sqrt{20}} < 1.96$$

$$12.33457 < \bar{x} < 15.66543$$

$$\therefore 12.3 < \bar{x} < 15.7 \quad (\text{Corr to 3 Sig fig})$$

$$\therefore \{\bar{x} \in \mathbb{R} : 12.3 < \bar{x} < 15.7\}$$

(iii) If $\bar{x} = 15.8$, at the 5% significance level, H_0 is rejected. There is sufficient evidence to indicate that the squirrels on the island do not have the same mean tail length as the species known to her.

7 (i). P (sis are next to each other)

$$= \frac{14! \times 2!}{15!} = \frac{2}{15} \#$$

(ii). P(the brothers are not all next to one another)

$$= 1 - \frac{13! \times 3!}{15!}$$
$$= \frac{34}{35} \#$$

(iii) P (the sisters are next to each other and the brothers are all next to one another).

$$= \frac{12! \times 2! \times 3!}{15!}$$

$$= \frac{2}{455} \#$$

(iv) Let event A be the sisters are next to each other.
event B be the brothers are all next to another.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$
$$= \frac{2}{15} + \left[1 - \frac{34}{35} \right] - \frac{2}{455}$$
$$= \frac{43}{273} \#$$

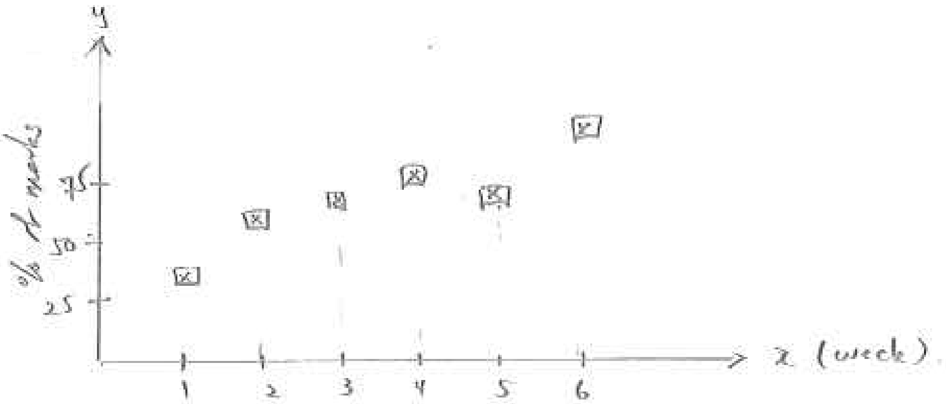
(v) P (sisters are next to each other in a circle)

$$= \frac{13! \times 2!}{14!}$$

$$= \frac{1}{7} \#$$

2012 H2 Maths Paper 2.

8 (i)



(ii) The drop in marks in Week 5, which broke the upward trend of Amy's marks, may be due to that week's practice paper being of a level of higher difficulty.

(iii) A linear model is not appropriate since it is not possible to score more than 100 % marks even as the weeks increase.
A quadratic model is also not appropriate since Amy's performance may not necessarily drop after she reaches a certain peak.

(iv). For $L = 91$, $r = -0.9297441$
≈ -0.929744 (corr to 6 dec pl)

(v) $L = 92$ is the most appropriate value since it's magnitude indicates a stronger negative correlation between x and y than $L = 91$ or $L = 93$.

(vi) Using $L = 92$, from GC
we have; $\ln(92 - y) = 4.104510 - 0.279599x$
$\ln(92 - y) = 4.10 - 0.280x$

$\therefore a = 4.10$, $b = -0.280$.

sub $y = 90$, $\ln(92 - 90) = 4.104510 - 0.279599x$.
$x = 12.2009$
≈ 12.2 (corr to 3 sig fig).

Amy will obtain her 1st mark of at least 90%
at week 13 #

(vii) L represents the mark Amy will likely score in her exam based on her current practice performance.

2012 H2 Maths Paper 2

9 (i) Each voter must be independent and not influenced by other voters in his/her vote.
The probability of each voter supporting the Alliance Party is the same

(ii). Given that $p = 0.15$

$$P(A=3 \text{ or } 4) = P(A=3) + P(A=4)$$
$$= \text{binompdf}(30, 0.15, 3) + \text{binompdf}(30, 0.15, 4)$$
$$= 0.1702593 + 0.2028089$$
$$= 0.3730682$$
$$\approx 0.373 \text{ (corr to 3 sig fig)}$$

(iii) (a) $np = 30(0.55) = 16.5$
$nq = 13.5$
Since $np > 5$ & $nq > 5$, we can approximate the distribution of A with a normal distribution.

