

**MATHEMATICS (H2)**  
 Paper 2 Suggested Solutions

**9740/02**  
**October/November 2008**

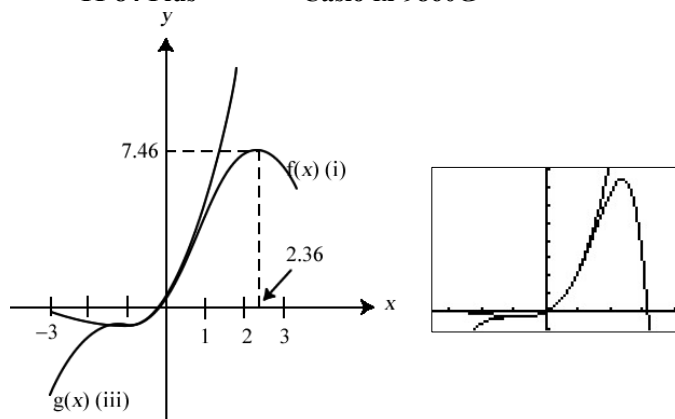
1. Topic : Functions and Graphs

(i)



TI-84 Plus

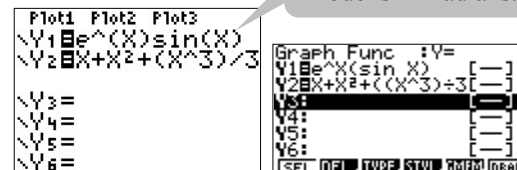
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(ii)  $f(x) = e^x \sin x$   
 $= (1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots) (x - \frac{x^3}{6} + \dots)$   
 $= x - \frac{x^3}{6} + x^2 + \frac{x^3}{2} + \dots$   
 $= x + x^2 + \frac{x^3}{3} + \dots$   
 $g(x) = x + x^2 + \frac{x^3}{3}$

Refer to formula list under Maclaurin's expansion

(iii)



TI-84 Plus

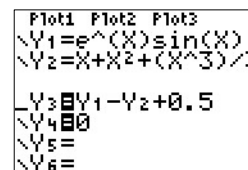
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Ensure calculator's mode is in Radians.

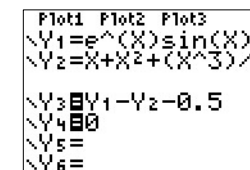
Refer to part (i)

(iv)

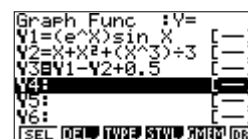
$|f(x) - g(x)| < 0.5$   
 $|e^x \sin x - (x + x^2 + \frac{x^3}{3})| < 0.5$   
 $-0.5 < e^x \sin x - (x + x^2 + \frac{x^3}{3}) < 0.5$   
 $e^x \sin x - (x + x^2 + \frac{x^3}{3}) + 0.5 > 0$  or  $e^x \sin x - (x + x^2 + \frac{x^3}{3}) - 0.5 < 0$   
 $x < 1.56$   $x > -1.96$



TI-84 Plus



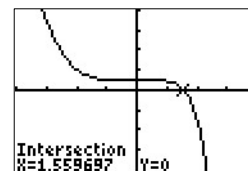
TI-84 Plus



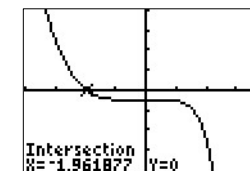
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$\therefore -1.96 \leq x \leq 1.56$



2. **Topic : Definite Integrals**

(i)  $y^2 = x(1-x)^{\frac{1}{2}}$   
 $y = x^{\frac{1}{2}}(1-x)^{\frac{1}{4}}$

Area of R =  $2 \int_0^1 x^{\frac{1}{2}}(1-x)^{\frac{1}{4}} dx$   
 = 2 (0.49944)  
 = 0.9988  
 ≈ **0.999 (3 sig. fig.)**

