2012 H1 Maths Paper V 1.1

1 $3e^{2x} = 4(e^{-2x} - 1)$.

Let $u = e^{2x}$

$3u = 4\left(\frac{1}{u} - 1\right)$.

$3u^2 = 4 - 4u$

$3u^2 + 4u - 4 = 0$.

$(3u-2)(u+2) = 0$

$\therefore u = \frac{2}{3}$, $u = -2$.

$e^{2x} = \frac{2}{3}$, $e^{2x} = -2$ (reject cos $e^{2x} > 0$ for all real values of x)

$2x = \ln\left(\frac{2}{3}\right)$

$x = \frac{1}{2}\ln\left(\frac{2}{3}\right)$ #

2. Given total length of the fencing $= 100$ m.

$x + y + x + x + 20 + x + y = 100$ m.

$4x + 2y = 80$.

$2x + y = 40$.

$y = 40 - 2x$. —— ①

Given Area of rect HBCG $= 3 \times$ Area of rect DEFG.

$y \times (x + 20) = 3 \times x(20)$ —— ②.

sub ① into ②.

$(40-2x)(x+20) = 60x$.

$40x + 800 - 2x^2 - \cancel{40x} = 60x$.

$2x^2 + 60x - 800 = 0$

$x^2 + 30x - 400 = 0$ (shown).

$(x+40)(x-10) = 0$.

$x = -40$ or $x = 10$.

Since $x > 0$, $\therefore x = 10$ cm is the solution.

$HF = y + x = (40 - 2x) + x$

$= 40 - x$.

$= 40 - 10$

$HF = 30$ cm #.

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3 Given curve $y = k^2 - x^2$ —— ①
Line $y = \frac{3}{4}k^2$ —— ②. where k is a positive constant.

$$\frac{3}{4}k^2 = k^2 - x^2$$
$$x^2 = \frac{k^2}{4}$$
$$x = -\frac{k}{2} \quad \text{or} \quad x = \frac{k}{2}$$

Area of the finite region between C and L

$$= \int_{-k/2}^{k/2} \left[(k^2 - x^2) - \frac{3}{4}k^2 \right] dx.$$

$$= \int_{-k/2}^{k/2} \left[\frac{1}{4}k^2 - x^2 \right] dx.$$

$$= \left[\frac{1}{4}k^2 x - \frac{x^3}{3} \right]_{-k/2}^{k/2}.$$
$$= \left[\frac{k^3}{8} - \frac{k^3}{24} + \frac{k^3}{8} - \frac{k^3}{24} \right]$$
$$= \frac{1}{6}k^3 \#$$

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4(i)(a) $\frac{d}{dx}\left[2\ln(3x+2)\right] = 2\times\frac{1}{3x+2}\times 3. = \frac{6}{3x+2}.$

(b) $\frac{d}{dx}\left[\frac{4}{2x+1}\right] = -\frac{4}{(2x+1)^2}\times 2. = -\frac{8}{(2x+1)^2}$

(ii). $\int_2^4 \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx = \int_2^4 (x-2+x^{-1})\,dx.$

$= \left[\frac{x^2}{2} - 2x + \ln|x|\right]_2^4$

$= 8 - 8 + \ln 4 - 2 + 4 - \ln 2.$

$= 2 + \ln 2$ #

5. (i) Refer to attached (Graph)

(ii). Using G.C at $x = 1.5$, $\frac{dy}{dx} = -1.039484$
≈ -1.0395 ((corr to 4 dec pl)

(iii) when $x = 1.5$, $y = 2^{1.5} - (1.5)^2.$

Equation of the tangent:
$y - (2^{1.5} - (1.5)^2) = (-1.039484)(x - 1.5).$
$y = -0.71421x + (1.039484)(1.5) + (2^{1.5} - (1.5)^2)$
$y = -1.039484x + 2.13765$
$y \approx -1.0395x + 2.1377$ (corr to 4 dec pl).

(iv) When $x = 0$, $y = 2.1377$
Coordinates of A is $(0, 2.1377)$.

