ELEMENTARY MATHEMATICS 4016/01
Paper 1 Suggested Solutions October/November 2009

1. **Topic: Algebra**

(a) $20xy - 5y = \mathbf{5y(4x - 1)}$

(b) $4x - 4(x + 3) = 4x - 4x - 12$

$= \mathbf{-12}$

Answer (a) $5y(4x - 1)$ [1]
(b) -12 [1]

2. **Topic: Arithmetic (Percentages)**

$100\% \rightarrow \$48$

$1\% \rightarrow \frac{\$48}{100}$

$235\% \rightarrow \frac{\$48}{100} \times 235$

$= \$112.80$

$\therefore$ Selling price $= \$112.80$

Selling price
= Cost price (100%) + Profit (135%)
= 235% × Cost price

Answer $ 112.80 [2]

3. **Topic: Statistics (Frequency Table & Pie Chart)**

(a) Modal colour = **Blue**

Modal colour
= Mode of frequency table
= Colour of car that appears most frequently

(b) Angle representing the colour green $= \frac{5}{9+5+1+6+3} \times 360°$

$= \mathbf{75°}$

∠ of sector in pie chart
∝ its given data

Answer (a) Blue [1]
(b) 75° [1]

4. **Topic: Arithmetic (Percentages & Fractions)**

(a) $\frac{17}{24} \times 100\% = \mathbf{70\frac{5}{6}\%}$

(b) Fraction of candidates who were not awarded an A or B grade

$= 1 - \frac{1}{3} - \frac{1}{4}$

$= \mathbf{\frac{5}{12}}$

Answer (a) $70\frac{5}{6}$ % [1]
(b) $\frac{5}{12}$ [1]

5. **Topic: Algebra (Indices)**

(a) $2^p \times 5 = 40$

$2^p = \frac{40}{5}$

$2^p = 8$

$2^p = 2^3$

$\therefore \boldsymbol{p = 3}$

(b) $1 \div x^{-4} = 1 \div \frac{1}{x^4}$
$= 1 \times \frac{x^4}{1}$
$= \boldsymbol{x^4}$

$\frac{1}{a^m} = a^{-m}$

Answer (a) $p =$ 3 [1]
(b) x^4 [1]

6. **Topic: Angle Properties of Polygon**

(a) $x° + \angle EDC = 180°$ (int. $\angle$s, AE // DC)
$\therefore x = 180 - 130$
$= \mathbf{50}$

(b) $\angle EAB = 180° - x°$
$= 130°$ (int. $\angle$s)

Sum of interior $\angle$s $= (5-2) \times 180°$
$x° + \angle EAB + y° + 80° + 130° = 3 \times 180°$
$50° + 130° + y° + 80° + 130° = 540°$
$\boldsymbol{y = 150}$

Sum of int. $\angle$s of an n-sided polygon = $(n-2) \times 180°$

Answer (a) $x =$ 50 [1]
(b) $y =$ 150 [1]

7. **Topic: Linear Inequalities**

$-2 < 2x - 5 < 7$
$-2 + 5 < 2x < 7 + 5$
$\therefore \frac{3}{2} < x < 6$

Answer (a) $\frac{3}{2} < x < 6$ [2]

8. **Topic: Arithmetic (Compound Interest)**

Total amount $= 5000\left(1 + \frac{4.8}{100}\right)^6$
$= \$6624.265$
$\therefore$ Interest $= \$6624.265 - \$5000$
$= \$1624.265$
$\approx$ **1624.27 (2 d.p.)**

Given in formula sheet (compound interest):
Total amount $= P\left(1 + \frac{r}{100}\right)^n$

Total interest
= Total amt. – Principal amt.

