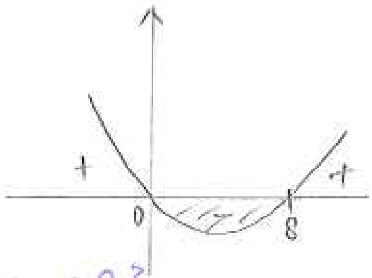
2011 H1 Maths.

1 $x^2 + (k-2)x + (k+1) > 0$

Then

$$b^2 - 4ac < 0.$$
$$(k-2)^2 - 4(1)(k+1) < 0.$$
$$k^2 - 4k + 4 - 4k - 4 < 0.$$
$$k^2 - 8k < 0$$
$$k(k-8) < 0.$$
$$\therefore 0 < k < 8$$



$\therefore$ The set of values of $k = \{ k \in \mathbb{R} \mid 0 < k < 8 \}$ #

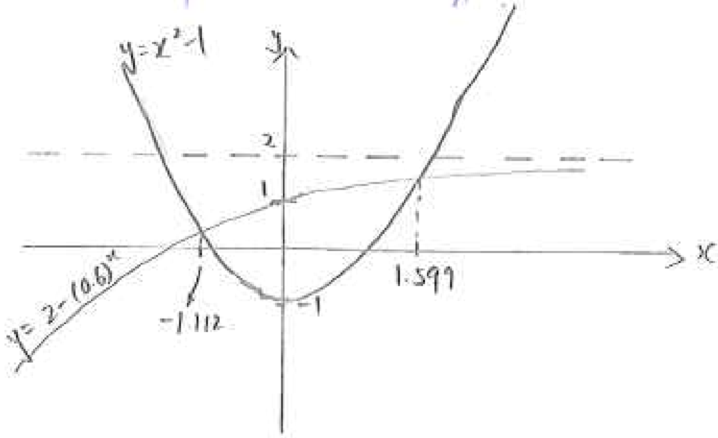
2(i). For $y = x^2 - 1$.

when $x = 0$, $y = -1$.

For $y = 2 - (0.6)^x$

when $x = 0$, $y = 2 - 1$

$y = 1$.



(ii) $y = x^2 - 1$ —— ①

$y = 2 - (0.6)^x$ —— ②.

① = ②.

$$x^2 - 1 = 2 - (0.6)^x.$$

From G.C, we have.

$x = -1.11157$

≈ -1.1116 # (corr to 4 dec pl)

$x = 1.599457$

≈ 1.5995 # (corr to 4 dec pl)

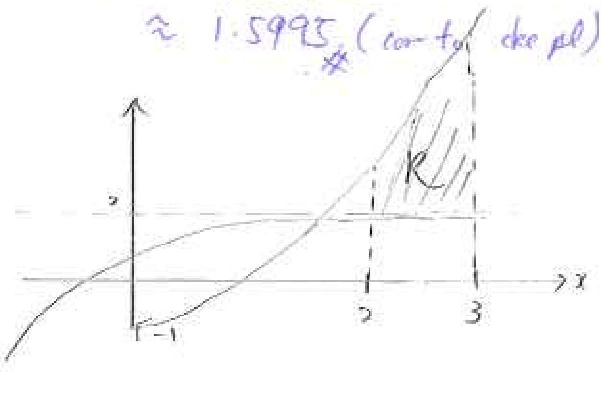
(iii). Area of R

$= \int_2^3 \{ (x^2 - 1) - [2 - (0.6)^x] \} \, dx$

$= \int_2^3 \{ x^2 - 3 + 0.6^x \} \, dx.$

$= 3.6152$

≈ 3.615 ((corr to 3 dec pl) #



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3 (i) $\int e^{3x+2}\,dx = \frac{1}{3}e^{3x+2} + C$.

(ii) $\int_4^9 3\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx = 3\int_4^9 \left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right) dx$

$= 3\left[\frac{2}{3}x^{3/2} - 2x^{\frac{1}{2}}\right]_4^9$

$= 3\left[\frac{2}{3}(9)^{3/2} - 2(9)^{1/2} - \frac{2}{3}(4)^{3/2} + 2(4)^{1/2}\right]$

$= 3\left[\frac{2}{3}(27) - 2(3) - \frac{2}{3}(8) + 2(2)\right]$

$= 32$ #.

4 $\therefore SR = (2-2x)$ m.

$QR = (2-2x)$ m.

$\therefore$ Volume of the box, $V = (2-2x)(2-2x)x$

$= (4 - 8x + 4x^2)x$.

$= (4x^3 - 8x^2 + 4x)$ m^3 (shown).

