2012 H2 Maths Paper 1 V1.1

1 Let \$$x$ be the cost of a ticket under 16 yrs old.
\$$y$ be the cost of a ticket between 16 and 65 yrs old
\$$z$ be the cost of a ticket Over 65 years old.

Then we have

$$9x + 6y + 4z = 162.03$$
$$7x + 5y + 3z = 128.36$$
$$10x + 4y + 5z = 158.50$$

Using G.C, we have

$$x = \$7.65 \#$$
$$y = \$9.85 \#$$
$$z = \$8.52 \#$$

2 (i). $\int \frac{x^3}{1+x^4}\,dx = \frac{1}{4}\int \frac{4x^3}{1+x^4}\,dx = \frac{1}{4}\ln|1+x^4| + C,$

where C is a constant.

(ii). Let $u = x^2$
$du = 2x\,dx$.

$$\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\int \frac{2x}{1+x^4}\,dx$$
$$= \frac{1}{2}\int \frac{1}{1+u^2}\,du$$
$$= \frac{1}{2}\tan^{-1}u + C$$
$$= \frac{1}{2}\tan^{-1}(x^2) + C \#$$

, where C is a constant.

(iii). $\int_0^1 \left(\frac{x}{1+x^4}\right)^2 dx = 0.18594$ (using G.C)

≈ 0.186 (corr to 3 dec pl).

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3 Given $u_1 = 2$ and $u_{n+1} = \frac{3u_n - 1}{6}$ for $n \geq 1$.

(i) when $n=1$, $u_2 = \frac{3u_1 - 1}{6} = \frac{3(2)-1}{6} = \frac{5}{6}$ #.

when $n=2$, $u_3 = \frac{3u_2 - 1}{6} = \frac{3(\frac{5}{6})-1}{6} = \frac{1}{4}$ #.

(ii) Given $u_n \to l$ as $n \to \infty$

$$\therefore \quad l = \frac{3l - 1}{6}$$
$$6l = 3l - 1.$$
$$3l = -1$$
$$l = -\frac{1}{3} \text{ \#}.$$

(iii). For $l = -\frac{1}{3}$.

Let P_n be the statement s.t.

$$u_n = \frac{14}{3}\left(\frac{1}{2}\right)^n - \frac{1}{3} \quad , n \geq 1.$$

When $n=1$, L.H.S $= u_1 = 2$.

RHS $= \frac{14}{3}\left(\frac{1}{2}\right)^1 - \frac{1}{3} = 2$

$\therefore$ L.H.S = RHS.

P_1 is true.

Assume P_k is true, s.t

$$u_k = \frac{14}{3}\left(\frac{1}{2}\right)^k - \frac{1}{3} \quad , \text{ for some } k \in \mathbb{Z}^+$$

Required to Prove u_{k+1} is true,

$$\Rightarrow u_{k+1} = \frac{14}{3}\left(\frac{1}{2}\right)^{k+1} - \frac{1}{3}$$

$$\begin{aligned}
\text{L.H.S} &= u_{k+1} \\
&= \frac{1}{6}(3u_k - 1) \\
&= \frac{1}{6}\left[14\left(\frac{1}{2}\right)^k - 1 - 1\right] \\
&= \frac{14}{6}\left(\frac{1}{2}\right)^k - \frac{1}{3} \\
&= \left(\frac{14}{3}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^k - \frac{1}{3} \\
&= \frac{14}{3}\left(\frac{1}{2}\right)^{k+1} - \frac{1}{3} = \text{RHS}.
\end{aligned}$$

ie P_{k+1} is true whenever P_k is true.

Since P_1 is true, by Mathematical Induction, P_n is true for all $n \in \mathbb{Z}^+$.

4 (i) Given $\angle BAC = \theta$ rad and $\angle ABC = \frac{3}{4}\pi$.