Answer $ 1624.27 [2]

9. **Topics: Trigonometry and Mensuration**

(a) Area of $\Delta ABC = \frac{1}{2}ab \sin c$
$= \frac{1}{2}(7.43)(7.43) \sin 38°$
$= 16.993$
$\approx$ **17.0 cm^2 (3 sig. fig.)**

(b) Volume of the prism = Base area × height
= Area of $\Delta ABC \times 20$
= 16.993×20
= 339.87
≈ **340 cm^2 (3 sig. fig.)**

Answer (a) 17.0 cm^2 [1]
(b) 340 cm^2 [1]

10. **Topic: Number Patterns**

(a) 1st term = 38
2nd term = 38 – 7
= **31**
3rd term = 31 – 7
= **24**

(b) $T_1 = 38$
T_2: $31 = 38 - 7$
T_3: $24 = 38 - 7 - 7$
T_n: $38 - 7(n - 1) = 45 - 7n$
n^{th} term = **$45 - 7n$**

Answer (a) 31 , 24 [1]
(b) $45 - 7n$ [1]

11. **Topic: Arithmetic (Application of Mathematics in Practical Situations)**

Small tin: 415 g → \$1.04
1 g → $\$\frac{1.04}{415}$
= \$0.002506/g
≈ \$0.00251/g (3 sig. fig.)

Large tin: 815 g → \$1.98
1 g → $\$\frac{1.98}{815}$
= \$0.002429 /g
≈ \$0.00243 /g (3 sig. fig.)

∴ the large tin gives better value because it costs less per gram.

Answer The large tin gives better value [2]

12. **Topic: Kinematics**

(a) Acceleration during the 1st 40 seconds = $\frac{24}{40}$
= **0.6 m/s^2**

Acceleration = $\frac{\text{Speed}}{\text{Time}}$

(b) Total distance travelled = Total area under the graph
= $\frac{1}{2}(60)(24)$
= **720 m**

Answer (a) 0.6 m/s^2 [1]
(b) 720 m [1]

13. **Topic: Volumes of Similar Figures**

(a) Let w be the width of the prism.

Let v_1 = volume of water when $d = 12$

and v_2 = volume of water when $d = 24$ cm

$$\frac{v_1}{v_2} = \frac{(\text{Base area when } d=12) \times w}{(\text{Base area when } d=24) \times w}$$

$$\frac{v_1}{v_2} = \left(\frac{12}{24}\right)^2 \times \frac{w}{w}$$

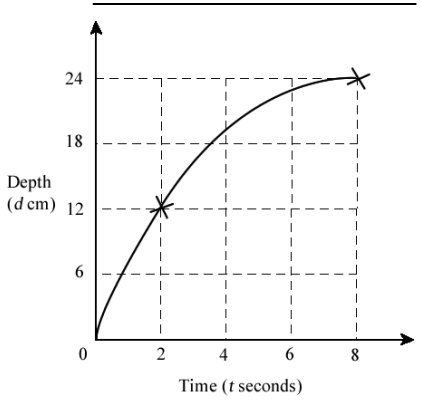
$$v_1 = \frac{1}{4} v_2$$

$\therefore$ since v_2 takes 8 seconds, v_1 takes 2 seconds

$\therefore$ $\boldsymbol{t = 2}$ when $d = 12$

Answer (a) $t =$ 2 [1]

(b)



[1]

14. **Topic: Mensuration (Surface area)**

Surface area of hemisphere
$= \frac{1}{2} \times$ surface area of sphere (from formula sheet)
$= \frac{1}{2}(4\pi r^2) = 2\pi r^2$

Curved surface area of a cone
$= \pi rl$ (from formula sheet)

Surface area of the toy $= 2\pi r^2 + \pi rl$

$= 2\pi(2.8)^2 + \pi(2.8)(7.2)$

≈ 112.59

$\approx$ **113 cm^2 (3 sig. fig.)**

Answer 113 cm^2 [1]