$\frac{dV}{dx} = 12x^2 - 16x + 4$.

For max value of V, $\frac{dV}{dx} = 0$.

$12x^2 - 16x + 4 = 0$.

$3x^2 - 4x + 1 = 0$.

$(3x-1)(x-1) = 0$.

$\therefore x = \frac{1}{3}$ or $x = 1$.

Using 2nd derivative test:

$\frac{d^2V}{dx^2} = 24x - 16$.

For $x = 1$, $\frac{d^2V}{dx^2} = 24(1) - 16 = 8 > 0$ which gives minimum.

For $x = \frac{1}{3}$, $\frac{d^2V}{dx^2} = 24(\frac{1}{3}) - 16 = -8 < 0$ which gives max V.

For maximum value of V, sub $x = \frac{1}{3}$ into V.

$\therefore V_{max} = 4\left(\frac{1}{3}\right)^3 - 8\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right)$

$= \frac{16}{27}$ m^3 #

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5 $y = x - \ln(2x+1)$. —— ①

$\frac{dy}{dx} = 1 - \frac{1}{2x+1} \cdot 2$.
$= 1 - \frac{2}{2x+1}$ —— ②.

At P, when $x = 2$, $y = 2 - \ln[2(2)+1]$.
$= 2 - \ln 5$.

Gradient of tangent, $m_T = 1 - \frac{2}{2(2)+1}$
$= 1 - \frac{2}{5}$
$= \frac{3}{5}$.

∴ Gradient of normal, $m_{AB} = -\frac{1}{m_T}$
$= -\frac{5}{3}$.

∴ Equation of normal AB:
$y - (2 - \ln 5) = -\frac{5}{3}(x-2)$.
$y - 2 + \ln 5 = -\frac{5}{3}x + \frac{10}{3}$.
$y = -\frac{5}{3}x + \frac{16}{3} - \ln 5$. —— ③.

(i). Using ②, $\frac{dy}{dx} = 0$.
$1 - \frac{2}{2x+1} = 0$.
$2x+1 = 2$.
$x = \frac{1}{2}$.

Sub $x = \frac{1}{2}$ into ①, $y = \frac{1}{2} - \ln[2(\frac{1}{2})+1]$.
$= \frac{1}{2} - \ln[1+1]$
$= \frac{1}{2} - \ln 2$.

Diff ②, $\frac{d^2y}{dx^2} = \frac{2}{(2x+1)^2}(2) = \frac{4}{(2x+1)^2}$

For $(\frac{1}{2}, \frac{1}{2} - \ln 2)$, $\frac{d^2y}{dx^2} = \frac{4}{(2(\frac{1}{2})+1)^2} = 1 > 0$ gives min pt.

∴ $(\frac{1}{2}; \frac{1}{2} - \ln 2)$ gives min pt on C.

5 (ii). Using (3),

sub $x = 0$, $y = \frac{16}{3} - \ln 5$. $\therefore B(0, \frac{16}{3} - \ln 5)$.

sub $y = 0$

$$0 = -\frac{5}{3}x + \frac{16}{3} - \ln 5.$$
$$\frac{5}{3}x = \frac{16}{3} - \ln 5.$$
$$5x = 16 - 3\ln 5.$$
$$x = \frac{16}{5} - \frac{3}{5}\ln 5.$$

$\therefore A(\frac{16}{5} - \frac{3}{5}\ln 5, 0)$.

Area of $\triangle OAB$

$= \frac{1}{2}(OA)(OB)$

$= \frac{1}{2}\left[\frac{16}{5} - \frac{3}{5}\ln 5\right]\left[\frac{16}{3} - \ln 5\right]$

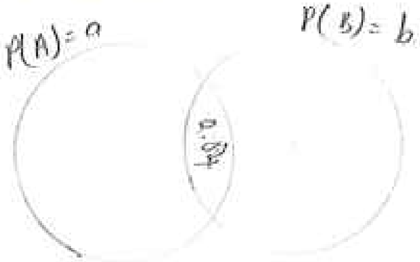
$= \frac{1}{2}(\frac{1}{5})(16 - 3\ln 5)\ \frac{1}{3}[16 - 3\ln 5]$

$= \frac{1}{30}(16 - 3\ln 5)^2$.

$\therefore p = 16$, $q = 3$ #.

2011 H1 Maths

6 Given $P(A) = a$ and $P(B) = b$.
$P(A \cup B) = 0.46$ and $P(A \cap B) = 0.04$.



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
$0.46 = a + b - 0.04$.
$a + b = 0.5$ —— ①.

Given A and B are independent events.

