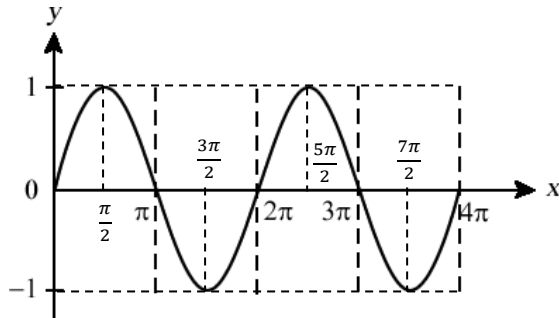


MATHEMATICS (H1)
 Paper 1 Suggested Solutions

8863/01
 October/November 2008

1. **Topic: Graphs**

$y = \sin x, 0 \leq x \leq 4\pi$



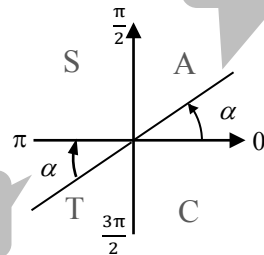
$\sin \alpha = c$

(i) $\sin(2\pi + \alpha) = \sin \alpha = c$

(ii) $\sin(3\pi + \alpha) = -\sin \alpha = -c$

$\sin x = -c$
 $x = \pi + \alpha$ or $2\pi - \alpha$

3π + x will end up at 3rd quad



2. **Topic: Simultaneous Equations**

$x + y = 20$
 $x = 20 - y$ (1)
 $x^2 + y^2 = 300$ (2)

Sub (1) into (2),
 $(20 - y)^2 + y^2 = 300$

$400 - 40y + y^2 + y^2 = 300$
 $2y^2 - 40y + 100 = 0$
 $y^2 - 20y + 50 = 0$

$y = \frac{20 \pm \sqrt{20^2 - 4(1)(50)}}{2}$
 $y = \frac{20 \pm \sqrt{200}}{2}$
 $y = 10 + 5\sqrt{2}$ (rejected) or $10 - 5\sqrt{2}$

$\therefore x = 20 - (10 - 5\sqrt{2})$
 $= 10 + 5\sqrt{2}$

3. **Topic: Simultaneous Equations, Integration**

$y = 2x^2$ (1)
 $y = x^2 + k^2$ (2)

(1) = (2),
 $2x^2 = x^2 + k^2$
 $x^2 = k^2$
 $x = k$ or $-k$

Graphs are symmetrical about the y-axis.

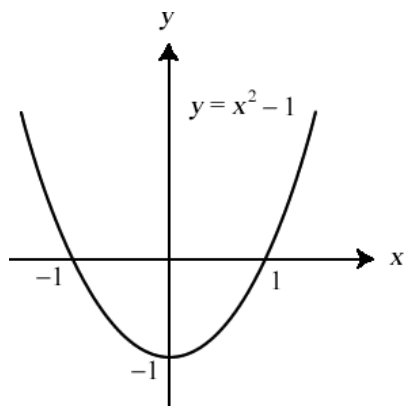
\therefore x-coordinates of P & Q are k or -k (Shown)

Shaded area between c_1 & $c_2 = 2 \int_0^k (x^2 + k^2 - 2x^2) dx$
 $= 2 \int_0^k (k^2 - x^2) dx$
 $= 2 \left[k^2x - \frac{x^3}{3} \right]_0^k$
 $= 2 \left[k^3 - \frac{k^3}{3} \right]$
 $= 2 \left[\frac{2}{3} k^3 \right]$
 $= \frac{4}{3} k^3$