15. **Topic: Areas & Volumes of Similar Figures**

(a) (i) $\frac{v_S}{v_L} = \left(\frac{R_S}{R_L}\right)^3$

$\frac{640}{1250} = \left(\frac{R_S}{R_L}\right)^3$

$\therefore \frac{R_S}{R_L} = \sqrt[3]{\frac{640}{1250}}$

$= \frac{4}{5}$

Ratio of the smaller radius to the larger radius = **4 : 5**

(ii) $\frac{A_S}{A_L} = \left(\frac{R_S}{R_L}\right)^2$

$= \left(\frac{4}{5}\right)^2$

$= \frac{16}{25}$

$\therefore$ Ratio of the surface area to the larger surface area = **16 : 25**

(b) $\frac{M_S}{M_L} = \left(\frac{R_S}{R_L}\right)^3$

$\frac{M_S}{25} = \left(\frac{4}{5}\right)^3$

$M_S = \frac{64}{125} \times 25$

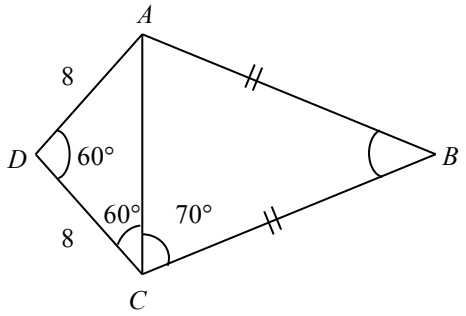
$= 12.8$ kg

$\therefore$ Mass of the smaller sphere = **12.8 kg**

Answer(a)(i) 4 : 5 [1]
(ii) 16 : 25 [1]
(b) 12.8 kg [1]

16. **Topic: Geometry**

(a) (i)



$\angle DAC = \angle ACD = 60°$

$\therefore \Delta DAC$ is an equilateral $\Delta \Rightarrow$ Length of AC = **8 cm**

(ii) Since $\angle ACD = 60°$, $\angle ACB = 130° - 60°$

$= 70°$

$\therefore \angle ABC = 180° - 2(70°)$

$=$ **40°** (sum of $\angle$s in Δ)

Answer(a)(i) $AC =$ 8 cm [1]
(ii) $\angle ABC =$ 40° [1]

(b) (i) $\angle POT = 2 \times \angle PQO$ ($\angle$ at centre = 2 × $\angle$s at circumference)

$= 2(32°)$

$=$ **64°**

(ii) $\angle OPT = 90°$ (tan $\perp$ radius)

$\therefore \angle OTP = 180° - 90° - 64°$

$=$ **26°** (sum of $\angle$s in Δ)

Answer(b)(i)$\angle POT =$ 64° [1]
(ii)$\angle OTP =$ 26° [1]

17. **Topic: Algebra**

(a) (i) $2x^2 + kx - 15 = 0$ ..... (1)

Sub $x = 3$,

$2(3)^2 + 3k - 15 = 0$

$18 + 3k - 15 = 0$

$3k = -3$

$\boldsymbol{k = -1}$

(ii) Sub $k = -1$ into (1),

$$2x^2 - x - 15 = 0$$
$$(2x + 5)(x - 3) = 0$$
$$2x + 5 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -\tfrac{5}{2} \qquad x = 3 \text{ (given)}$$
$$= \mathbf{-2.5}$$

(b) $6p^2 - 3pq - 10ap + 5a = 3p(2p - q) - 5a(2p - q)$
$= \mathbf{(2p - q)(3p - 5a)}$

Answer (a)(i) $k =$ -1 [1]
(ii) $x = -2.5$ [1]
(b) $(2p - q)(3p - 5a)$ [2]

18. **Topic: Factors and Multiples**

(a) $150 = 2 \times 75$
$= 2 \times 3 \times 25$
$= \mathbf{2 \times 3 \times 5^2}$

(b) $150 = 2 \times 3 \times 5^2$
$48 = 2^4 \times 3$
$\text{HCF} = 2 \times 3$
$= \mathbf{6}$

(c) LCM of 48 and 150 $= 2^4 \times 3 \times 5^2$
$= 1200$
Least number of chocolate bars he could have bought $= \frac{1200}{150}$
$= \mathbf{8}$

Answer (a) 150 = $2 \times 3 \times 5^2$ [1]
(b) 6 [1]
(c) 8 [2]