$P(A \cap B) = 0.04$
$P(A) \cdot P(B) = 0.04$.
$ab = 0.04$.
$b = 0.04/a$ —— ②.

Sub ② into ①.

$a + \frac{0.04}{a} = 0.5$.
$a^2 + 0.04 = 0.5a$.
$a^2 - 0.5a + 0.04 = 0$
$a = \frac{0.5 \pm \sqrt{0.5^2 - 4(1)(0.04)}}{2}$
$= \frac{0.5 \pm \sqrt{0.09}}{2}$
$= 0.1$ # or 0.4 #.

Hence possible values of $P(A) = 0.1$ or 0.4 #

7(i) A random sample is one where each student within this sample population is chosen randomly and entirely by chance, with each having equal probability of being picked.

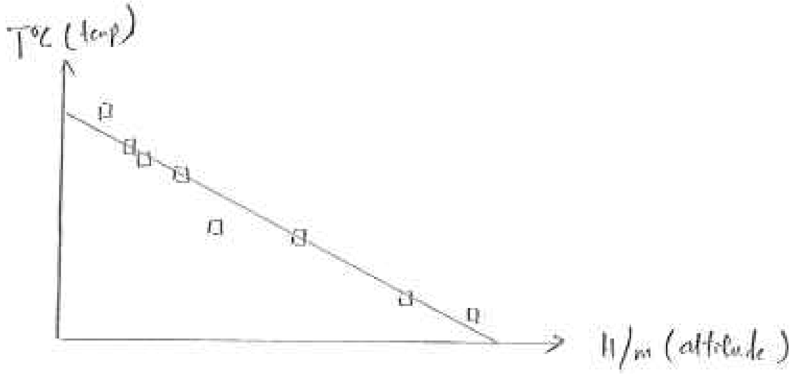
(ii) The three strata are
① Students traveling by car.
② Students traveling by bicycle.
③ Students traveling on foot

To obtain a stratified sample of 100 students.
Since student travel by car made up $\frac{440}{2000}$ of the student population.
no. of students to take part in survey $= \frac{440}{2000} \times 100 = 22.$

Using similar calculation:
Student by bicycle $= \frac{760}{2000} \times 100 = 38.$
Student on foot $= \frac{800}{2000} \times 100 = 40.$

Therefore 22 students (by cars), 38 students (by bicycle) and 40 students (on foot) are to be selected to take part in the survey.

8 (i).



(ii) Using G.C: $r = -0.96700$
≈ -0.967 (corr to 3 sig fig)

The strong negative product moment coefficient indicates that as the altitude increases, the temperature will drop correspondingly.

(iii) Using G.C: $T = 27 - 0.0147H$.

(iv) When $H = 1000\text{ m}$,
we have $T = 27 - 0.0147(1000)$
$= 12.3°C$

Since the altitude of 1000 m is within the given range of data value 200 m to 1550 m, the estimate is reliable as we are interpolating within the data range.

9 Let the random variable X be the mean lifetime of a light bulb.
Given $E(X) = 12000$
$Var(X) = 1400^2$.

$\therefore \bar{X} \sim N\left(12{,}000, \frac{1400^2}{50}\right)$ approximately by Central limit thm since $n \geq 50$.

Sample mean $\bar{x} = 11{,}500$
$H_0 : \mu = 12{,}000$
$H_1 : \mu < 12{,}000$

From G.C, $p = 0.0057 < 0.01$
We reject H_0 at the 1% level significance, there is sufficient evidence to indicate that this particular batch is substandard.

Given $\bar{x} = T$

$$\frac{T-\mu}{\frac{1400}{\sqrt{50}}} > \text{InvN}(5\%).$$

$$\sqrt{50}\left(\frac{T-12000}{1400}\right) > -1.6448.$$

$$T > 11674.34.$$

$\therefore$ The least possible value of $T = 11675$ hrs.

10(i) Let the r.v X be the no of days Jon complete a puzzle.
$X \sim Bin(7, 0.8)$.

(a) $P(X=3) = \text{binompdf}(7, 0.8, 3)$
$= 0.028672$
$\approx 0.0287_{\#}$ ((corr to 3sig fig)

(b) $P(X \geq 5) = 1 - P(X \leq 4)$
$= 1 - \text{binomcdf}(7, 0.8, 4)$
$= 0.851968$
$\approx 0.852_{\#}$ (corr to 3sig fig)

(ii) Let the r.v Y be the no of weeks out of 10 weeks, s.t. Jon complete the puzzle at least 5 times.