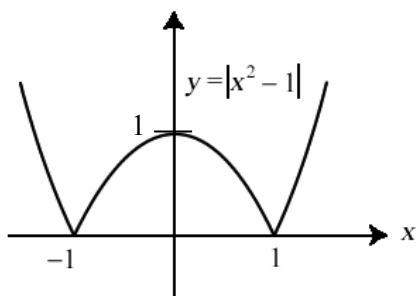
$\int x^n dx = \frac{x^{n+1}}{n+1} + c$

4. **Topics: Functions**

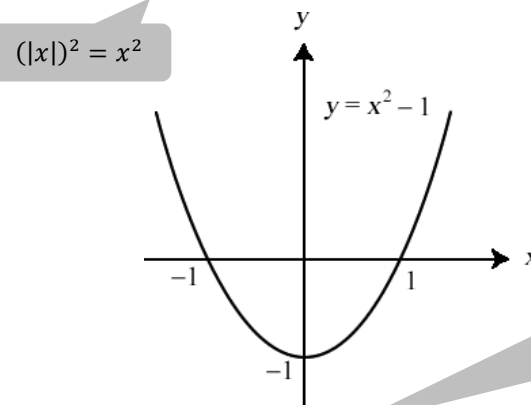
(i) (a) $y = f(x) = x^2 - 1$



(b) $y = gf(x) = g(x^2 - 1) = |x^2 - 1|$



(c) $y = fg(x)$
 $= f(|x|)$
 $= (|x|)^2 - 1$
 $= x^2 - 1$



f has to be a one-one function since $x \leq a$.

(ii) For f to have an inverse \Rightarrow Greatest value of $a = 0$

$D_{f^{-1}} = R_f = [-1, \infty]$
 Let $y = x^2 - 1$
 $y + 1 = x^2$
 $x = \pm \sqrt{y + 1}$
 $\therefore f^{-1}(x) = -\sqrt{x + 1}$

Take the $-ve$ part of the graph since $x \leq a$

5. **Topic: Differentiation**

$$x = t^3 - 12t^2 + kt$$

$$\frac{dx}{dt} = 3t^2 - 24t + k$$

Since x is an increasing function of t

$$\Rightarrow \frac{dx}{dt} > 0$$

$$(3t^2 - 24t + k) > 0$$

$$\Rightarrow b^2 - 4ac < 0$$

$$(-24)^2 - 4(3)(k) < 0$$

$$576 - 12k < 0$$

$$576 < 12k$$

$$k > 48$$

$ax^2 + bx + c > 0$ for all real values of x
 $\Leftrightarrow b^2 - 4ac < 0$ and $a > 0$

(i) $x = t^3 - 12t^2 + 36t$
 $= t(t^2 - 12t + 36)$
 $= t(t - 6)^2$

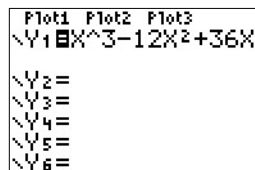
$$\frac{dx}{dt} = 3t^2 - 24t + 36$$

When $\frac{dx}{dt} = 0$

$$t^2 - 8t + 12 = 0$$

$$(t - 6)(t - 2) = 0$$

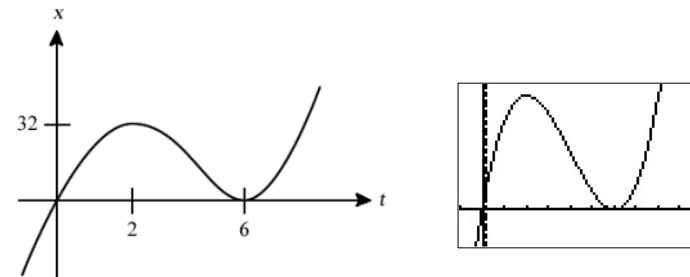
$$t = 2 \text{ or } t = 6$$



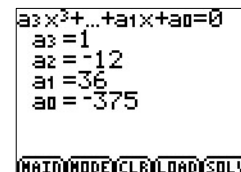
TI-84 Plus



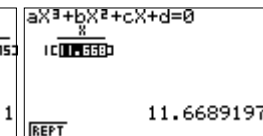
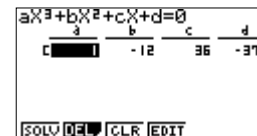
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(ii) $t^3 - 12t^2 + 36t = 375$
 $t^3 - 12t^2 + 36t - 375 = 0$
 $t = 11.66892$
 $t \approx 11.7$ (3 sig. fig.)