(ii) Let  $u = 1 - x$   
 $\therefore du = -dx$

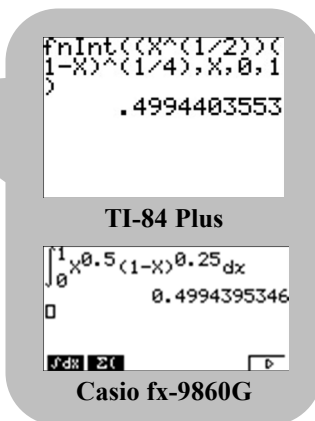
When  $x = 0$ ,  $u = 1$

When  $x = 1$ ,  $u = 0$

Volume =  $\pi \int_0^1 y^2 dx$   
 =  $\pi \int_0^1 x \sqrt{1-x} dx$   
 =  $\pi \int_1^0 -(1-u)\sqrt{u} du$   
 =  $\pi \int_0^1 (u^{\frac{1}{2}} - u^{\frac{3}{2}}) du$   
 =  $\pi \left[ \frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{u^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^1$   
 =  $\pi \left[ \frac{2}{3} - \frac{2}{5} \right]$   
 =  $\frac{4}{15} \pi \text{ units}^3$

Reversing limits:  
 $\int_a^b f(x) dx$   
 $= -\int_b^a f(x) dx$

(iii)  $y^2 = x \sqrt{1-x}$   
 $2y \frac{dy}{dx} = x \left( \frac{1}{2} \right) (1-x)^{-\frac{1}{2}} (-1) + \sqrt{1-x}$   
 $2y \frac{dy}{dx} = \frac{1}{2} (1-x)^{-\frac{1}{2}} [-x + 2(1-x)]$   
 $\frac{dy}{dx} = \left( \frac{1}{4} \right) \frac{2-3x}{y \sqrt{1-x}}$



To find max point, let  $\frac{dy}{dx} = 0$

$\left( \frac{1}{4} \right) \frac{2-3x}{y \sqrt{1-x}} = 0$   
 $2 - 3x = 0$   
 $\therefore x = \frac{2}{3}$

3. **Topic : Complex numbers (polar form)**

(a)  $w = re^{i\theta}$   
 $w^* = re^{i(-\theta)}$   
 $p = \frac{w}{w^*}$   
 $= \frac{r e^{i\theta}}{r e^{i(-\theta)}}$   
 $p = e^{i2\theta}$

$|p| = 1$ ,  $\arg(p) = 2\theta$

$p^5 = (e^{i2\theta})^5$   
 $= e^{i10\theta}$   
 $= \cos 10\theta + i \sin 10\theta$

Given that  $p^5$  is positive and real,

$p^5 = \cos 10\theta + i(0)$

$\Rightarrow \sin 10\theta = 0$

Basic  $\angle = 0$

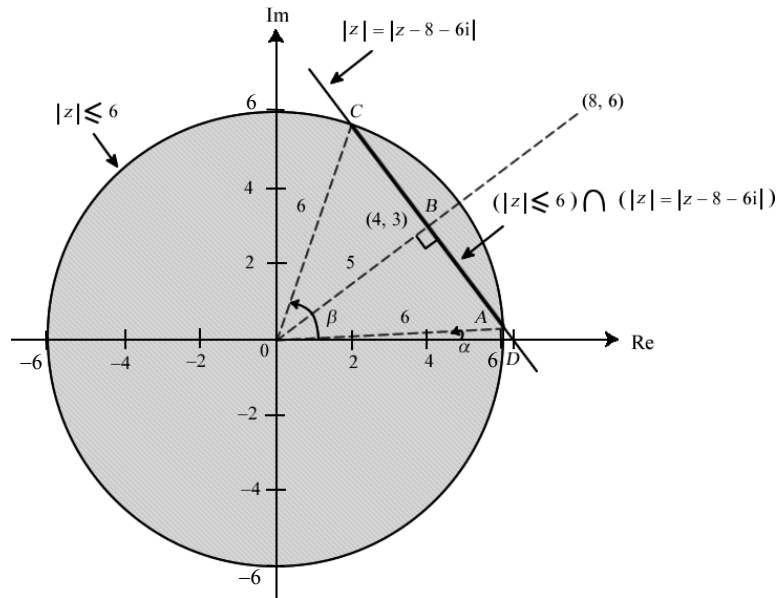
$10\theta = 2\pi, 4\pi$

$\therefore \theta = \frac{2\pi}{10}, \frac{4\pi}{10}$   
 $= \frac{\pi}{5}, \frac{2\pi}{5}$

For a complex number  $z = x + iy$ ,  
 when  $z$  is real, imaginary part  $y = 0$

Given that  $0 < \theta < \frac{\pi}{2}$ ,  
 $0 < 10\theta < 5\pi$

(b) (i)