$Y \sim Bin(10, 0.851968)$

$P(Y=10) = (0.851968)^{10} = 0.20148$
$\approx 0.201_{\#}$ (corr to 3sig fig)

(iii). 10 week = 70 days.
Let the r.v W be the no of puzzle completed in 10 weeks
$W \sim Bin(70, 0.8)$.
$np = 70(0.8) = 56$
$nq = 70(0.2) = 14$.

Note: $npq = 70(0.8)(0.2) = 11.2$.

Since $np > 5$ and $nq > 5$, we can approximate
$W \sim N(56, 11.2)$ $E(W) = 56_{\#}$ and $Var(W) = 11.2$.

$P(W \geq 50) \xrightarrow{\text{Continuity Correction}} P(W > 49.5) = 0.97394$
≈ 0.974 (corr to 3sig fig)

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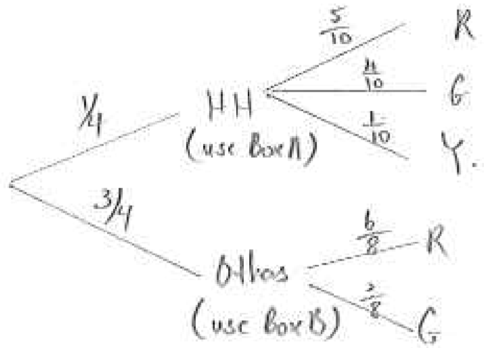
11 $P(\text{two heads}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

(i)(a).



(i)(b) $P(\text{red ball is chosen}) = \frac{1}{4} \times \frac{5}{10} + \frac{3}{4} \times \frac{6}{8}$
$= \frac{11}{16}$ #

(i)(c). Let X be ball came from box A.
Y be ball is red.

$$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{\frac{1}{4} \times \frac{5}{10}}{11/16} = \frac{2}{11}.$$

(ii) $P(\text{boths balls are the same color})$.
$= P(RR \text{ from box A}) + P(GG \text{ from box A}) + P(RR \text{ from box B}) + P(GG \text{ from box B})$
$= \frac{1}{4} \times \frac{5}{10} \times \frac{4}{9} + \frac{1}{4} \times \frac{4}{10} \times \frac{3}{9} + \frac{3}{4} \times \frac{6}{8} \times \frac{5}{7} + \frac{3}{4} \times \frac{2}{8} \times \frac{1}{7}$.

$= \frac{163}{315}$ #.

12. $B \sim N(60, 12^2)$.
$G \sim N(50, 10^2)$.

(i) $P(50 < B < 70) = \text{normalcdf}(50, 70, 60, \sqrt{12^2})$
$= 0.59534$
≈ 0.595 (corr to 3 sig figs).

(ii). $E(B-G) = E(B) - E(G) = 60 - 50 = 10$
$Var(B-G) = Var(B) + Var(G) = 12^2 + 10^2 = 244$.

$\therefore B - G \sim N(10, 244)$

$P(B > G) = P(B - G > 0)$
$= \text{normalcdf}(0, E99, 10, \sqrt{244})$
$= 0.73897$
≈ 0.739 (corr to 3 sig figs)

(iii) Let T be the total mass of 3 boys and 2 girls.
Let $T = B_1 + B_2 + B_3 + G_1 + G_2$.
$E(T) = E(B_1 + B_2 + B_3 + G_1 + G_2)$
$= 60 + 60 + 60 + 50 + 50$
$= 280$.
$Var(T) = Var(B_1 + B_2 + B_3 + G_1 + G_2)$
$= 12^2 + 12^2 + 12^2 + 10^2 + 10^2$
$= 632$.
$\therefore T \sim N(280, 632)$.

$P(T < 300) = \text{normalcdf}(-e99, 300, 280, \sqrt{632})$
$= 0.78685$
≈ 0.787 (corr to 3 sig figs).

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12 (iv) Let $R = B_1 + B_2 + B_3 + B_4 + B_5 + B_6$.

$E(R) = 60 + 60 + 60 + 60 + 60 + 60 = 360$

$Var(R) = 12^2 + 12^2 + 12^2 + 12^2 + 12^2 + 12^2 = 864$.

$R \sim N(360, 864)$.

$P(R < L) = 0.95$.

$P\left(\frac{R-360}{\sqrt{864}} < \frac{L-360}{\sqrt{864}}\right) = 0.95$.

$P\left(Z < \frac{L-360}{\sqrt{864}}\right) = 0.95$.

$\frac{L-360}{\sqrt{864}} = \text{InvN}(0.95)$

$\frac{L-360}{\sqrt{864}} = 1.64485$

$L = 408.348$.

$\therefore L \approx 408$ (corr to 3sig fig).