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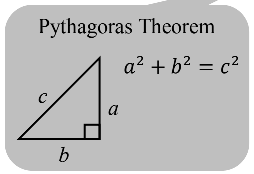
ELEMENTARY MATHEMATICS 4016/02

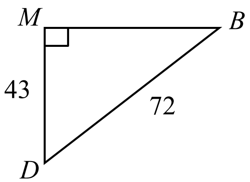
Paper 2 Suggested Solutions — October/November 2010

1. **Topics: Trigonometry**
   (Trigonometric Ratios, Pythagoras' Theorem, Bearings)

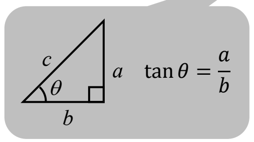
(a) (i)



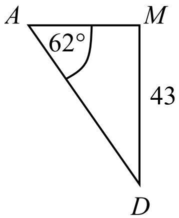
$$\begin{aligned} BD^2 &= DM^2 + MB^2 \\ MB^2 &= BD^2 - DM^2 \\ &= 72^2 - 43^2 \\ MB &= \sqrt{3335} \\ &= 57.749 \\ &\approx \mathbf{57.7\ m\ (3\ sig.\ fig.)} \end{aligned}$$



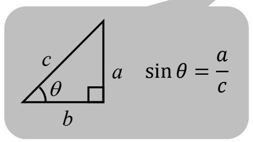
(ii)



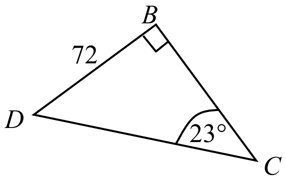
$$\begin{aligned} \tan 62° &= \frac{43}{AM} \\ AM &= \frac{43}{\tan 62°} \\ &= 22.863 \\ \therefore AB &= AM + MB \\ &= 22.863 + \sqrt{3335} \\ &\approx \mathbf{80.6\ m\ (3\ sig.\ fig.)} \end{aligned}$$



(iii)



$$\begin{aligned} \sin 23° &= \frac{72}{CD} \\ CD &= \frac{72}{\sin 23°} \\ &= 184.269 \\ &\approx \mathbf{184\ m\ (3\ sig.\ fig.)} \end{aligned}$$

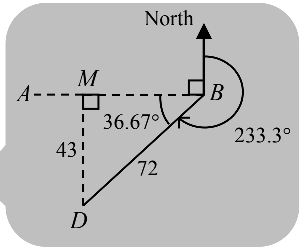


(b)

$$\begin{aligned} \sin M\hat{B}D &= \frac{43}{72} \\ M\hat{B}D &= \sin^{-1}\frac{43}{72} \\ &= 36.67° \end{aligned}$$

"B is due east of A" $\Rightarrow$ bearing of A from $B = 270°$

$$\begin{aligned} \therefore \text{Bearing of } D \text{ from } B &= 270° - 36.67° \\ &\approx \mathbf{233.3°\ (1\ d.p.)} \end{aligned}$$



2. **Topic: Algebra (Solutions to Quadratic Equations, Formulae)**

(a)

$$\begin{aligned} \frac{x}{8} &= \frac{50}{x} \\ x^2 &= 8(50) \\ x &= \pm\sqrt{400} \\ &= \mathbf{20\ or\ -20} \end{aligned}$$

(b)

$$\begin{aligned} \frac{t+p}{4} &= \frac{q}{5} \\ 5(t+p) &= 4q \\ 5t + 5p &= 4q \\ 5t &= 4q - 5p \\ \boldsymbol{t} &= \boldsymbol{\frac{4q-5p}{5}} \end{aligned}$$

(c) $y = a + \frac{600}{x}$ — y: cost per copy; x: total no. of copies

(i) Sub $x = 50$, $y = 17$,

$$\begin{aligned} 17 &= a + \frac{600}{50} \\ a &= 17 - 12 \\ &= \mathbf{5} \end{aligned}$$

(ii) Sub $x = 100$,

$$\begin{aligned} y &= 5 + \frac{600}{100} \\ &= 11 \end{aligned}$$

$\therefore$ When 100 copies are printed, the cost of each copy is \$11.