When $y = x$, $x = -1.039484x + 2.13765$
$x = 1.04813$
Coordinates of B is $(1.04813, 1.04813)$.
$AB = \sqrt{(1.04813-0)^2 + (1.04813 - 2.1377)^2}$
$= 1.51186$
≈ 1.51 (corr to 3 sig fig). #

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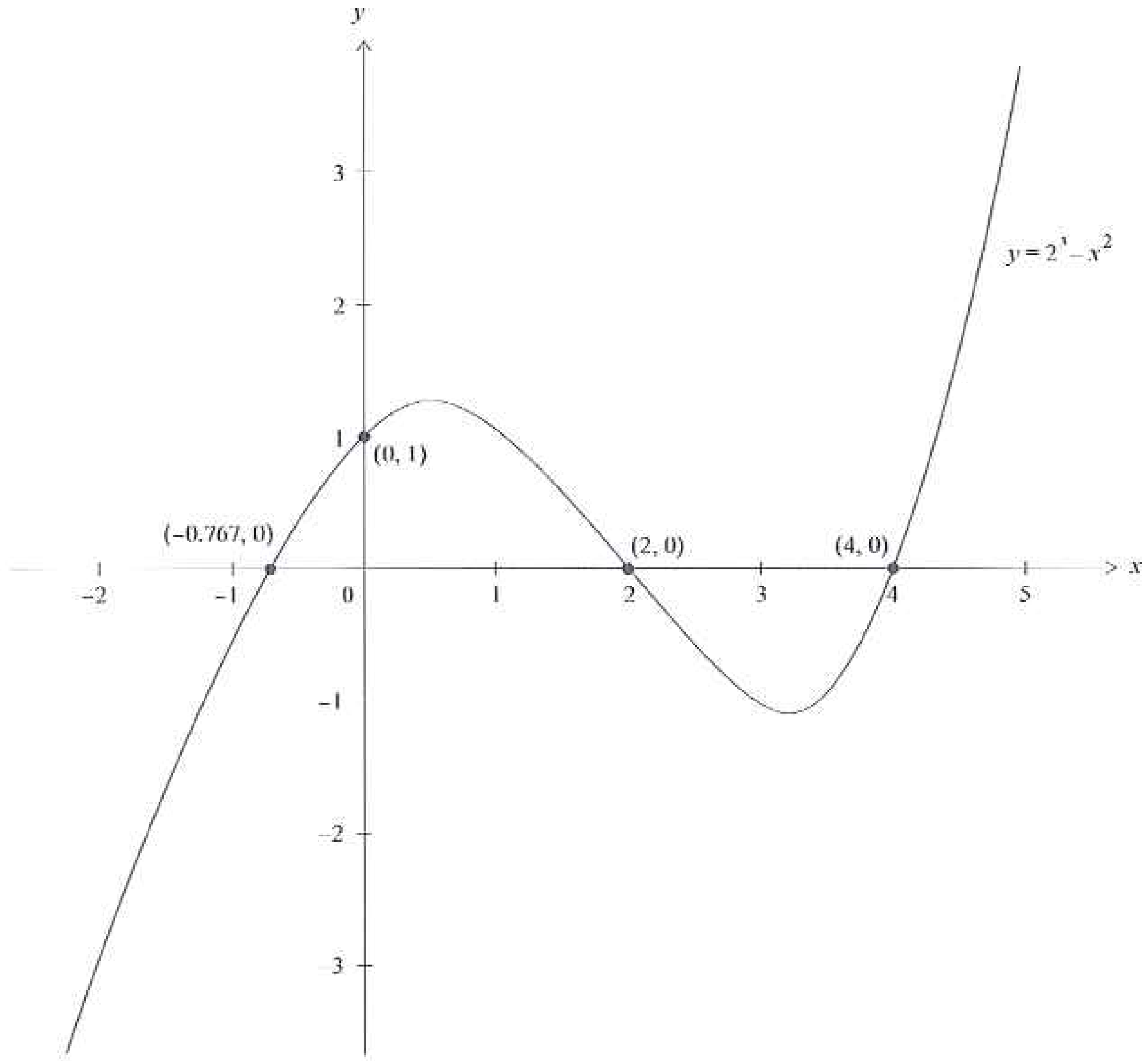
5 (i)



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6 (i) Systematic sampling is a systematic way of obtaining a sample, of a predetermined size ~~that~~ $1/k$ of the population size, from the population which involves selecting every kth member, starting from a randomly selected member as the start point.

(ii) An advantage of this procedure is that it is simple and easy to implement.

A disadvantage of this procedure is that the sample obtained during the survey period may not be represent the actual population of adults which may contain a hidden periodic bias, since a large proportion of working adults may not visit the supermarket at midday

(iii) The researcher might choose to conduct his survey at a busier time outside the supermarket when adults are not working.

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7. (i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A).P(B)$ { Given A & B are independent }.