(b) Since $p = 0.55$ is more than 0.1,
It is not possible to approximate the distribution of A with a poisson distribution.

2012 H2 Maths Paper 2:

9 (iv). Given $P(A=15) = 0.06864$.

$$^{30}C_{15}\, p^{15}(1-p)^{15} = 0.06864$$

$$[p(1-p)]^{15} = 0.06864 \div {}^{30}C_{15}$$

$$p(1-p) = [0.06864 \div {}^{30}C_{15}]^{1/15}$$
$$p(1-p) = 0.237899$$
$$p(1-p) \approx 0.238 \quad (\text{Corr to 3 sig fig}).$$

$\therefore k = 0.238$.

From G.C, we have
$p = 0.38999$ or $p = 0.61000$

Given that $p < 0.5$
$\therefore p = 0.39$ ((Corr to 2 dec pl).

2012 H2 Maths Paper 2.

10 (i) - A gold coin is found randomly within $1m^2$ area and the average number of gold coins found within a $1m^2$ area is constant and known.

- The number of gold coins found in a $1m^2$ area is independent from the number of gold coins found in another $1m^2$ area.

(ii). Let the random variable X be the no. of gold coins found in 1 square meter, $X \sim Po(0.8)$

$$P(X \geq 3) = 1 - P(X \leq 2)$$
$$= 0.047422$$
$$\approx 0.0474 \quad \text{(corr to 3 sig fig)}$$

(iii) Let the random variable Y be the no of gold coins found in x square meter is. $Y \sim Po(0.8x)$

$$P(Y=1) = 0.2$$
$$e^{-0.8x}(0.8x) = 0.2$$
$$e^{0.8x}x - 4 = 0$$

From G.C, $x = 0.32376$
≈ 0.324 (corr to 3 sig fig)

2012 H2 Maths Paper 2:

10 (iv) $\therefore X_{100} \sim Po(0.8 \times 100)$
$X_{100} \sim Po(80)$.

Since $\lambda = 80$ which is > 10, we can approximate
$X_{100} \sim N(80, 80)$.

$\therefore P(X_{100} \geqslant 90) \xrightarrow[\text{correction}]{\text{continuity}} P(X_{100} > 89.5)$

$= \text{normalcdf}(89.5, e^{99}, 80, \sqrt{80})$

$= 0.144087$

≈ 0.144 ((corr to 3 sig figs).

(v) $X_{50} \sim Po(0.8 \times 50)$
$X_{50} \sim Po(40)$. $\Rightarrow X_{50} \sim N(40, 40)$ approximately since $\lambda > 40$.

Let the number of pottery shards in 1 square metre be $S \sim Po(3)$

$S_{50} \sim Po(3 \times 50)$
$S_{50} \sim Po(150) \Rightarrow S_{50} \sim N(150, 150)$ approximately since $\lambda > 150$.

$\Rightarrow X_{50} + S_{50} \sim N(190, 190)$

$P(X_{50} + S_{50} \geqslant 200) \xrightarrow[\text{correction}]{\text{continuity}} P(X_{50} + S_{50} > 199.5)$
$= 0.24534$
≈ 0.245 ((corr to 3 sig figs)

2012 H2 Maths Paper 2

10 (vi) $X_{50} \sim Po(40)$. $\Rightarrow X_{50} \sim N(40, 40)$. approximately since $\lambda > 40$.

$S_{50} \sim Po(150) \Rightarrow S_{50} \sim N(150, 150)$ approximately since $\lambda > 150$.

$$E(S_{50} - 3X_{50}) = E(S_{50}) - 3E(X_{50})$$
$$= 150 - 3(40)$$
$$= 30$$

$$Var(S_{50} - 3X_{50}) = Var(S_{50}) + 9\,Var(X_{50})$$
$$= 150 + 9(40)$$
$$= 510.$$

$S_{50} - 3X_{50} \sim N(30, 510)$

$$\therefore P(X_{50} + S_{50} \geq 4X_{50}) = P(S_{50} - 3X_{50} \geq 0)$$
$$\xrightarrow[\text{Correction}]{\text{Continuity}} P(S_{50} - 3X_{50} > -0.5)$$
$$= \text{Normalcdf}(-0.5, e^{99}, 30, \sqrt{510})$$
$$= 0.91158$$
$$\approx 0.912 \text{ (Corr to 3 sig fig)}$$