(iii) Sub $x = 300$, $y = 5 + \frac{600}{300} = 7$ — Cost per copy

$$\begin{aligned} \therefore \text{Total cost} &= 7 \times 300 \\ &= \mathbf{\$2100} \end{aligned}$$

(iv) Sub $y = 5.20$,

$$\begin{aligned} 5.20 &= 5 + \frac{600}{x} \\ \frac{600}{x} &= 0.2 \\ x &= \frac{600}{0.2} \\ &= \mathbf{3000} \end{aligned}$$

$\therefore$ 3000 copies were printed.

3. **Topic: Arithmetic (Application of Mathematics in Practical Situations)**

(a) (i) Total amount Alan will pay for the computer

$= \frac{1}{3}(1299) + 24(40.30)$

Deposit

24 monthly instalments

$=$ **$1400.20**

(ii) Extra cost of computer (as % of cash price)

$= \frac{\$1400.20 - \$1299}{\$1299} \times 100\%$

$= 7.7906\%$

$\approx$ **7.79% (3 sig. fig.)**

"extra cost as % of cash price":
$\frac{\text{hire purchase price} - \text{cash price}}{\text{cash price}} \times 100\%$

(b) Total amount Betty will pay

$= 1299\left(1 + \frac{6}{100}\right)^3$

$= \$1547.129$

Given in formula sheet (compound interest):
Total amount $= P\left(1 + \frac{r}{100}\right)^n$

Interest Betty will pay $= \$1547.129 - \$1299$

$= \$248.129$

$\approx$ **$248.13 (2 d. p.)**

Total interest = Total amt. – Principal amt.

(c) $115\% \rightarrow \$759$

$1\% \rightarrow \$\frac{759}{115}$

$100\% \rightarrow \$\frac{759}{115} \times 100 = \$660$

$\therefore$ **The trader paid $660 for the camera**.

Selling price
= Cost price (100%) + Profit (15%)
= 115% × Cost price

4. **Topic: Coordinate Geometry; Vectors in Two Dimensions**

(a) Equation of AB:

$\frac{y-4}{x-(-5)} = \frac{4}{3}$

$y - 4 = \frac{4}{3}(x+5)$

$y - 4 = \frac{4}{3}x + \frac{20}{3}$

$y = \frac{4}{3}x + \frac{20}{3} + 4$

$y = \frac{4}{3}x + 10\frac{2}{3}$

$\mathbf{3y = 4x + 32}$

Equation of straight line passing through $A(x_1, y_1)$ and $B(x_2, y_2)$:
$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = \text{gradient } m$$

Alternative Method

Equation of straight line with gradient m & y-intercept c:

$y = mx + c$

$y = \frac{4}{3}x + c$

Sub $(-5, 4)$,

$4 = \frac{4}{3}(-5) + c$

$c = \frac{32}{3}$

$\therefore y = \frac{4}{3}x + 10\frac{2}{3}$

(b) From (a), $3y = 4x + 32$ – (1)

Given $2x + 9y = 68$,

$x = \frac{68 - 9y}{2}$ – (2)

Sub (2) into (1), $3y = 4\left(\frac{68-9y}{2}\right) + 32$

$3y = 136 - 18y + 32$

$21y = 168$

$y = 8$

Sub $y = 8$ into (2), $x = \frac{68 - 9(8)}{2} = -2$

$\therefore$ **Coordinates of B = (–2, 8)**

(c) (i) $|\overrightarrow{AE}| = \sqrt{6^2 + 1^2}$

$= \sqrt{37}$

$\approx$ **6.08 (3 sig.fig.)**

Magnitude of $\begin{pmatrix} u \\ v \end{pmatrix}$:
$$\left|\begin{pmatrix} u \\ v \end{pmatrix}\right| = \sqrt{u^2 + v^2}$$