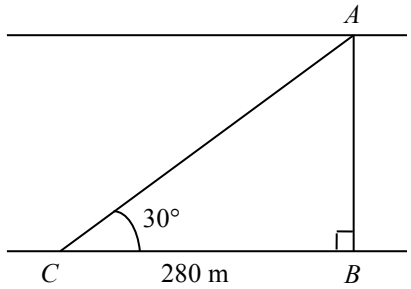
19. **Topics: Approximation & Estimation, Trigonometry**

(a) (i) $\frac{494.6}{56.33 \times 98.12} = \frac{494.6}{5527.0996}$
$= \mathbf{0.08948635556}$

(ii) $0.08948635556 \approx \mathbf{0.1 \text{ (1 d.p.)}}$

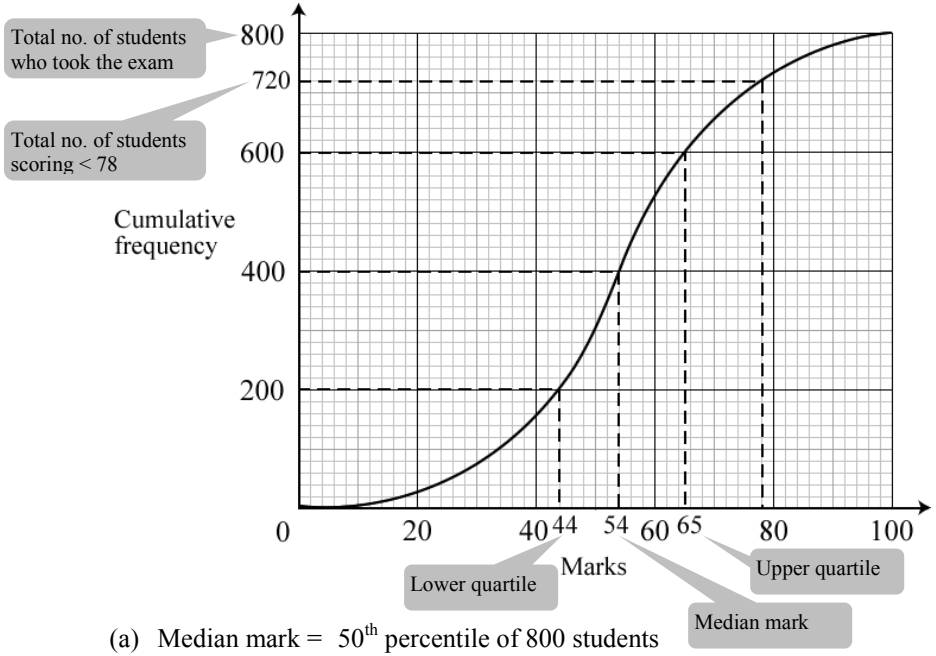
Answer (a)(i) 0.08948635556 [1]
(ii) 0.1 [1]

(b) $\tan 30° = \frac{AB}{280}$
$AB = 280 \tan 30°$
$= 161.65$
$\approx \mathbf{162 \text{ m (3 sig. fig.)}}$



Answer (b) 162 m [2]

20. **Topic: Statistics (Cumulative Frequency)**



(a) Median mark = 50th percentile of 800 students
= **54**

(b) Interquartile range = Upper quartile – Lower quartile
= 65 – 44
= **21**

(c) Number of students who are awarded a grade A = 800 – 720
= **80**

Answer (a) 54 [1]
(b) 21 [2]
(c) 80 [1]

21. **Topic: Standard Form**

(a) $1.32 \times 10^9 - 832 \times 10^6 = 10^6[1.32 \times 10^3 - 832]$
$= 10^6[1320 - 832]$
$= 488 \times 10^6$
$= \mathbf{4.88 \times 10^8}$