(ii)

$$\begin{aligned} \cos \angle AOB &= \frac{5}{6} \\ \angle AOB &= \cos^{-1} \left( \frac{5}{6} \right) \\ &= 0.58569 \\ \sin \angle BOD &= \frac{3}{5} \\ \angle BOD &= \sin^{-1} \left( \frac{3}{5} \right) \\ &= 0.64350 \\ \angle COB &= \angle AOB \\ &= 0.58569 \end{aligned}$$

$$\begin{aligned} \therefore \text{least possible value of arg } z &= \alpha \\ &= \angle BOD - \angle AOB \\ &= 0.64350 - 0.58569 \end{aligned}$$

$$\begin{aligned} &= 0.05781 \\ &\approx \mathbf{0.058 \text{ rad (3 d.p.)}} \end{aligned}$$

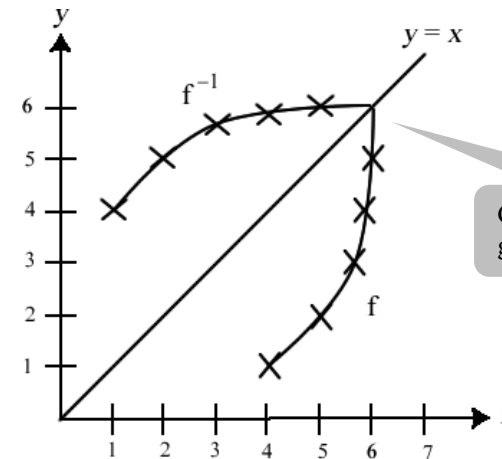
∴ greatest possible value of arg  $z = \beta$

Greatest arg  $z$  occurs at  $\angle COD = \beta$

$$\begin{aligned} &= \angle COB + \angle AOB + \alpha \\ &= 0.58569 + 0.58569 + 0.05781 \\ &= 1.2292 \\ &\approx \mathbf{1.229 \text{ rad (3 d.p.)}} \end{aligned}$$

4. **Topic : Functions and inverse functions**

(i)



Graph of  $f^{-1}$  is a reflection of graph of  $f$  in the line  $y = x$ .

(ii) Let  $y = (x - 4)^2 + 1$   
 $(x - 4)^2 = y - 1$   
 $x - 4 = \pm\sqrt{y - 1}$   
 $x = \pm\sqrt{y - 1} + 4$   
 $\therefore f^{-1}(x) = 4 + \sqrt{x - 1}, x > 1$

$D_{f^{-1}} = (1, \infty)$    $D_{f^{-1}} = R_f$

(iv) Equation of the line  $\Rightarrow y = x$

$f(x) = f^{-1}(x) = x$   Common point of intersection of the 3 graphs   
 $(x - 4)^2 + 1 = x$   
 $x^2 - 8x + 16 + 1 - x = 0$   
 $x^2 - 9x + 17 = 0$

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4(1)(17)}}{2}$$

$$= \frac{9 \pm \sqrt{81 - 68}}{2}$$

$$= \frac{9 \pm \sqrt{13}}{2}$$

$$= \frac{9 + \sqrt{13}}{2} \text{ or } \frac{9 - \sqrt{13}}{2} \text{ (rej.)}$$

$\therefore x = \frac{9 + \sqrt{13}}{2}$

**5. Topic : Sampling**

Call an assembly of the 950 pupils.

Start from a randomly selected student and pick every  $\frac{950}{50} = 19^{\text{th}}$  student.

As the different pupils belong to different groups and organizations that will use different sports facilities for different activities, a stratified sample put together from random samples taken from each group will provide a more accurate representation of the pupil population without any possible bias towards any groups.