(ii) $\overrightarrow{AE} = \overrightarrow{OE} - \overrightarrow{OA}$

$\overrightarrow{OE} = \overrightarrow{AE} + \overrightarrow{OA}$

$= \begin{pmatrix}6\\1\end{pmatrix} + \begin{pmatrix}-5\\4\end{pmatrix}$

$= \begin{pmatrix}1\\5\end{pmatrix}$

$\therefore$ **Coordinates of E=(1, 5)**

(iii) $\overrightarrow{OD} = \begin{pmatrix}4\\2\end{pmatrix}$

(a) $\overrightarrow{DE} = \overrightarrow{OE} - \overrightarrow{OD}$

From (c)(ii), $\overrightarrow{OE} = \begin{pmatrix}1\\5\end{pmatrix}$

$= \begin{pmatrix}1\\5\end{pmatrix} - \begin{pmatrix}4\\2\end{pmatrix}$

$= \begin{pmatrix}-3\\3\end{pmatrix}$

(b) $\overrightarrow{DB} = \overrightarrow{OB} - \overrightarrow{OD}$

From (b), $\overrightarrow{OB} = \begin{pmatrix}-2\\8\end{pmatrix}$

$= \begin{pmatrix}-2\\8\end{pmatrix} - \begin{pmatrix}4\\2\end{pmatrix}$

$= \begin{pmatrix}-6\\6\end{pmatrix}$

(iv) $\overrightarrow{DE} = 3\begin{pmatrix}-1\\1\end{pmatrix}$

$\overrightarrow{DB} = 6\begin{pmatrix}-1\\1\end{pmatrix}$

$\overrightarrow{AB} = k\overrightarrow{BC}$
$\Rightarrow A, B, C$ are collinear (straight line)

Since $\overrightarrow{DB} = 2\overrightarrow{DE} = k\overrightarrow{DE}$

$\Rightarrow$ ***D*, *E* and *B* are collinear** (i.e. they all lie on a straight line)

and since $\overrightarrow{DB} = 2\overrightarrow{DE}$

$\Rightarrow$ ***E* is the mid-point of *D* and *B*.**

5. **Topic: Angles of Polygon**

Each exterior $\angle$ of a regular n-sided polygon $= \frac{360°}{n}$
NOTE: X is not part of polygon $ABCDEF$ …! $\Rightarrow \angle XCD$ is an exterior $\angle$

(a) (i) $\angle XCD = \frac{360°}{15}$

$= \mathbf{24°}$

(ii) Since $ABCDEF$ … is a regular polygon and $\angle XCD$ & $\angle XDC$ are exterior angles,

$\Rightarrow \angle XCD = \angle XDC = 24°$

$\Rightarrow \Delta XCD$ is isosceles

$\Rightarrow \angle CXD = 180° - \angle XCD - \angle XDC$ (sum of $\angle$s in Δ)

$= 180° - 24° - 24°$

$= \mathbf{132°}$

(b) Given $BC = DE = a$

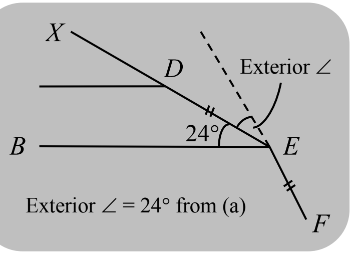
From (a)(ii), ΔXCD is a isosceles

$\Rightarrow XC = XD = b$

$XB = BC + CX = a + b$

$XE = DE + XD = a + b$

Hence $XB = XE$.