TI-84 Plus



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6. **Topic: Applications of Differentiation**

$$y = \ln(2x + 4)$$

$$\frac{dy}{dx} = \frac{2}{2x+4}$$

$$= \frac{1}{x+2}$$

When $x = 1$, $\frac{dy}{dx} = \frac{1}{3}$

Equation of tangent at P :

$$\Rightarrow y - \ln 6 = \frac{1}{3}(x - 1)$$

$$y = \frac{1}{3}x - \frac{1}{3} + \ln 6$$

When $y = 0$,

$$\frac{1}{3}x - \frac{1}{3} + \ln 6 = 0$$

$$x - 1 + 3\ln 6 = 0$$

$$x = 1 - 3\ln 6$$

\therefore x -coordinate of $T = 1 - 3\ln 6$ (**Shown**)

Gradient of $PN = -3$

Equation of PN :

$$\Rightarrow y - \ln 6 = -3(x - 1)$$

$$y = -3x + 3 + \ln 6$$

When $y = 0$,

$$-3x + 3 + \ln 6 = 0$$

$$3x = 3 + \ln 6$$

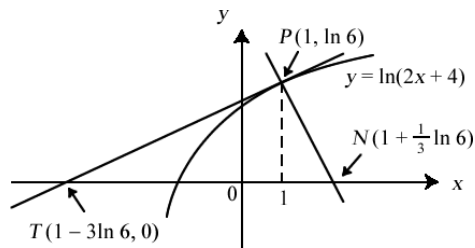
$$x = 1 + \frac{1}{3}\ln 6$$

\therefore x -coordinate of $N = 1 + \frac{1}{3}\ln 6$

$$\text{Area of } \Delta PTN = \frac{1}{2}(\ln 6) \left[1 + \frac{1}{3}\ln 6 - (1 - 3\ln 6) \right]$$

$$= \frac{1}{2}(\ln 6) \left[\frac{1}{3}\ln 6 + 3\ln 6 \right]$$

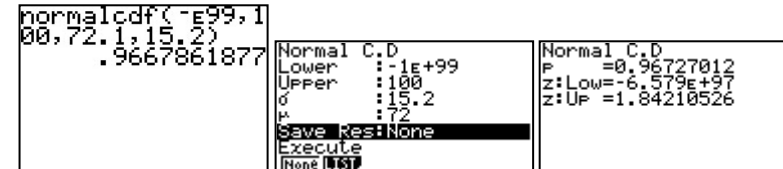
$$= \frac{5}{3}(\ln 6)^2 \text{ units}^2$$



Gradient of $TP = \frac{1}{3}$
 \Rightarrow Gradient $PN = -\frac{1}{\frac{1}{3}} = -3$ ($TP \perp PN$)

7. **Topic: Normal Distribution**

Using this distribution, we have $P(X > 100) = 1 - 0.967 = 0.033$ which is impossible since the maximum marks is 100. Hence, normal distribution is not a good approximation.



TI-84 Plus

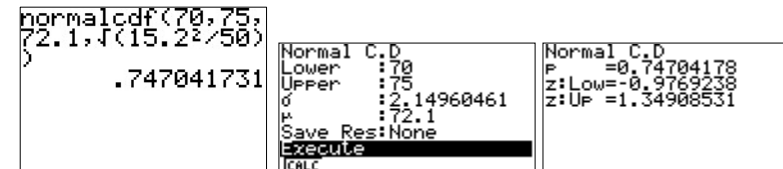
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Let random variable X be the mean mark for all candidates, i.e. $X \sim N(72.1, 15.2^2)$

For a random sample of 50 candidates, by Central Limit Theorem:

$$\bar{X} \sim N\left(72.1, \frac{15.2^2}{50}\right)$$

$P(70 < \bar{X} < 75) = 0.747$ (3 sig. fig.)