$\angle BCA = \pi - \theta - \frac{3}{4}\pi$ (sum of $\angle$s in Δ)

$= \frac{\pi}{4} - \theta$

Using Sine Rule in ΔABC.

$$\frac{AC}{\sin\angle ABC} = \frac{AB}{\sin\angle BCA}$$

$$AC = \frac{1}{\sin(\frac{\pi}{4}-\theta)} \times \sin\frac{3\pi}{4}$$

$$= \frac{1}{\sin\frac{\pi}{4}\cos\theta - \cos\frac{\pi}{4}\sin\theta} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta\right]}$$

$$= \frac{1}{\cos\theta - \sin\theta} \quad \text{(shown)}.$$

(ii) Given that θ is sufficiently small,

$\sin\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{\theta^2}{2}$.

$$AC = (\cos\theta - \sin\theta)^{-1}$$

$$\approx \left(1 - \frac{\theta^2}{2} - \theta\right)^{-1}$$

$$= \left(1 - \theta - \frac{\theta^2}{2}\right)^{-1}$$

$$= 1 + \frac{-1}{1}\left(-\theta - \frac{\theta^2}{2}\right)^1 + \frac{(-1)(-2)}{2!}\left(-\theta - \frac{\theta^2}{2}\right)^2 + \ldots$$

$$= 1 + \theta + \frac{1}{2}\theta^2 + \theta^2 + \ldots$$

$$= 1 + \theta + \frac{3}{2}\theta^2 + \ldots$$

$$\approx 1 + \theta + \frac{3}{2}\theta^2$$

$\therefore a = 1$, $b = \frac{3}{2}$ #.

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5. Given $\vec{OA} = \underset{\sim}{a} = \begin{pmatrix}1\\-1\\1\end{pmatrix}$. $\vec{OB} = \underset{\sim}{b} = \begin{pmatrix}1\\2\\0\end{pmatrix}$.

$$\vec{OC} = \underset{\sim}{c} = \lambda \underset{\sim}{a} + \mu \underset{\sim}{b}$$
$$= \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix} + \mu\begin{pmatrix}1\\2\\0\end{pmatrix}$$
$$= \begin{pmatrix}\lambda+\mu\\-\lambda+2\mu\\\lambda\end{pmatrix}$$

(i) Given area of $\Delta OAC = \sqrt{126}$.

$$\frac{1}{2}\left|\vec{OA} \times \vec{OC}\right| = \sqrt{126}$$
$$\left|\begin{pmatrix}1\\-1\\1\end{pmatrix} \times \begin{pmatrix}\lambda+\mu\\-\lambda+2\mu\\\lambda\end{pmatrix}\right| = 2\sqrt{126}$$

$$\left|\begin{pmatrix}-\lambda+\lambda-2\mu\\-(\lambda-\lambda-\mu)\\-\lambda+2\mu+\lambda+\mu\end{pmatrix}\right| = 2\sqrt{126}$$

$$\left|\begin{pmatrix}-2\mu\\\mu\\3\mu\end{pmatrix}\right| = 2\sqrt{126}$$

$$\sqrt{4\mu^2+\mu^2+9\mu^2} = 2\sqrt{126}$$
$$14\mu^2 = 4\times126$$
$$\mu^2 = 36$$

$\mu = 6$ or $\mu = -6$ (reject as μ are positive constants.)

(ii) Given $\mu = 4$, then $\vec{OC} = \begin{pmatrix}\lambda+4\\-\lambda+8\\\lambda\end{pmatrix}$

Given $OC = 5\sqrt{3}$

$$\sqrt{(\lambda+4)^2+(-\lambda+8)^2+\lambda^2} = 5\sqrt{3}$$
$$\lambda^2+8\lambda+16+\lambda^2-16\lambda+64+\lambda^2 = 25\times3$$
$$3\lambda^2-8\lambda+5 = 0$$
$$(3\lambda-5)(\lambda-1) = 0$$
$$\lambda = \frac{5}{3} \text{ or } \lambda = 1$$

When $\lambda = \frac{5}{3}$, $\vec{OC} = \begin{pmatrix}17/3\\19/3\\5/3\end{pmatrix} = \frac{1}{3}\begin{pmatrix}17\\19\\5\end{pmatrix}$ #

When $\lambda = 1$, $\vec{OC} = \begin{pmatrix}5\\7\\1\end{pmatrix}$ #

Possible Coordinates of C are $\left(\frac{17}{3}, \frac{19}{3}, \frac{5}{3}\right)$ and $(5, 7, 1)$.

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6 Given $Z = 1 + ic$, where c is a non-zero real nos.