1 million = 1×10^6
1 billion = 1×10^9

(b) Average number of per square kilometer living in Africa

$= \frac{832 \times 10^6}{26.6 \times 10^6}$

= **31.3 people per sq. km**

(c) $\frac{\text{Number of people living in Singapore}}{\text{Number of people living in China}} = \frac{4.48 \times 10^6}{1.32 \times 10^9}$
$= \frac{4.48}{1.32 \times 10^3}$
$= \frac{4.48}{1320}$
$= \frac{14}{4125}$

Ratio of no. of people living in Singapore : no. of people living in China
= 14 : 4125
= $\mathbf{1 : 294\frac{9}{14}}$

Answer (a) 4.88×10^8 [2]
(b) 31.3 [1]
(c) $1 : 294\frac{9}{14}$ [1]

22. **Topics: Mensuration and Trigonometry**

θ must be in radians

(a) Length of the major arc AB $= r\theta$
$= 10(2\pi - 2.3)$
$= 39.83$
$\approx$ **39.8 cm (3 sig. fig.)**

$\sin\theta = \sin(\pi - \theta)$

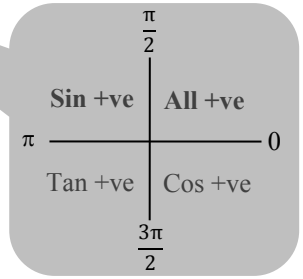
(b) $\sin x = \frac{1}{2}$

$\because \sin\frac{\pi}{6} = \frac{1}{2}$ and $\sin\left(\pi - \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

$\therefore x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$= 0.5235$ or 2.617

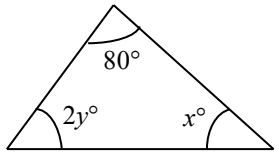
$\approx$ **0.524 or 2.62 (3 sig. fig.)**



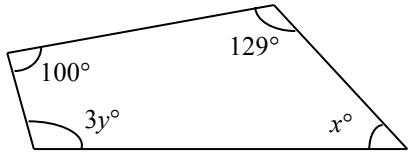
Answer (a) 39.8 cm [2]
(b) $x =$ 0.524 or 2.62 [2]

23. **Topics: Angle Properties of Polygons and Simultaneous Equations**

(a)



$2y° + 80° + x° = 180°$
$\boldsymbol{x = 100 - 2y}$ ..... (1)



$100° + 129° + 3y° + x° = 360°$ (int. ∠s of quadrilaterals)
$\boldsymbol{3y + x = 131}$ ..... (2)

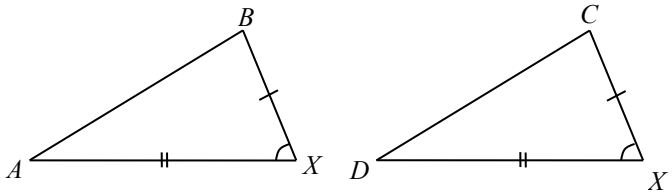
(b) Sub (1) into (2):

$3y + 100 - 2y = 131$
$y = 31$
Sub $y = 31$ into (1), $x = 100 - 2(31)$
$= 38$
$\therefore \boldsymbol{x = 38}$ and $\boldsymbol{y = 31}$

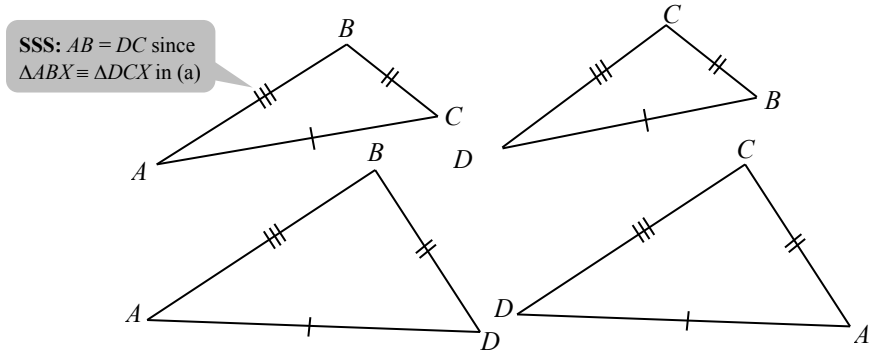
Answer (a) $x = 100 - 2y$
$3y + x = 131$ [2]
(b) $x =$ 38
$y =$ 31 [2]