$\frac{5}{9} = p + p - p(p)$

$\therefore p^2 - 2p + \frac{5}{9} = 0.$

(ii) $p = \frac{2 \pm \sqrt{4 - 4(1)(\frac{5}{9})}}{2}$

$= \frac{5}{3}$ (reject). or $\frac{1}{3}$

$\therefore P(A \cap B) = p^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}.$

8 (i) P (voted for A and is male) $= \frac{1}{2} \times \frac{6}{10} = 0.3$ #.

(ii) P(V is female) = P(female voted A) + P(female voted B) + P(female voted C).

$= \frac{1}{2}(0.4) + 0.35 \times 0.6 + 0.15 \times 0.8$

$= 0.53$ #.

(iii) P(V voted for C, given V is male)

$= \frac{P(\text{male voted for } C)}{P(V \text{ is male})} = \frac{0.15 \times 0.2}{1 - 0.53}$

$= 0.063829$

≈ 0.0638 # (Corr to 3 sig fig)

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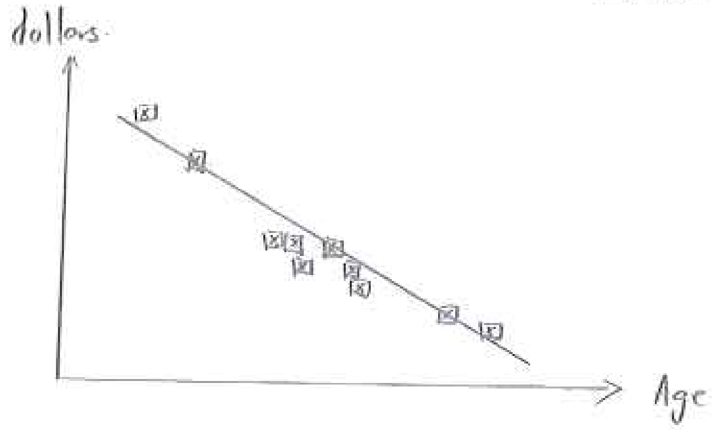
9. (i)



(ii) From G.C, $r = -0.98402$
$= -0.984$.

The r-value of -0.984 indicates that there exists a strong negative linear correlation between the car's age and its selling price i.e. the advertised price decreases as the car's age increases in a linear fashion.

(iii) From G.C, we have

$$y = 183.11584 + (-19.21307)x$$
$$y \approx 183.12 - 19.21x \quad (\text{corr to 2 dec pl.})$$

(iv) When $x = 4$, $y = 183.11584 + (-19.21307)(4)$
$= 106.26$
≈ 106 ((corr to 3 sig fig)

$\therefore$ The advertised prise is 106 hundreds of dollars.

When $x = 9$, $y = 183.11584 - 19.21307(9)$
$= 10.198$
≈ 10.2

$\therefore$ The advertised prise is 10.2 hundreds of dollars.

(v) Since a 4-year-old car falls within the data range, the estimate in (a) is reliable as we are interpolating within the range of data values.
Since there is no data for cars that are more than 9 years old, The estimate in (b) is unreliable as we are extrapolating the data beyond the range of data values.

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10 (i) Let the random variable S be the no. of sunbrite plant that will flower out of 12 plants.

$S \sim Bin(12, 0.8)$.

$P(S=10) = \text{binompdf}(12, 0.8, 10) = 0.28346$

≈ 0.283 ((corr to 3 sig fig)

(ii) $P(S<8) = P(S \leq 7)$

$= \text{binomcdf}(12, 0.8, 7)$

$= 0.072555$

≈ 0.0726 ((corr to 3 sig fig)

(iii) 8 trays of Plants $= 8 \times 12 = 96$ plants.

Let Random variable Y be the no of plants that will flower, out of 96 plants.

$Y \sim Bin(96, 0.8)$.

Since $np > 5$, $nq > 5$

$Y \sim N(96(0.8), 96(0.8)(0.2))$

$\sim N(76.8, 15.36)$ approximately

$P(Y > 75)$

$\xrightarrow{c.c} P(Y > 75.5) = \text{normalcdf}(75.5, e^{99}, 76.8, \sqrt{15.36})$

$= 0.62994$

≈ 0.630 ((corr to 3 sig fig)

(iv) P(at least two of the three gardeners have more than 75 of their plants flowered)

= P(Two gardeners have more than 75 of their plants flowered) + P(Three gardeners have more than 75 of their plants flowered)

$= (0.62994)^2(1-0.62994) \times \frac{3!}{2!} + (0.62994)^3$

$= 0.690522$

≈ 0.691 ((corr to 3 sig fig).#

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11 (i) Unbiased estimates of the population mean = 299 m.