(i) $Z^3 = (1+ic)(1+ic)(1+ic)$
$= [1-c^2+2ci][1+ic]$
$= 1-c^2 + c[1-c^2]i + 2ci - 2c^2$
$= 1-3c^2 + [3c - c^3]i$

(ii) Given that Z^3 is real:
then $3c - c^3 = 0$
$c[c^2-3] = 0$
$c = 0$ or $c = -\sqrt{3}$ or $c = \sqrt{3}$.
Since c is non-zero real nos.
$\therefore\ c = -\sqrt{3}$ or $\sqrt{3}$.

Possible values of Z:
$Z = 1 - \sqrt{3}i$ or $Z = 1 + \sqrt{3}i$.

(iii) Using $c<0$, $Z = 1 - \sqrt{3}i$
$= 2e^{i(-\frac{\pi}{3})}$

For $|Z^n| > 1000$.
$2^n |e^{i(-n\pi/3)}| > 1000$.
Note $|e^{i\theta}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$.
$\therefore\ 2^n > 1000$.
Since $2^9 = 512$, $2^{10} = 1024$
$\Rightarrow\ n > 9$
$\therefore$ smallest integer $n = 10$ #.

Modulus of $Z^n = 2^{10} = 1024$.
Argument of $Z^n = \frac{2\pi}{3}$.

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7 (i) Given $g(x) = \frac{x+k}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$.
$= 1 + \frac{k+1}{x-1}$ where k is a constant, $k \neq -1$.

$\therefore R_g = \mathbb{R}/\{1\}$ ——— ⊛

Let $y = g(x)$ s.t $x = g^{-1}(y)$.

$y = \frac{x+k}{x-1}$.

$xy - y = x + k$.

$xy - x = y + k$.

$x(y-1) = y + k$.

$x = \frac{y+k}{y-1}$

$\therefore g^{-1}(y) = \frac{y+k}{y-1}$, $y \in \mathbb{R}$, $y \neq 1$. {from ⊛}.

$\therefore g^{-1}(x) = \frac{x+k}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$.

$\therefore g(x) = g^{-1}(x)$.

It shows that g is self-inverse.

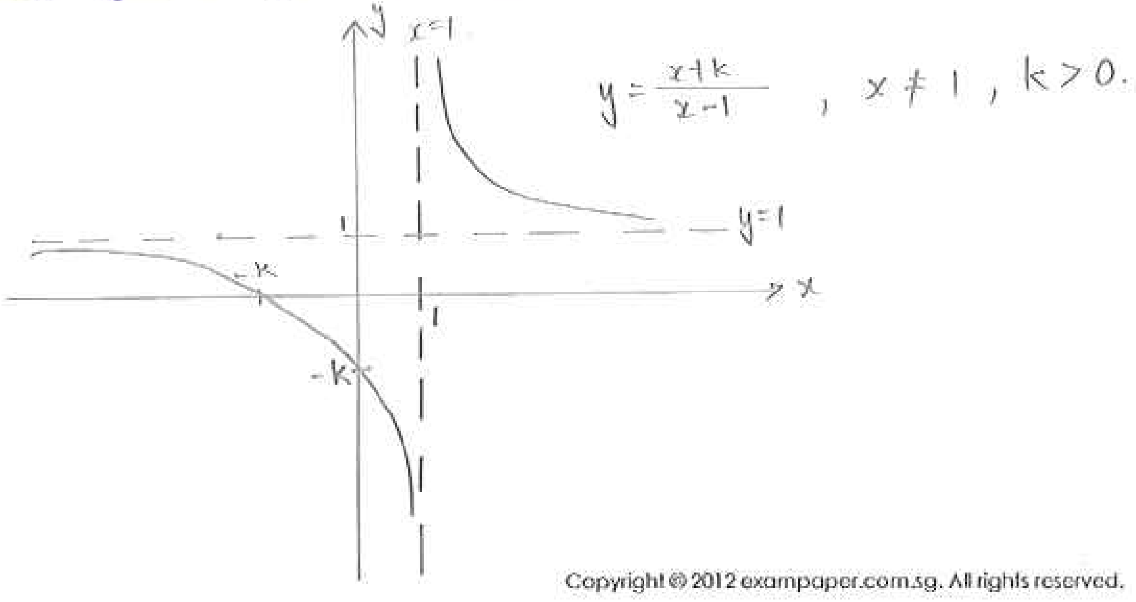
(ii) Given that $k > 0$.

vertical asymptotes : $x = 1$.

horizontal asymptotes : $y = 1$.

when $x = 0$, $g(0) = -k$.

when $g(x) = 0$, $x = -k$.