(c) From (b), ΔXBE is isosceles,

$\Rightarrow \angle XBE = \angle XEB = \frac{180° - \angle CXD}{2} = 24°$

$\angle BEF = 180° -$ exterior $\angle - \angle XEB$

$= 180° - 24° - 24°$

$= \mathbf{132°}$

<u>**Alternative Method**</u>

Each interior $\angle$ of a regular n-sided polygon $= \frac{(n-2)\times 180°}{n}$

$\angle BEF = \angle DEF - \angle XEB$

$= \frac{(15-2)\times 180°}{15} - 24°$

$= \mathbf{132°}$

6. **Topics: Solutions to Quadratic Equations**

(a) Number of hours. John took $= \frac{42}{x}$

(b) Number of hours Peter took $= \frac{42}{x-\frac{1}{2}}$

Time taken $= \frac{\text{Distance}}{\text{Speed}}$

(c)
$$\frac{42}{x-\frac{1}{2}} - \frac{42}{x} = \frac{10}{60}$$
$$\frac{84}{2x-1} - \frac{42}{x} = \frac{1}{6}$$
$$\frac{84x-42(2x-1)}{(2x-1)x} = \frac{1}{6}$$
$$\frac{84x-84x+42}{2x^2-x} = \frac{1}{6}$$
$$42\,(6) = 2x^2-x$$
$$2x^2-x-252 = 0 \textbf{ (Shown)}$$

(d)
$$2x^2-x-252 = 0$$
$$x = \frac{1\pm\sqrt{(-1)^2-4(2)(-252)}}{2(2)}$$
$$= \frac{1\pm\sqrt{2017}}{4}$$
$$= 11.4777 \text{ or } -10.9777$$
$$\approx \mathbf{11.478 \text{ or } -10.978 \text{ (3 d.p.)}}$$

General solution to a quadratic equation ax^2+bx+c:
$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Question simply asks to solve the equation. Do NOT reject the negative value of x here!

(e) Taking $x = 11.4777$ from (d), time that John took to complete the race

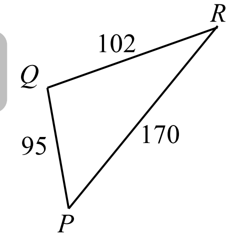
$= \frac{42}{11.4777}$

$= 3.65925$ hours

$= 3$ hrs 39.555 min

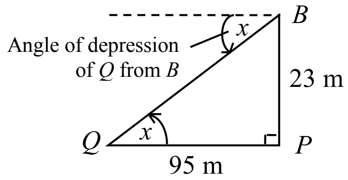
≈ 3 hrs 39 min 33 seconds

7. **Topic: Trigonometry**



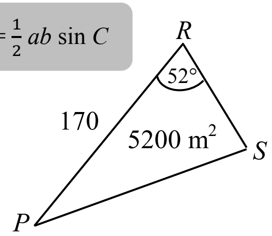
(a)
$$\cos P\hat{Q}R = \frac{95^2+102^2-170^2}{2(95)(102)}$$
$$= -0.48869$$
$$P\hat{Q}R = \cos^{-1}(-0.48869)$$
$$= 119.255$$
$$\approx \mathbf{119.3° \text{ (1 d.p.)}}$$

Cosine rule:
$c^2 = a^2 + b^2 - 2ab\cos C$



(b) In ΔBPQ,
$$\tan x = \frac{23}{95}$$
$$x = 13.609°$$
$$\approx 13.6° \text{ (1d.p.)}$$

Angle of depression of Q from B = **13.6°**



(c)
$$\text{Area of } \Delta PRS = 5200 \text{ m}^2$$
$$\tfrac{1}{2}(PR)(RS)\sin 52° = 5200$$
$$\tfrac{1}{2}(170)(RS)\sin 52° = 5200$$
$$RS = 77.634$$
$$\approx \mathbf{77.6 \text{ m (3 s.f.)}}$$

Area of $\Delta = \frac{1}{2}ab\sin C$

(d) (i) $\frac{77.634}{3} = 25.87$

∴ Number of panels that needs to be bought = **26**

Rounded up to 26 ∵ need to buy 26 × panels (of unit length 3 m) to fence up the full distance of *RS*.

(ii) Number of posts required = 27

∴ Total cost of the panels and post

$= 26 \times 28.50 + 27 \times 14.95$

= $1144.65

26 posts for each of the panels + 1 extra post at the end.