**6. Topic : Hypothesis Testing**

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{1026}{15}$$

$$= 68.4$$

$$S^2 = \frac{1}{n-1} [\sum x^2 - \frac{(\sum x)^2}{n}]$$

$$= \frac{1}{14} [77265.90 - \frac{1026^2}{15}]$$

$$= 506.25 = (22.5)^2$$

$$\bar{X} \sim N(78, \frac{S^2}{n})$$

$$\bar{X} \sim N(78, \frac{506.25}{15})$$

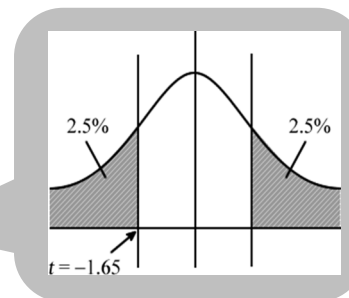
$H_0: \mu = 78$

$H_1: \mu \neq 78$

From G.C,

$$t = -1.6524$$

$$p = 0.121$$



Since  $p = 0.121 \geq 0.05$ , we do not reject  $H_0$ . We conclude that at the 5% level, there is insufficient evidence to suggest that the mean mass of calcium in a bottle has changed.

```
T-Test
Inpt:Data 58303
μ0:78
x̄:68.4
Sx:22.5
n:15
μ:μ0 <μ0 >μ0
Calculate Draw
```

```
T-Test
μ≠78
t=-1.652472894
p=.1206798869
x̄=68.4
Sx=22.5
n=15
```

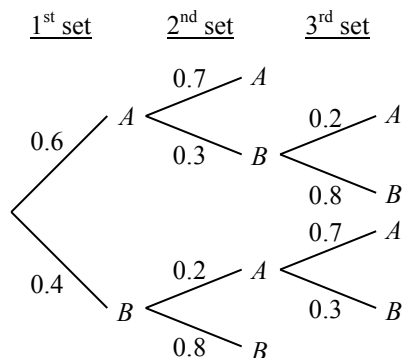
TI-84 Plus

1-Sample tTest	
Data	Variable
$\mu_0$	70
$\bar{x}$	60.4
$s_{\text{dev}}$	16.5
$n$	15
List Var	

1-Sample tTest	
$t$	-1.6524729
$p$	0.12067989
$x/\bar{n}$	68.4
$s_{\text{dev}}$	16.5
$n$	15

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7. Topic : Probability



(i)  $P(A \text{ wins in the second set}) = 0.6(0.7) + 0.4(0.2)$   
 $= 0.5$  AA + BA

(ii)  $P(A \text{ wins the match}) = 0.6(0.7) + 0.4(0.2)(0.7) + 0.6(0.3)(0.2)$   
 $= 0.42 + 0.056 + 0.036$   
 $= 0.512$  AA + BAA + ABA

\*Note that this is a conditional probability question.

(iii)  $P(B \text{ won the first set} | A \text{ won the match})$   
 $= \frac{P(B \text{ wins 1st set and } A \text{ wins the match})}{P(A \text{ wins the match})}$   
 $= \frac{0.4(0.2)(0.7)}{0.512}$  BAA  
 $= \frac{7}{64}$  from (ii)

8. Topic : Correlation coefficient and linear regression

(i) From G.C,  $r = 0.970$  (3 sig. fig.)

Since  $r$  is close to 1, this indicates that there is a strong positive linear correlation between  $x$  and  $t$ .

L1	L2	L3	1
2.2	2.2	-----	
2.7	4.5		
3.8	7.6		
4.8	9.9		
5.6			
6.9			
L1(D)=1.2			

LinReg	
$y=a+bx$	
$a=1.739267374$	
$b=1.263893644$	
$r^2=.9399848275$	
$r=.9695281468$	

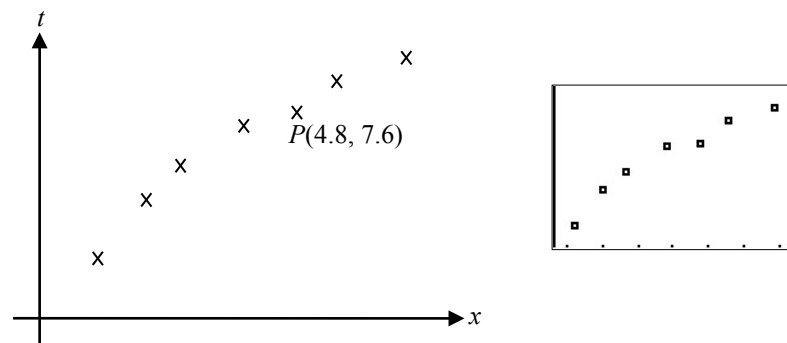
TI-84 Plus

List 1		List 2		List 3		List 4	
2.2	4.5	1.2					
2.7	7.6						
3.8	9.9						
4.8							
5.6							
6.9							
L1(D)=1.2							

LinearReg	
$a=0.74372146$	
$b=-1.0620433$	
$r=.96952814$	
$r^2=.93998482$	
$MSe=0.30052168$	
$y=ax+b$	

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(ii)



(iii) As the remaining points' scattering pattern appears to follow that of a logarithmic data set, the scatter diagram may be modeled by a straight line  $t = a + b \ln x$ .