Unbiased estimates of population variance

$= \frac{1}{n-1}\left[\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right]$

$= \frac{1}{100-1}\left[(1240) - \frac{(-60)^2}{100}\right]$

$= 12.161616$

$\approx 12.2\ m^2$ ((corr to 3 sig fig).

(ii) Let $\bar{X}$ be the sample mean of 100 balls of string and the lengths of string per ball.

$\bar{X} \sim N\left(300, \frac{12.2}{100}\right)$

$H_0 : \mu = 300$

$H_1 : \mu < 300$.

$Z_{cal} = \frac{\bar{X} - \mu}{s/\sqrt{n}}$ where $\bar{x} = \frac{\Sigma x}{n} + 300$

$= \frac{-60}{100} + 300$

$= 299.4$

$= \frac{299.4 - 300}{\frac{\sqrt{12.2}}{10}}$

$= -1.71779$

≈ -1.72 ((corr to 3 sig fig).

At 5% significance level, $Z_{critical} = -1.645$.

Since $Z_{cal} < Z_{critical}$, we reject H_0 at 5% significance level, we have sufficient evidence to show that the manager's claim is invalid.

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11 (iii). Let $\overline{X}_1$ be the sample mean of 100 balls of string and the length of string per ball with process improved.

$\overline{X}_1 \sim N\left(300, \frac{12.1}{100}\right)$.

$\therefore \overline{x} = k$

$\therefore Z_{critical} = \text{invNorm}(0.1) = -1.282$

For the manager's claim to be valid
$\Rightarrow \quad Z_{cal} > Z_{critical}$.

$$\frac{k-300}{\frac{\sqrt{12.1}}{10}} > -1.282.$$

$$k > 300 - 1.282\left(\frac{\sqrt{12.1}}{10}\right)$$

$$k > 299.55.$$

$\therefore$ Min $k = 299.56$ (corr to 2 dec pl.) #

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12 $A \sim N(0.25, 0.02^2)$ and $B \sim N(0.35, 0.03^2)$

(i) $A_1 + A_2 + \dots + A_{10} \sim N(10(0.25), 10(0.02)^2)$
$N(2.5, \; 4 \times 10^{-3})$

{Note total mass > 0}
$P(A_1 + A_2 + \dots + A_{10} < 2.4) = \text{normalcdf}(0, 2.4, 2.5, \sqrt{4 \times 10^{-3}})$
$= 0.056923$
≈ 0.0569 (corr to 3 sig fig).

(ii) $A_1 + A_2 + A_3 + \dots + A_6 - (B_1 + \dots + B_5) \sim N\left(6(0.25) - 5(0.35), 6(0.02)^2 + 5(0.03)^2\right)$
$\sim N(-0.25, 6.9 \times 10^{-3})$

$P\left(-0.2 \leq \left[(A_1 + A_2 + \dots + A_6) - (B_1 + B_2 + \dots + B_5)\right] \leq 0.2\right)$

$= P\left(\frac{-0.2 - (-0.25)}{\sqrt{6.9 \times 10^{-3}}} \leq Z \leq \frac{0.2 - (-0.25)}{\sqrt{6.9 \times 10^{-3}}}\right)$

$= \text{normalcdf}(0.60193, 5.41736, 0, 1)$

$= 0.27361$
≈ 0.274 ((corr to 3 sig fig).

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12 (iii) Let Mrs Woo's expense be

$W = 1.5A_1 + 1.5A_2 + 1.5A_3 + 2.4B_1 + 2.4B_2 + 2.4B_3.$

$W \sim N(1.5 \times 3 \times 0.25 + 2.4 \times 3 \times (0.35),\ 1.5^2(3)(0.02)^2 + 2.4^2(3)(0.03)^2)$

$\sim N(1.125 + 2.52,\ 2.7 \times 10^{-3} + 0.015552)$

$\sim N(3.645,\ 0.018252)$

Let Mr Tan's expense be.

$T = 1.5A_1 + 1.5A_2 + \ldots + 1.5A_{10}$

$T \sim N(1.5(10)(0.25),\ 1.5^2(10)(0.02)^2)$

$\sim N(3.75,\ 9 \times 10^{-3})$

$W - T \sim N(3.645 - 3.75,\ 0.018252 + 9 \times 10^{-3})$

$\sim N(-0.105,\ 0.027252)$

$P(W > T)$

$= P(W - T > 0)$

$= P\left(Z > \frac{0.105}{\sqrt{0.027252}}\right)$

$= 0.26237.$

≈ 0.262 (corr. to 3 sig fig)