8. **Topics: Trigonometry, Mensuration**

(a) (i)

$$\text{Perimeter of sector} = 44 \text{ m}$$
$$\Rightarrow \text{Length of major arc } PRQ + 2 \times \text{radius } (r) = 44 \text{ m}$$
$$r\theta + 2r = 44$$
$$8\theta + 2(8) = 44$$
$$\theta = \frac{28}{8}$$
$$= \mathbf{3.5 \text{ radians}}$$

Arc length = $r\theta$
Note: θ must be in radians and can be reflex.

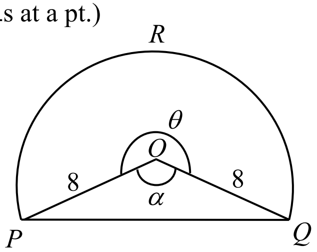
(ii) Obtuse $\angle POQ$ (α) = 2π – reflex $\angle POQ$ (θ) ($\angle$s at a pt.)

$\Rightarrow$ $\alpha = 2\pi - 3.5$ radians

$$\text{Area of } \Delta POQ = \frac{1}{2}(OP)(OQ)\sin\alpha$$
$$= \frac{1}{2}(8)^2\sin(2\pi - 3.5)$$
$$= 11.225$$
$$\approx \mathbf{11.2 \text{ m}^2 \text{ (3 s.f.)}}$$

Area of $\Delta = \frac{1}{2}ab\sin C$

Calculator must be in RAD mode to perform this sin operation!



(iii) Area of major sector
$$= \frac{1}{2}r^2\theta$$
$$= \frac{1}{2}(8)^2(3.5)$$
$$= 112 \text{ m}^2$$

Total area of the cross-section of the tunnel

= Area of major sector + Area of ΔPOQ

= 112 + 11.225

= 123.225

≈ **123 m^2 (3 s.f.)**

(b) (i) Volume of the bollard

= Volume of pyramid + Volume of cuboid

$= \frac{1}{3}(10)(10)(12) + (10)(10)(30)$

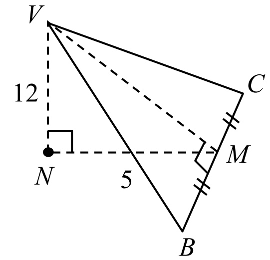
= **3400 cm^3**

Volume of pyramid
$= \frac{1}{3} \times$ base area $\times$ height

(ii) Let M be the midpoint of BC.

Using Pythagoras' Theorem in ΔVNM,

$$VM = \sqrt{VN^2 + NM^2}$$
$$= \sqrt{12^2 + 5^2}$$
$$= 13 \text{ cm}$$



$\therefore$ Surface area of the bollard

= [4 × Area of ΔVBC] + [Perimeter of $ABCD \times AE$]

$= 4 \times \left(\frac{1}{2} \times 10 \times 13\right) + (10 + 10 + 10 + 10) \times 30$