- (iv) Using G.C,  
 $t = 1.42 + 4.40 \ln x$   
 $\therefore a = 1.42, b = 4.40$

```
LinReg
y=a+bx
a=1.424721784
b=4.396562723
r2=.9999552585
r=.999977629
```

TI-84 Plus

```
LinearReg
a = 4.39656272
b = 1.42472178
r = 0.99997762
r2 = 0.99995525
MSE = 4.6324E-04
y = ax + b
```

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- (v)  $t = 1.42 + 4.40 \ln x$   
 When  $x = 4.8$ ,  
 $t = 1.42 + 4.40 \ln 4.8$   
 $= 8.32$  (3 sig. fig.)

Remove coordinates of  $P$  and  $\ln$  values of  $x$ .

- (vi) Since  $x = 8$  is out of the given data set's range, the value of  $t$  obtained may not be accurate.

9. **Topic : Poisson Distribution**

Let r.v.  $X$  = number of grand pianos sold in a week

$$X \sim \text{Po}(1.8)$$

$$\begin{aligned} P(X \geq 4) &= 1 - P(X \leq 3) \\ &= 1 - 0.89129 \\ &= 0.1087 \\ &\approx 0.109 \text{ (3 sig. fig.)} \end{aligned}$$

```
Poissoncdf(1.8,3)
)
.8912916054
```

TI-84 Plus

```
Poisson C.D
Data :Variable
x :3
μ :1.8
Save Res:None
Execute
```

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```
Poisson C.D
P=0.8912916
```

Let r.v.  $Y$  = number of upright pianos sold in a week,  $Y \sim \text{Po}(2.6)$

$X + Y$  = total number of pianos sold in a week

$$\begin{aligned} X + Y &\sim \text{Po}(1.8 + 2.6) \\ &\sim \text{Po}(4.4) \end{aligned}$$

$$\begin{aligned} P(X + Y = 4) &= 0.1917 \\ &\approx 0.192 \text{ (3 sig. fig.)} \end{aligned}$$

```
PoissonPdf(4.4,4)
)
.1917360358
```

TI-84 Plus

```
Poisson P.D
Data :Variable
x :4
μ :4.4
Save Res:None
Execute
```

```
Poisson P.D
P=0.19173603
```

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$$\begin{aligned} \text{Mean number of pianos sold in a year} &= 50 \times 1.8 \\ &= 90 \end{aligned}$$

Let  $A$  = number of grand pianos sold in a year.  
 $A \sim \text{Po}(90)$

Since mean  $> 10$ ,  $A \sim N(90, 90)$  approximately

$$\begin{aligned} P(A < 80) &= P(A < 79.5) \text{ [Continuity Correction]} \\ &= 0.1341 \\ &\approx 0.134 \text{ (3 sig. fig.)} \end{aligned}$$

Use continuity correction to approximate a discrete distribution (i.e Poisson) by a continuous distribution (i.e normal).

```
normalcdf(-E99,79.5,90,√(90))
)
.134190856
```

TI-84 Plus

```
Normal C.D
Lower :-1E+99
Upper :79.5
σ :9.48683298
μ :90
Save Res:None
Execute
```

```
Normal C.D
P = 0.13419081
z:Low = -1.054E+98
z:UP = -1.1067972
```

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The sales of the pianos are likely to follow demand trends across the different months of the year. Hence a piano sale should not be considered random event within the interval of a year.

10. **Topic : Combinations**

(i)  ${}^3C_2 {}^4C_3 {}^5C_3 = 120$

Choose 2 from 3 K  
 3 from 4 L  
 3 from 5 M

(ii)  ${}^9C_8 = 9$

Choose 8 from the total of 9 from L & M

(iii)  ${}^5C_4 {}^7C_4 + {}^5C_5 {}^7C_3 = 175 + 35 = 210$

4 from M and 4 from remaining 7 (K + L)  
**OR**

5 from M and 3 from remaining 7 (K + L)

(iv)  ${}^{12}C_8 - {}^9C_8 - {}^8C_8 - 0 = 495 - 9 - 1 - 0 = 485$

${}^{12}C_8$ : total no. of ways with no restriction  
 ${}^9C_8$ : group without K  
 ${}^8C_8$ : group without L

11. **Topic : Normal distributions**

(i)  $X_1 + X_2 \sim N(50 \times 2, 8^2 \times 2) \sim N(100, 128)$

0: group without M (impossible as  $K + L = 7$ )

$P(X_1 + X_2 > 120) = 0.03854$   
 $\approx 0.0385$  (3 sig. fig.)

$X_1 = X_2 \sim N(\mu, \sigma^2)$   
 $E(X_1 + X_2) = \mu + \mu = 2\mu$   
 $\text{Var}(X_1 + X_2) = \sigma^2 + \sigma^2 = 2\sigma^2$

