2013 AM Paper 1 v1.0

1 Given $y = |2x-3| - 2$

For y-intercept, $x = 0$

$$y = |2(0)-3| - 2$$
$$= |-3| - 2$$
$$= 3 - 2$$
$$= 1 \quad \Rightarrow (0, 1).$$

For x-intercept, $y = 0$.

$$0 = |2x-3| - 2$$
$$|2x-3| = 2.$$

$2x - 3 = 2$ or $2x - 3 = -2$.

$2x = 5$ $\quad$ $2x = 1$

$x = 2.5$ $\quad$ $x = 0.5$.

$\therefore (2.5, 0)$ and $(0.5, 0)$.

$\therefore (0,1)$, $(2.5, 0)$ and $(0.5, 0)$ are the solutions #

2 $$\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right) = 2\sin\left[\frac{1}{2}\left(\frac{7\pi}{12}+\frac{\pi}{12}\right)\right]\cos\left[\frac{1}{2}\left(\frac{7\pi}{12}-\frac{\pi}{12}\right)\right]$$
$$= 2\sin\frac{\pi}{3}\cos\frac{\pi}{4}$$
$$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{6}}{2} \ \#$$

2013 AM Paper 1 v1.0

3 $y = x^3 + px^2 + qx + 10$

$\frac{dy}{dx} = 3x^2 + 2px + q$

Given y is decreasing function ($\frac{dy}{dx} < 0$) when $3 < x < 7$.
Then

$(x-3)(x-7) < 0.$
$x^2 - 10x + 21 < 0.$
$3x^2 - 30x + 63 < 0.$

Since $\frac{dy}{dx} < 0$ when $3 < x < 7$.
$3x^2 + 2px + q < 0.$

By comparing, $2p = -30$ and $q = 63$.
$p = -15$ #

$\therefore$ $p = -15$ and $q = 63$ #

2013 AM Paper 1 v1.0

4 (i) $$\frac{(\sin A-\cos A)(1+\sin A\cos A)}{\cos^3 A} = \tan^3 A - 1$$

$$\begin{aligned}
\text{L.H.S} &= \frac{(\sin A-\cos A)(1+\sin A\cos A)}{\cos^3 A} \\
&= \frac{\sin A + \sin^2 A\cos A - \cos A - \sin A\cos^2 A}{\cos^3 A} \\
&= \frac{\sin A(1-\cos^2 A) - \cos A(1-\sin^2 A)}{\cos^3 A} \\
&= \frac{\sin A\sin^2 A - \cos A\cos^2 A}{\cos^3 A} \\
&= \frac{\sin^3 A}{\cos^3 A} - \frac{\cos^3 A}{\cos^3 A} \\
&= \tan^3 A - 1 \\
&= \text{RHS (shown)}.
\end{aligned}$$

(ii) $(\sin A - \cos A)(1+\sin A\cos A) = 2\cos^3 A$.

$$\frac{(\sin A-\cos A)(1+\sin A\cos A)}{\cos^3 A} = 2$$

$$\begin{aligned}
\tan^3 A - 1 &= 2 \quad \{\text{use result (i)}\}. \\
\tan A &= \sqrt[3]{3} \\
\text{basic } \angle &= \tan^{-1}(\sqrt[3]{3}).
\end{aligned}$$

Given A is acute $\angle$,

Then $A = \tan^{-1}(\sqrt[3]{3})$

$= 0.96453$ or $(55.264°)$

≈ 0.964 rad # or $(\approx 55.3°)$ #

2013 AM Paper 1 v1.0

5 $(r+1)$th term of $(a-x)^5 = \binom{5}{r} a^{5-r} (-1)^r x^r$.

Coeff of x^2 in $(a-x)^5 = \binom{5}{2} a^3 (-1)^2 = 10a^3$

Coeff of x^3 in $(a-x)^5 = \binom{5}{3} a^2 (-1)^3 = -10a^2$.

$(r+1)$th term of $(2+x)^6 = \binom{6}{r} 2^{6-r} x^r$

Coeff of x^2 in $(2+x)^6 = \binom{6}{2} 2^4 = 240$.

Coeff of x^3 in $(2+x)^6 = \binom{6}{3} 2^3 = 160$.

(i) Given coeff of x^3 in $(a-x)^5 + (2+x)^6 = 70$.
Then $-10a^2 + 160 = 70$.

$10a^2 = 90$.

$a^2 = 9$

$a = \pm 3$.

Given a is positive constant, $a = 3$ #.

(ii) Coeff of x^2 in $(a+x)^5 + (2+x)^6 = 10a^3 + 240$.

$= 10(3)^3 + 240$ (use result (i))

$= 510$ #

2013 AM Paper 1. v1.0

6 Let $\dfrac{7x+2}{(x^2+4)(x-2)} = \dfrac{Ax+B}{x^2+4} + \dfrac{C}{x-2}$

$7x+2 = (Ax+B)(x-2) + C(x^2+4)$.

when $x=2$

$7(2)+2 = 0 + C(4+4)$

$C = 2$.

Compare coeff of x^2:

$0 = A + C$.

$A = -2$.

Compare constant term:

$2 = -2B + 4C$.

$2B = 8-2$.

$B = 3$.

$\therefore \dfrac{7x+2}{(x^2+4)(x-2)} = \dfrac{3-2x}{x^2+4} + \dfrac{2}{x-2}$ #.

2013 AM Paper 1 v1.0

7 Given $2x^2 + px - 1 = 0$ and α, β are the roots.
then

$$\alpha + \beta = -\frac{p}{2} \quad \text{—— (1)}$$
$$\alpha\beta = -\frac{1}{2}. \quad \text{—— (2)}$$

Given $x^2 - 5x + q = 0$ and $\frac{1}{\alpha^2}$, $\frac{1}{\beta^2}$ are the roots.