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7 (iii) Equation of line of symmetry in (ii) : $y = x$.

$f(x) = \frac{1}{x} \rightarrow g(x) = \frac{x+k}{x-1} = 1 + \frac{k+1}{x-1} = (k+1)f(x-1) + 1$

Step 1 : A translation of the graph $f(x)$ along the x-axis by 1 unit to the right.

Step 2 : Stretch the graph of $f(x-1)$ by a factor $(k+1)$ parallel to the y-axis.

Step 3 : A translation of the graph $(k+1)f(x-1)$ along the y-axis by 1 unit upwards.

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8 $\quad x - y = (x+y)^2$.

(i) Diff wrt x,

$$1 - \frac{dy}{dx} = 2(x+y)\left[1 + \frac{dy}{dx}\right].$$

$$1 - \frac{dy}{dx} = 2(x+y) + 2(x+y)\frac{dy}{dx}.$$

$$[2(x+y)+1]\frac{dy}{dx} = 1 - 2x - 2y.$$

$$\frac{dy}{dx} = \frac{1-2x-2y}{2x+2y+1}$$

$$\frac{dy}{dx} = \frac{-(2x+2y+1)+2}{2x+2y+1}.$$

$$\frac{dy}{dx} = -1 + \frac{2}{2x+2y+1}.$$

$$1 + \frac{dy}{dx} = \frac{2}{2x+2y+1} \quad \text{(shown).} \quad \text{—— (1)}$$

(ii). Diff (1) wrt x

$$\frac{d^2y}{dx^2} = -\frac{2}{(2x+2y+1)^2}\left[2 + 2\frac{dy}{dx}\right]$$

$$= -\frac{4}{(2x+2y+1)^2} \times \left[1 + \frac{dy}{dx}\right].$$

$$= -\frac{4}{(2x+2y+1)^2} \times \frac{2}{(2x+2y+1)}.$$

$$= -\frac{8}{(2x+2y+1)^3}$$

$$= -\left(1 + \frac{dy}{dx}\right)^3 \qquad \{\text{use result (1)}\}.$$

(iii). Given C has only one turning pt.

Sub $\frac{dy}{dx} = 0$,

ie. $\frac{d^2y}{dx^2} = -(1+0)^3$

$= -1 < 0$.

The turning pt is a max pt.

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9(i). Given $\vec{OA} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$ $\vec{OB} = \begin{pmatrix} -1 \\ -8 \\ 1 \end{pmatrix}$

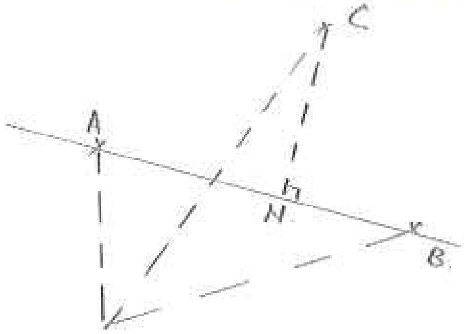
Vector eqn of the line AB:

$$\underset{\sim}{r} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} + \lambda \left[\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} - \begin{pmatrix} -1 \\ -8 \\ 1 \end{pmatrix} \right]$$, where λ is a parameter.

$$\underset{\sim}{r} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} + \lambda \begin{bmatrix} 8 \\ 16 \\ 8 \end{bmatrix}$$

$$\therefore \underset{\sim}{r} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$, where $\mu = 8\lambda$, μ is a parameter

(ii). Given $\vec{OC} = \begin{pmatrix} 1 \\ 8 \\ 3 \end{pmatrix}$.