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8. **Topic: Normal Distribution, Hypothesis Testing**

Let the random variable X be the number of loaves of bread, out of 6 loaves, which are crusty.

$$X \sim B(6, 0.6)$$

$$P(X=3) = 0.2764$$

$$\approx 0.276 \text{ (3 sig. fig.)}$$

<pre>binompdf(6,0.6,3)) .27648</pre>	<pre>Binomial P.D Data :Variable X :3 Numtrial:6 p :0.6 Save Res:None Execute</pre>	<pre>Binomial P.D P=0.27648</pre>
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Let the random variable Y be the number of loaves of bread, out of 40 loaves, which are crusty.

$$n = 40, p = 0.6, q = 0.4$$

Since n is large, $np = 24 > 5$ and $nq = 16 > 5$

$$\therefore Y \sim B(40, 0.6)$$

$$\sim N(40 \times 0.6, 40 \times 0.6 \times 0.4) \text{ approximately}$$

$$\sim N(24, 9.6)$$

$$P(Y \geq 20) = P(Y > 19.5) \text{ [Continuity Correction]}$$

$$= 0.927$$

<pre>normalcdf(19.5, E 99, 24, 9.6) .9268004102</pre>	<pre>Normal C.D Lower :19.5 Upper :1e+99 σ :3.09838667 μ :24 Save Res:None Execute</pre>	<pre>Normal C.D P =0.92680045 z:Low=-1.4523688 z:Up =3.2275e+98</pre>
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Use continuity correction to approximate a discrete distribution (i.e binomial) by a continuous distribution (i.e normal).

Let the random variable M be the mass of a loaf.

$$M \sim N(1.24, \sigma^2)$$

$$P(M < 1) = 0.04$$

$$P\left(Z < \frac{1-1.24}{\sigma}\right) = 0.04$$

$$\frac{1-1.24}{\sigma} = -1.75068$$

$$-0.24 = -1.75068\sigma$$

$$\sigma = 0.13708$$

$$\sigma \approx 0.137 \text{ (3 sig. fig.)}$$

```
invNorm(0.04,0,1)
)
-1.750686071
```

TI-84 Plus

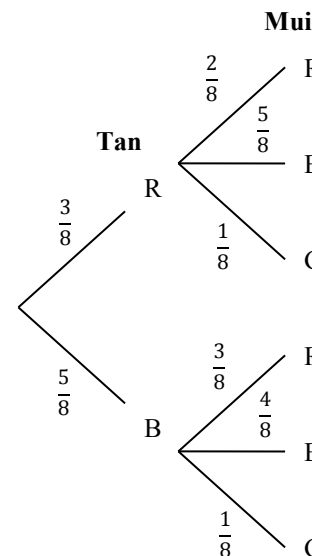
```
Inverse Normal
Left:Left
Area :0.04
σ :1
μ :0
Save Res:None
Execute
LEFT RIGHT CNTR
```

```
Inverse Normal
x=-1.7506861
```

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9. **Topic: Probability**

(i)



$$\begin{aligned} \text{(ii) } P(\text{Mui's pen is blue} \mid \text{Tan's pen is red}) &= \frac{P(\text{Tan's pen red AND Mui's pen blue})}{P(\text{Tan's pen red})} \\ &= \frac{\frac{3}{8} \left(\frac{5}{8} \right)}{\frac{3}{8}} \\ &= \frac{5}{8} \end{aligned}$$

$$\begin{aligned} \text{(iii) } P(\text{Mui's pen is red}) &= \frac{3}{8} \left(\frac{2}{8} \right) + \frac{5}{8} \left(\frac{3}{8} \right) \\ &= \frac{21}{64} \end{aligned}$$

RR + BR

$$\begin{aligned} \text{(iv) } P(\text{Tan's pen is red} \mid \text{Mui's pen is blue}) &= \frac{P(\text{Tan's pen red AND Mui's pen blue})}{P(\text{Mui's pen blue})} \\ &= \frac{\frac{3}{8} \left(\frac{5}{8} \right)}{\frac{3}{8} \left(\frac{5}{8} \right) + \frac{5}{8} \left(\frac{3}{8} \right)} \\ &= \frac{3}{7} \end{aligned}$$