```
normalcdf(120, E99, 100, sqrt(128))
.0385498886
```

TI-84 Plus

```
Normal C.D
Lower : 120
Upper : 1E+99
σ : 11.3137084
μ : 100
Save Res:None
Execute
```

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```
Normal C.D
P = 0.03854993
z:Low=1.76776695
z:Up = 8.8388E+97
```

(ii)  $X_1 - X_2 \sim N(0, 128)$

$P(X_1 > X_2 + 15) = P(X_1 - X_2 > 15)$   
 $= 0.0924$  (3 sig. fig.)

$E(X_1 - X_2) = \mu - \mu = 0$   
 $\text{Var}(X_1 - X_2) = \sigma^2 + \sigma^2 = 2\sigma^2$

```
normalcdf(15, E99, 0, sqrt(128))
.0924488624
```

TI-84 Plus

```
Normal C.D
Lower : 15
Upper : 1E+99
σ : 11.3137084
μ : 0
Save Res:None
Execute
[None] [LIST]
```

```
Normal C.D
P = 0.09244879
z:Low=1.32582521
z:Up = 8.8388E+97
```

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Consider,  $Y \sim N(\mu, \sigma^2)$

$P(Y < 74) = 0.0668$

$P(Z < \frac{74-\mu}{\sigma}) = 0.0668$

$\Rightarrow \frac{74-\mu}{\sigma} = -1.5 \quad -(1)$

Change to standard normal

$X \sim N(\mu, \sigma^2)$

$z = \frac{x-\mu}{\sqrt{\sigma^2}}$

$= \frac{x-\mu}{\sigma} \sim N(0, 1)$

```
invNorm(0.0668, 0, 1)
-1.500055602
```

TI-84 Plus

```
Inverse Normal
tail : Left
Area : 0.0668
σ : 1
μ : 0
Save Res:None
Execute
[LEFT] [RIGHT] [ENTER]
```

```
Inverse Normal
x=-1.5000556
```

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$P(Y > 146) = 0.0668$

$P(Y < 146) = 1 - 0.0668 = 0.9332$

$P(Z < \frac{146-\mu}{\sigma}) = 0.9332$

$\Rightarrow \frac{146-\mu}{\sigma} = 1.5 \quad -(2)$

```
invNorm(0.9332, 0, 1)
1.500055602
```

TI-84 Plus

```
Inverse Normal
tail : Left
Area : 0.9332
σ : 1
μ : 0
Save Res:None
Execute
[LEFT] [RIGHT] [ENTER]
```

```
Inverse Normal
x=1.5000556
```

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(1) - (2),

$$-\frac{72}{\sigma} = -3$$
$$\sigma = 24$$

sub  $\sigma = 24$  into (1),

$$74 - \mu = -1.5(24)$$
$$\mu = 110$$

$$\therefore \mu = 110, \sigma = 24$$

$$\therefore E(Y) = 110$$

$$\text{Var}(Y) = 24^2$$
$$= 576$$

$$E(aX + b) = 110$$

$$aE(X) + b = 110$$

$$50a + b = 110$$

$$b = 110 - 50a \quad - (3)$$

$$\text{Var}(aX + b) = 24^2$$

$$a^2 \text{Var}(X) = 576$$

$$a^2(64) = 576$$

$$a^2 = 9$$

$$a = 3 \text{ or } -3 \text{ (reject)}$$

sub  $a = 3$  into (3),

$$b = 110 - 50(3)$$
$$= -40$$

$$\therefore a = 3, b = -40$$

From qn,  
 $E(X) = 50$