Since N lies on AB,

$$\vec{ON} = \begin{pmatrix} 7+\mu \\ 8+2\mu \\ 9+\mu \end{pmatrix}.$$

$$\vec{CN} = \begin{pmatrix} 7+\mu \\ 8+2\mu \\ 9+\mu \end{pmatrix} - \begin{pmatrix} 1 \\ 8 \\ 3 \end{pmatrix} = \begin{pmatrix} 6+\mu \\ 2\mu \\ 6+\mu \end{pmatrix}$$

Given CN $\perp$ AB, then

$$\begin{pmatrix} 6+\mu \\ 2\mu \\ 6+\mu \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 0.$$

$$6+\mu + 4\mu + 6+\mu = 0.$$

$$6\mu = -12$$

$$\mu = -2.$$

$$\therefore \vec{ON} = \begin{pmatrix} 5 \\ 4 \\ 7 \end{pmatrix}$$

$$\vec{AN} = \vec{ON} - \vec{OA} = \begin{pmatrix} 5 \\ 4 \\ 7 \end{pmatrix} - \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \\ -2 \end{pmatrix} = -2 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$\vec{NB} = \vec{OB} - \vec{ON} = \begin{pmatrix} -1 \\ -8 \\ 1 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} -6 \\ -12 \\ -6 \end{pmatrix} = -6 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$\frac{AN}{NB} = \frac{|\vec{AN}|}{|\vec{NB}|} = \frac{2 \left| \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \right|}{6 \left| \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \right|} = \frac{1}{3} \#$$

AN : NB = 1 : 3.

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9 (iii) Let C' be the image of pt C under reflection in the line AB. Then

$$\vec{OM} = \frac{1}{2}\left[\vec{OC} + \vec{OC'}\right]$$

$$2\begin{pmatrix}5\\4\\7\end{pmatrix} = \begin{pmatrix}1\\8\\3\end{pmatrix} + \vec{OC'}$$

$$\vec{OC'} = \begin{pmatrix}9\\0\\11\end{pmatrix}.$$

$$\vec{AC'} = \vec{OC'} - \vec{OA} = \begin{pmatrix}9\\0\\11\end{pmatrix} - \begin{pmatrix}7\\8\\9\end{pmatrix} = \begin{pmatrix}2\\-8\\2\end{pmatrix} = 2\begin{pmatrix}1\\-4\\1\end{pmatrix}.$$

Equation of the line AC' {which is a reflection of the line AC in the line AB}.

$$\underset{\sim}{r} = \begin{pmatrix}7\\8\\9\end{pmatrix} + \alpha\begin{pmatrix}1\\-4\\1\end{pmatrix}. \quad , \text{ where } \alpha \text{ is a parameter.}$$

Let $\underset{\sim}{r} = \begin{pmatrix}x\\y\\z\end{pmatrix}$,

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}7\\8\\9\end{pmatrix} + \alpha\begin{pmatrix}1\\-4\\1\end{pmatrix}.$$

$\therefore$ Cartesian Equation of the line AC'

$$\frac{x-7}{1} = \frac{y-8}{-4} = \frac{z-9}{1}$$

$$x-7 = \frac{8-y}{4} = z-9 \;\#$$

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10 (i) Given volume of the model, $V = k\ cm^3$, where k is a constant

$$\frac{1}{2} \times \frac{4}{3}\pi r^3 + \pi r^2 h = k$$

$$2\pi r^3 + 3\pi r^2 h = 3k$$

$$h = \frac{3k - 2\pi r^3}{3\pi r^2} \quad \text{——— (1)}$$

Let $A\ cm^2$ be the external surface area of the model

$$A = \frac{1}{2} \times 4\pi r^2 + 2\pi r h + \pi r^2$$

$$= 3\pi r^2 + 2\pi r h$$

$$= 3\pi r^2 + 2\pi r \left(\frac{3k - 2\pi r^3}{3\pi r^2} \right) \quad \{ \text{use (1)} \}$$

$$= 3\pi r^2 + \frac{2}{3}\left[\frac{3k - 2\pi r^3}{r} \right]$$

$$= 3\pi r^2 + 2k r^{-1} - \frac{4}{3}\pi r^2$$

$$= \frac{5}{3}\pi r^2 + 2k r^{-1} \quad \text{——— (2)}$$

$$\frac{dA}{dr} = \frac{10}{3}\pi r - 2k r^{-2}$$

$$\frac{d^2A}{dr^2} = \frac{10}{3}\pi + 4k r^{-3}$$

When $\frac{dA}{dr} = 0$,

$$\frac{10}{3}\pi r - 2k r^{-2} = 0$$

$$\frac{10}{3}\pi r = \frac{2k}{r^2}$$

$$r^3 = \frac{3k}{5\pi}$$

$$r = \sqrt[3]{\frac{3k}{5\pi}}\ cm$$

when $r = \sqrt[3]{\frac{3k}{5\pi}}$ cm, $\frac{d^2A}{dr^2} = \frac{10}{3}\pi + 4k\left(\frac{5\pi}{3k}\right)$

$$= \frac{10}{3}\pi + \frac{20}{3}\pi$$

$$= 10\pi > 0.$$

$\therefore$ $r = \sqrt[3]{\frac{3k}{5\pi}}$ cm gives minimum external surface area.