10. **Topic: Hypothesis Testing**

$$\begin{aligned} \bar{x} &= \frac{\sum x}{n} \\ &= \frac{10317}{70} \\ &= 147.38 \end{aligned}$$

σ_x = unbiased estimate of population variance

s_x = sample variance

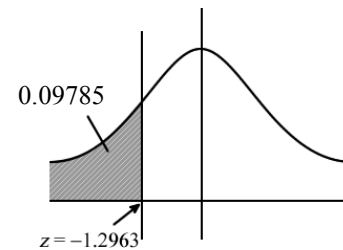
$$\begin{aligned} \sigma_x^2 &= \frac{n}{n-1} s_x^2 \\ &= \frac{n}{n-1} \left[\frac{\sum x^2}{n} - (\bar{x})^2 \right] \\ &= \frac{70}{69} \left[\frac{1540231}{70} - (147.3857)^2 \right] \\ &= 284.824 \end{aligned}$$

$$\begin{aligned} \bar{X} &\sim N\left(\mu, \frac{\sigma_x^2}{n}\right) \\ &\sim N\left(150, \frac{284.824}{70}\right) \end{aligned}$$

$$H_0: \mu = 150$$

$$H_1: \mu < 150$$

$$\begin{aligned} z &= \frac{\bar{x} - \mu}{\frac{\sigma_x}{\sqrt{n}}} \\ &= \frac{147.385 - 150}{\sqrt{\frac{284.824}{70}}} \\ &= -1.2963 \end{aligned}$$



From G.C., p -value = 0.09785

\Rightarrow 9.7% of the sample batteries size of 70 has lifetime less than 150 hours

<pre>Z-Test Inpt:Data μ₀:150 σ:16.876729540... x̄:147.39 n:70 μ≠μ₀ Calculate Draw</pre>	<pre>Z-Test μ<150 z=-1.293901561 p=.0978497778 x=147.39 n=70</pre>
---	---

TI-84 Plus

```

i-Sample ZTest
μ < μ0
μ0 : 150
σ : 19.8767295
x̄ : 147.39
n : 70
Save Res:None
(Home LIST)
    
```

```

i-Sample ZTest
μ < μ0
μ0 : 150
σ : 19.8767295
x̄ : 147.39
n : 70
    
```

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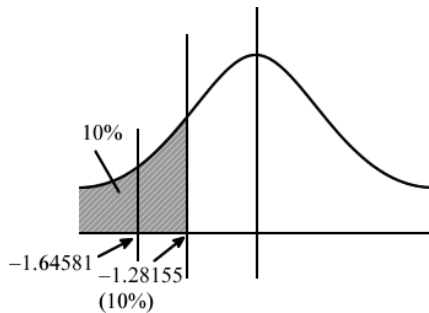
$$\begin{aligned} \text{Mean} &= \frac{\Sigma x + \Sigma y}{n_1 + n_2} \\ &= \frac{10317 + 7331}{70 + 50} \\ &= 147.067 \end{aligned}$$

$$\begin{aligned} \text{Unbiased estimate of population variance} &= \frac{n}{n-1} \left[\frac{\Sigma x^2 + \Sigma y^2}{n_1 + n_2} - (\text{mean})^2 \right] \\ &= \frac{120}{119} \left[\frac{1540231 + 1100565}{120} - 147.067^2 \right] \\ &= 381.107 \end{aligned}$$

$$H_0 : \mu = 150$$

$$H_1 : \mu < 150$$

$$\begin{aligned} z &= \frac{147.067 - 150}{\sqrt{\frac{381.107}{120}}} \\ &= -1.64581 \end{aligned}$$



Since $p\text{-value} = 4.99\% < 10\%$, we reject H_0 .
 \therefore there is sufficient evidence, at the 10% significance level that $\mu < 150$.

```

Z-Test
Inpt:Data STAT
μ0:150
σ:19.521961991...
x̄:147.067
n:120
μ:≠μ0 < μ0 > μ0
Calculate Draw
    
```

```

Z-Test
μ < μ0
μ0 : 150
σ : 19.521961991...
x̄ : 147.067
n : 120
    
```