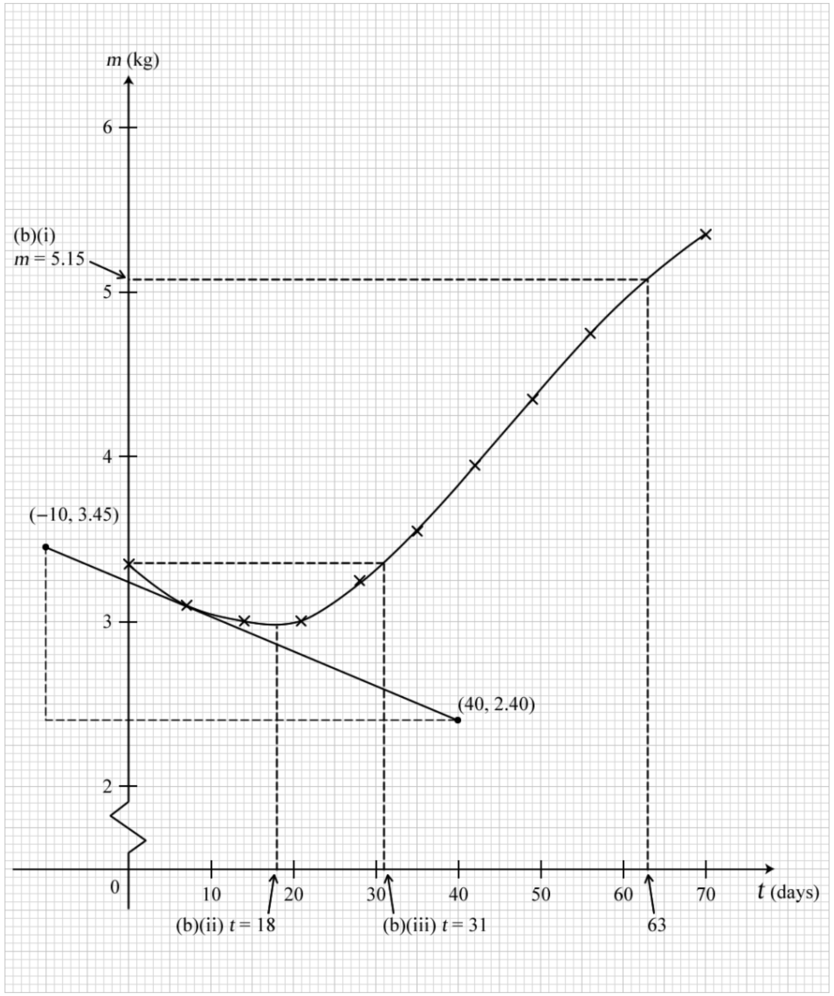
= **1460 cm^2**

Surface area of pyramid (excl. base)

Surface area of cuboid (excl. top &base)

9. **Topic: Graphical Solution of Equations**

(a)



(b) From the graph

(i) Mass of the baby after 63 days = **5.15 kg**

(ii) Days since birth when the baby's mass was least = **18 days**

(iii) Days since birth when the baby regained its birth mass = **31 days**

(c) (i) From the tangent drawn in the graph, gradient of the curve at (7 , 3.10)

$$= \frac{3.45 - 2.40}{-10 - 40}$$

$$= \mathbf{-0.0210\ (3\ s.f.)}$$

(ii) **This gradient represents the rate of change of the baby's mass at seven days since birth (i.e. *t* = 7).**

(d) **As the graph is non-linear, it is not appropriate to estimate the mass of the baby when it is 1 year old by extending the graph linearly up to *t* = 365.**

10. **Topics: Statistics, Simple Probability**

(a) (i)

$a = 28 \div 4 = \mathbf{7}$

$b = 60 - (12 + 15 + 10 + 7 + 4 + 0 + 2 + 1) = \mathbf{9}$

$c = 0 \times 12 = \mathbf{0}$

$d = 3 \times 9 = \mathbf{27}$

$e = 0 + 15 + 20 + 27 + 28 + 20 + 0 + 14 + 8 = \mathbf{132}$

(ii)

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{132}{60} = \mathbf{2.2}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} = \sqrt{\frac{510}{60} - (2.2)^2} = 1.9131 \approx \mathbf{1.91\ (3\ sig.\ fig.)}$$

$\frac{\Sigma fx}{\Sigma f}$ from (a)(ii)

(b) P(One pupil read exactly 6 books) $= \frac{0}{60} = \mathbf{0}$

$\frac{\text{no. of pupils who had read exactly 6 books}}{\text{total no. of pupils in group}}$

(c) P(Both had read more than 4 books) $= \left(\frac{7}{60}\right)\left(\frac{6}{59}\right) = \mathbf{\frac{7}{590}}$

P[1st pupil (chosen from the 60) had read > 4 books] AND
P[2nd pupil (chosen from the remaining 59) had read > 4 books]