and $h = \frac{1}{3\pi}\left(\frac{3k}{5\pi}\right)^{-\frac{2}{3}}\left[3k - 2\pi\left(\frac{3k}{5\pi}\right)\right]$

$$= \frac{1}{3\pi}\left(\frac{3k}{5\pi}\right)^{-\frac{2}{3}}\left(\frac{9k}{5}\right)$$

$$= \frac{3k}{5\pi}\left(\frac{3k}{5\pi}\right)^{-\frac{2}{3}} = \left(\frac{3k}{5\pi}\right)^{1/3} = \sqrt[3]{\frac{3k}{5\pi}}\ cm \ \#$$

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10 (ii). Given $k = 200 \text{ cm}^3$. and $A = 180 \text{ cm}^2$.

From ②.

$$180 = \frac{5}{3}\pi r^2 + \frac{400}{r}.$$

$$180r = \frac{5}{3}\pi r^3 + 400.$$

$$\frac{5}{3}\pi r^3 - 180r + 400 = 0.$$

Using GC; we have.

$$r = -6.758 \quad \text{or} \quad r = 3.7215 \quad \text{or} \quad r = 3.037$$

Since $r > 0$, two possible values of r:

$r \approx 3.72$ cm or 3.04 cm $_{*}$ (corr to 3 sig fig).

sub $r = 3.7215$ into ①

$$h = \left[3 \times 200 - 2\pi(3.7215)^3\right] / 3\pi(3.7215)^2$$

$$= 2.1156$$

and sub $r = 3.037$ into ①

$$h = \left[3 \times 200 - 2\pi(3.037)^3\right] / 3\pi(3.037)^2$$

$$= 4.8775$$

Given $r < h$, $\therefore$ $r = 3.04$ cm and $h \approx 4.88$ cm $_{*}$ ((corr to 3 sig fig)

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11 Given $x = \theta - \sin\theta$, $y = 1 - \cos\theta$, $0 \le \theta \le 2\pi$.

(i) $\frac{dx}{d\theta} = 1 - \cos\theta$ $\qquad \frac{dy}{d\theta} = \sin\theta$.

$$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{\sin\theta}{1-\cos\theta}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$

$$= \cot\frac{\theta}{2} \text{ (shown).}$$

Note:
$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$
$1-\cos\theta = 2\sin^2\frac{\theta}{2}$.

When $\theta = \pi$, $\frac{dy}{dx} = \frac{1}{\tan\frac{\theta}{2}} = 0$.