TI-84 Plus

```

i-Sample ZTest
μ < μ0
μ0 : 150
σ : 19.521961991...
x̄ : 147.067
n : 120
Save Res:None
    
```

```

i-Sample ZTest
μ < μ0
μ0 : 150
σ : 19.521961991...
x̄ : 147.067
n : 120
Save Res:None
    
```

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11. Topic: Correlation Coefficient and Linear Regression

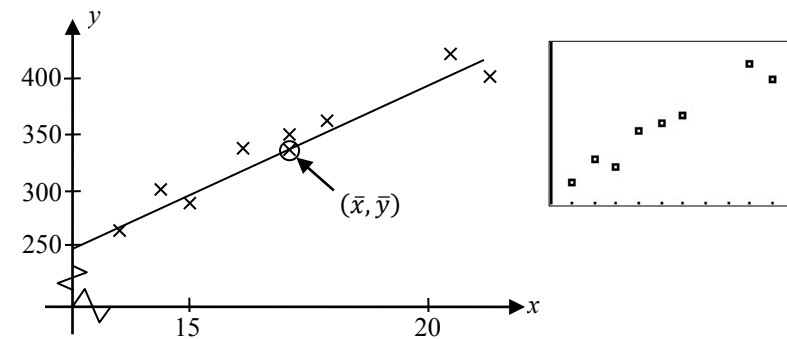
(i)

L1	L2	L3	Z
15	290	-----	
17	350		
13	270		
21	430		
16	340		
22	410		
14	300		

TI-84 Plus

Sub	List 1	List 2	List 3	List 4
1	15	290		
2	17	350		
3	13	270		
4	21	430		

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- (ii) From G.C.,
 $\bar{x} = 17$
 $\bar{y} = 343.75$

2-Var Stats $\bar{x}=17$ $\Sigma x=136$ $\Sigma x^2=2384$ $Sx=3.207134903$ $\sigma x=3$ $\downarrow n=8$	2-Var Stats $\uparrow \bar{y}=343.75$ $\Sigma y=2750$ $\Sigma y^2=967700$ $Sy=56.55275666$ $\sigma y=52.90025992$ $\downarrow \Sigma xy=47980$
--	--

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2-Variable $\bar{x}=17$ $\Sigma x=136$ $\Sigma x^2=2384$ $x\sigma n=3$ $x\sigma n=3.2071349$ $n=8$	2-Variable $\bar{y}=343.75$ $\Sigma y=2750$ $\Sigma y^2=967700$ $y\sigma n=52.9002599$ $y\sigma n=56.5527566$ $\Sigma xy=47980$
--	---

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- (iii) From G.C, $y = 53.3 + 17.1x$
 (iv) From G.C.,

Product moment correlation coefficient, $r = 0.969$
 Since r is close to 1, this confirms a strong positive correlation between x and y .

LinReg $y=a+bx$ $a=53.33333333$ $b=17.08333333$ $r^2=.9385817979$ $r=.9688043135$	LinearReg $a=17.08333333$ $b=53.33333333$ $r=.96880431$ $r^2=.93858179$ $MSe=229.166666$ $y=ax+b$
--	---

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- (v) $x = 20$,
 $y = 53.3 + 17.1(20)$
 $= 395$
 \therefore Corresponding profit = **395 (in thousand dollars)**

- (vi) As $x = 40$ falls outside the range $15 \leq x \leq 22$, we would be extrapolating and hence it would not give a reliable estimate of y .