24. **Topic: Congruency and Similarity**

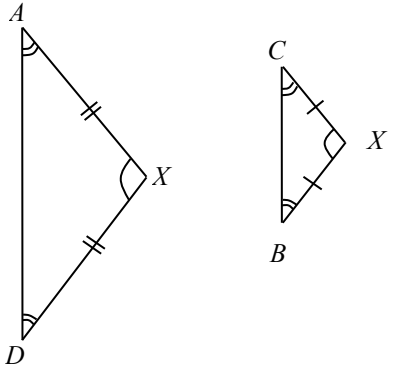
(a)



(b)



(c)



Answer (a) In triangles ABX and DCX,

$AX = DX$ (given)

$BX = CX$ (given)

$\angle AXB = \angle CXD$ (vertically opp. $\angle$s)

$\therefore$ By SAS property, ΔABX and ΔDCX are congruent. [2]

(b) Triangles *ABC* and *DCB*

or Triangles *ABD* and *DCA* [1]

(c) Triangles *ADX* and *CBX* [1]

25. **Topic: Coordinate Geometry**

(a) Gradient of $AB = \frac{3-1}{6-0}$

$= \frac{2}{6}$

$= \mathbf{\frac{1}{3}}$

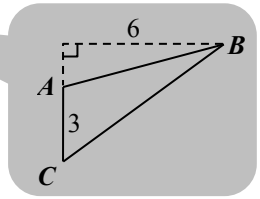
Gradient of straight line passing through $A(x_1, y_1)$ and $B(x_2, y_2)$
$= \frac{y_2-y_1}{x_2-x_1}$

(b) Equation of AB: $y = \frac{1}{3}x + c$

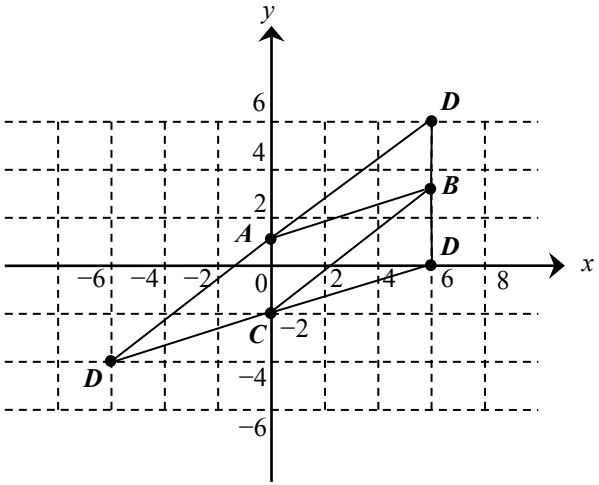
Sub (0, 1), $\therefore \boldsymbol{y = \frac{1}{3}x + 1}$

Equation of straight line with gradient m & y-intercept c:
$y = mx + c$

(c) Area of $\Delta ABC = \frac{1}{2} \times 3 \times 6$
$= \mathbf{9\ units^2}$



(d) **Two possible points of *D* are (−6, −4), (6, 0) or (6, 6) (Any two)**



Answer (a) $\frac{1}{3}$ cm [2]

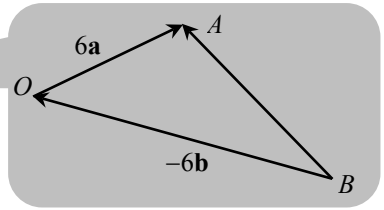
(b) $y = \frac{1}{3}x + 1$ [1]

(c) 9 unit^2 [1]

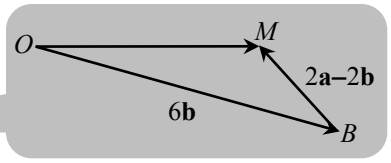
(d) (−6, −4)
(6, 0)
(6, 6) [2]