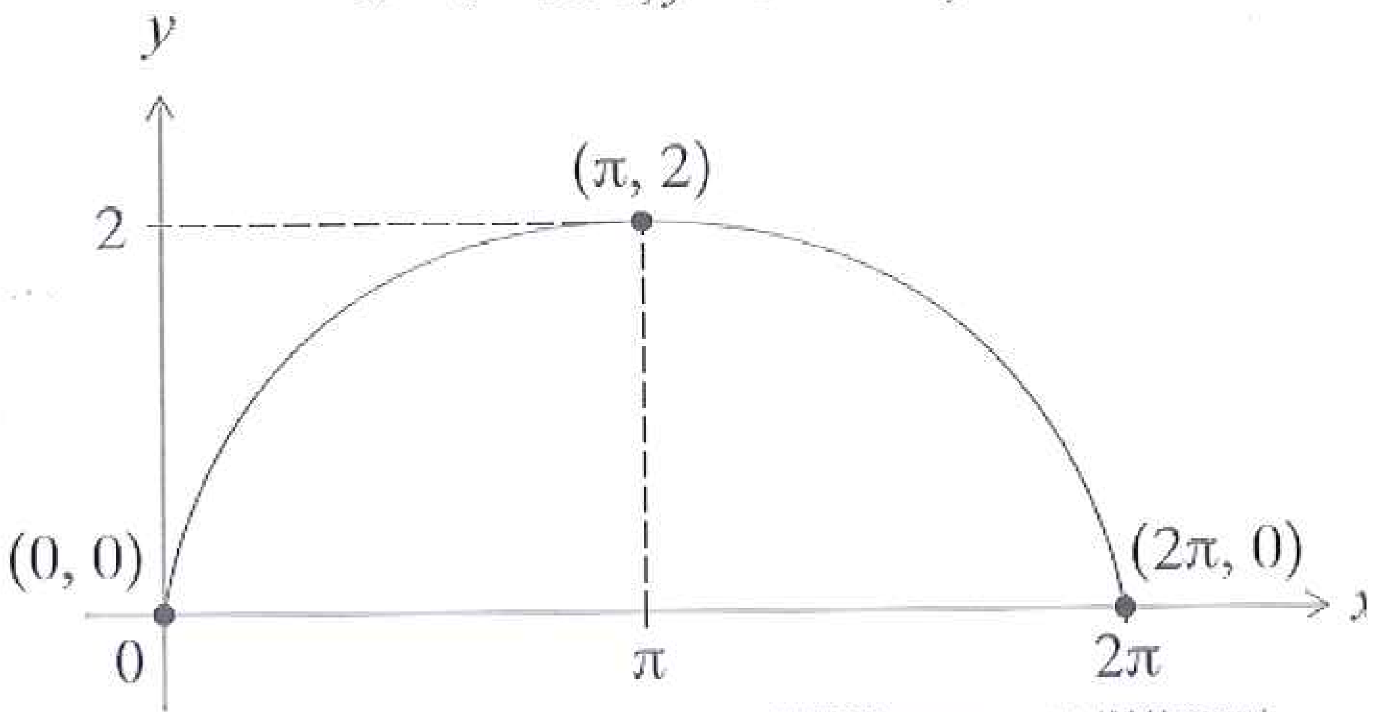
$\therefore$ at $\theta = \pi$ $\frac{dy}{dx} = 0$

As $\theta \to 2\pi$, $\frac{dy}{dx}$ decreases from 0 to $-\infty$. (tends to a vertical line with a negative gradient)

As $\theta \to 0$, $\frac{dy}{dx}$ increases from 0 to $+\infty$ (tends to a vertical line with a positive gradient

(ii)

$x = \theta - \sin\theta, y = 1 - \cos\theta, 0 \le \theta \le 2\pi$



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11 (iii) Area of the region bounded by C and the x-axis.

$= \int_0^{2\pi} y \, dx.$

$= \int_0^{2\pi} (1-\cos\theta)(1-\cos\theta)\, d\theta.$

$= \int_0^{2\pi} (1 - 2\cos\theta + \cos^2\theta)\, d\theta.$

$= \int_0^{2\pi} \left[1 - 2\cos\theta + \frac{1}{2}\cos 2\theta + \frac{1}{2} \right] d\theta.$

$= \left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta \right]_0^{2\pi}$

$= \left[\frac{3}{2}(2\pi) - 0 + 0 - 0 + 0 - 0 \right]$

$= 3\pi$ units^2 #

(iv) A point P with $\theta = p$.

$x = p - \sin p \quad , \quad y = 1 - \cos p.$

Gradient of normal at $p = -\frac{1}{\cot p/2} = -\tan\frac{p}{2}.$

Equation of Normal at P:

$y - 1 + \cos p = -\tan\frac{p}{2}(x - p + \sin p).$

When $y = 0$,

$-(1-\cos p) = -\tan\frac{p}{2}(x - p + \sin p).$

$2\sin^2\frac{p}{2} = \frac{\sin\frac{p}{2}}{\cos\frac{p}{2}}(x - p + 2\sin\frac{p}{2}\cos\frac{p}{2}).$

$2\sin\frac{p}{2}\cos\frac{p}{2} = x - p + 2\sin\frac{p}{2}\cos\frac{p}{2}.$

$0 = x - p$

$x = p$

$\therefore$ Normal to C at P crosses the x-axis at $(p, 0)$ # (shown)