12. **Topics: Normal Distributions**

Let M = Mass of a apple
 $M \sim N(0.234, 0.025^2)$

Let $S = M_1 + M_2 + M_3 + M_4 + M_5$
 i.e. $S \sim N(1.17, 3.125 \times 10^{-3})$

Let $T = M_1 + M_2 + \dots M_{10}$
 i.e. $T \sim N(2.34, 6.25 \times 10^{-3})$

$$P(M_1 + M_2 + \dots M_5 > 1.2 \text{ kg}) = P(S > 1.2)$$

$$= 0.2957$$

$$\approx \mathbf{0.296 \text{ (3 sig. fig.)}}$$

normalcdf(1.2, E9 $9, 1.17, \sqrt{3.125 \times 10^{-3}})$ $.2957524928$	Normal C.D Lower : 1.2 Upper : 1E+99 $\sigma : 0.05590169$ $\mu : 1.17$ Save Res: None Execute	Normal C.D $P = 0.29575251$ $z: \text{Low} = 0.53665631$ $z: \text{Up} = 1.7889E+99$
---	--	---

TI-84 Plus

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$$(S_1 + S_2) - T \sim N[2(1.17) - 2.34, 2 \times 0.00315 + 0.00625]$$

$$\sim N(0, 0.0125)$$

$$P(|(S_1 + S_2) - T| \leq 0.2) = P(-0.2 \leq [S_1 + S_2 - T] \leq 0.2)$$

$$= 0.9263$$

$$\approx \mathbf{0.926 \text{ (3 sig. fig.)}}$$




```
normalcdf(-0.2,0
2.0,√(0.0125))
.9263618329
```

TI-84 Plus

```
Normal C.D
Lower :-0.2
Upper :0.2
σ :0.11180339
μ :
Save Res:None
Execute
```

```
Normal C.D
P =0.92636173
z:Low=-1.7888544
z:Up =1.78885438
```

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$$P[(S_1 + S_2)1.5 - 1.2(T) \geq 0.5] = P\left[(S_1 + S_2) - 0.8T \geq \frac{1}{3}\right]$$

$$(S_1 + S_2) - 0.8T$$

$$\sim N[2 \times 1.17 - 0.8 \times 2.34, 2(3.125 \times 10^{-3}) + 0.8^2(6.25 \times 10^{-3})]$$

$$\sim N(0.468, 0.01025)$$

$$\therefore P\left[(S_1 + S_2) - 0.8T \geq \frac{1}{3}\right] = 0.908264$$

$$\approx \mathbf{0.908 \text{ (3 sig. fig.)}}$$

```
normalcdf(1/3, E9
9,0.468,√(0.0102
5))
.9082642789
```

TI-84 Plus

```
Normal C.D
Lower :0.33333333
Upper :1E+99
σ :0.10124228
μ :0.468
Save Res:None
Execute
None [16]
```

```
Normal C.D
P =0.90826434
z:Low=-1.3301425
z:Up =9.8773E+99
```

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Alternative Method:

Let Y be the random variable denoting the amount of money in \$ that Lee pays more than Foo.

$$Y = 1.5(S_1 + S_2) - 1.2T$$

$$\begin{aligned} E(Y) &= E[1.5(S_1 + S_2) - 1.2T] \\ &= 1.5[E(S_1) + E(S_2)] - 1.2E(T) \\ &= 1.5(1.17 + 1.17) - 1.2(2.34) \\ &= 3.51 - 2.808 \\ &= 0.702 \end{aligned}$$

$$\begin{aligned} \text{Var}(Y) &= \text{Var}[1.5(S_1 + S_2) - 1.2T] \\ &= 1.5^2[\text{Var}(S_1) + \text{Var}(S_2)] + 1.2^2 \text{Var}(T) \\ &= 1.5^2(0.003125 + 0.003125) + 1.2^2(0.00625) \\ &= 0.0140625 + 0.009 \\ &= 0.0230625 \end{aligned}$$

$$Y \sim N(0.702, 0.0230625).$$

Using G. C., $P(Y \geq 0.50) = 0.908264$
 $\approx \mathbf{0.908 \text{ (3 sig. fig.)}}$

```
normalcdf(0.5, E9
9,0.702,√(0.0230
625))
.9082642789
```

TI-84 Plus

```
Normal C.D
Lower :0.5
Upper :1E+99
σ :0.15186342
μ :0.702
Save Res:None
Execute
ICALC
```

```
Normal C.D
P =0.90826434
z:Low=-1.3301425
z:Up =6.5849E+99
```

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