26. **Topic: Vectors in Two Dimensions**

(a) $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$
$= -6\mathbf{b} + 6\mathbf{a}$

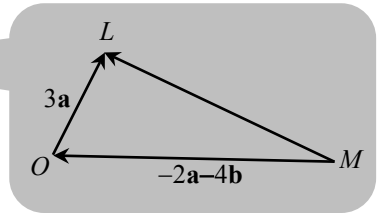


(i) $\overrightarrow{BM} = \frac{1}{3}\overrightarrow{BA}$
$= \frac{1}{3}[-6\mathbf{b} + 6\mathbf{a}]$
$= -2\mathbf{b} + 2\mathbf{a}$
$= 2\mathbf{a} - 2\mathbf{b}$



(ii) $\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM}$
$= 6\mathbf{b} + 2\mathbf{a} - 2\mathbf{b}$
$= 2\mathbf{a} + 4\mathbf{b}$

(iii) $\overrightarrow{ML} = \overrightarrow{MO} + \overrightarrow{OL}$
$= -2\mathbf{a} - 4\mathbf{b} + 3\mathbf{a}$
$= \mathbf{a} - 4\mathbf{b}$



(b) $\overrightarrow{LP} = 3\overrightarrow{LM}$
$\overrightarrow{OP} - \overrightarrow{OL} = 3(4\mathbf{b} - \mathbf{a})$
$\overrightarrow{OP} = 12\mathbf{b} - 3\mathbf{a} + 3\mathbf{a}$
$= 12\mathbf{b}$

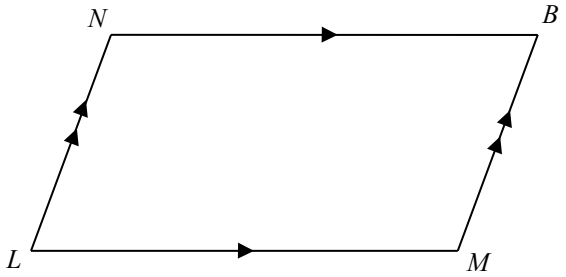
Expressing a vector in terms of its position vectors
$\overrightarrow{LP} = \overrightarrow{OP} - \overrightarrow{OL}$

(c) $\overrightarrow{OP} = 12\mathbf{b}$
$= 2(6\mathbf{b})$
$= 2\overrightarrow{OB}$

$\overrightarrow{AB} = k\overrightarrow{BC}$
$\Rightarrow$ *A*, *B* are collinear (straight line)

$\therefore$ *O*, *B* and *P* are collinear and *OP* is twice of *OB*.

(d)



Since $LMBN$ is a parallelogram,

$$\begin{aligned} \overrightarrow{NB} &= \overrightarrow{LM} \\ \overrightarrow{OB} - \overrightarrow{ON} &= 4\mathbf{b} - \mathbf{a} \\ \overrightarrow{ON} &= \overrightarrow{OB} - 4\mathbf{b} + \mathbf{a} \\ &= 6\mathbf{b} - 4\mathbf{b} + \mathbf{a} \\ &= 2\mathbf{b} + \mathbf{a} \\ &= \mathbf{a} + 2\mathbf{b} \end{aligned}$$

$\overrightarrow{NB}$ and $\overrightarrow{LM}$ are equal vectors
⇒ same direction and magnitude

Answer (a)(i) $\overrightarrow{BM}$ = $2\mathbf{a} - 2\mathbf{b}$ [2]

(ii) $\overrightarrow{OM}$ = $2\mathbf{a} + 4\mathbf{b}$ [1]

(iii) $\overrightarrow{ML}$ = $\mathbf{a} - 4\mathbf{b}$ [1]

(b) $\overrightarrow{OP} = 12\mathbf{b}$ [1]

(c) O, B and P are collinear, OP is twice of OB. [2]

(d) $\overrightarrow{ON} = \mathbf{a} + 2\mathbf{b}$ [1]