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 5. \quad \text{—— (3)}$$

$$\left(\frac{1}{\alpha^2}\right)\left(\frac{1}{\beta^2}\right) = q \quad \text{—— (4)}.$$

From (4)

$$\frac{1}{(\alpha\beta)^2} = q$$
$$q = \frac{1}{\left(-\frac{1}{2}\right)^2} = 4 \qquad \{\text{using (2)}\}.$$

From (3).
$$\frac{\beta^2 + \alpha^2}{(\alpha\beta)^2} = 5.$$

$$\alpha^2 + 2\alpha\beta + \beta^2 = 5(\alpha\beta)^2 + 2\alpha\beta.$$
$$(\alpha + \beta)^2 = 5\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) \qquad \{\text{using (2)}\}.$$
$$\left(-\frac{p}{2}\right)^2 = \frac{1}{4}$$
$$\frac{p^2}{4} = \frac{1}{4}$$
$$p^2 = 1$$
$$p = \pm 1$$

Given p is a positive constant,
$\therefore$ $p = 1$, $q = 4$ #.

2013 AM Paper 1 v1.0

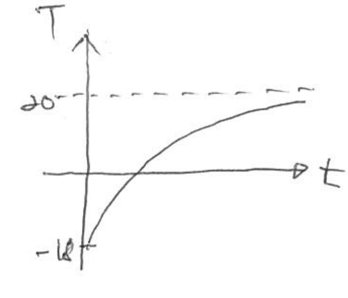
8 Given $T = 20 - 38e^{-0.6t}$

(i) when $t = 0$, $T = 20 - 38e^{0} = -18\,^{\circ}C$

(ii) when $t = 2$,
$$\begin{aligned} T &= 20 - 38e^{-0.6(2)} \\ &= 20 - 38e^{-1.2} \\ &= 8.5546 \\ &\approx 8.55\,^{\circ}C \end{aligned}$$

(iii)
$$\begin{aligned} T &= 20 - 38e^{-0.6t}. \\ \frac{38}{e^{0.6t}} &= 20 - T \\ e^{0.6t} &= \frac{38}{20 - T}. \\ \frac{3}{5}t &= \ln\left(\frac{38}{20-T}\right) \\ t &= \frac{5}{3}\ln\left(\frac{38}{20-T}\right). \end{aligned}$$

(iv)



When t is very large, $38^{-0.6t} \to 0$
$\therefore$ The value of T is 20
Hence the temp of the chicken can never reach $20\,^{\circ}C$.

OR from (iii), since t exists for $t > 0$.
then $20 - T > 0$.
$\Rightarrow 20 > T$
$\therefore T < 20\,^{\circ}C$.

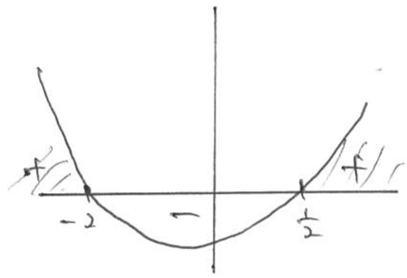
9 Given $y = 2x^2 + 3x - 5$.

(i) For $y + 3 > 0$.

$2x^2 + 3x - 5 + 3 > 0$.

$2x^2 + 3x - 2 > 0$.

$(2x - 1)(x + 2) > 0$.



$\therefore x < -2$ or $x > \frac{1}{2}$.

Set of values of $x = \{ x : x < -2 \text{ or } x > \frac{1}{2} \}$

(ii) $y = 2x^2 + 3x - 5$

$\frac{dy}{dx} = 4x + 3$.

Given $\frac{dx}{dt} = 0.04$ units/s and $\frac{dy}{dt} = 0.2$ units/s.

Then $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$.

$0.2 = (4x + 3) \times 0.04$

$4x + 3 = 5$.

$x = \frac{1}{2}$

When $x = \frac{1}{2}$, $y = 2(\frac{1}{2})^2 + 3(\frac{1}{2}) - 5$

$y = -3$.

$\therefore$ Coordinates of $P = (\frac{1}{2}, -3)$ #.

2013 AM Paper 1 v1.0

10(i) L_1 : $4y + x = 48$

$x = 48 - 4y$ — ①.

L_2 : $3y = 4x - 40$ — ②.

Sub ① into ②.

$3y = 4(48 - 4y) - 40.$

$3y = 192 - 16y - 40$

$19y = 152.$

$y = 8.$

Put $y = 8$ into ①, $x = 48 - 4(8) = 16.$

$\therefore A(16, 8)$

$M = \left(\frac{16+0}{2}, \frac{8+0}{2}\right) = (8, 4)$

Gradient OM $= m_{OM} = \frac{8}{4} = 2.$

From ②, sub $y = 0$,

$0 = 4x - 40.$

$x = 10.$

$C(10, 0).$

Gradient MC $= m_{MC} = \frac{4-0}{8-10} = -\frac{1}{2}.$

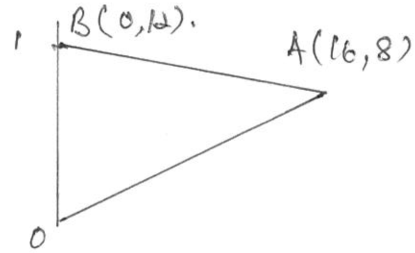
$\therefore m_{OM} \times m_{MC} = 2 \times (-\frac{1}{2}) = -1$

Hence $\angle OMC = 90°$ (shown).

2013 AM Paper 1

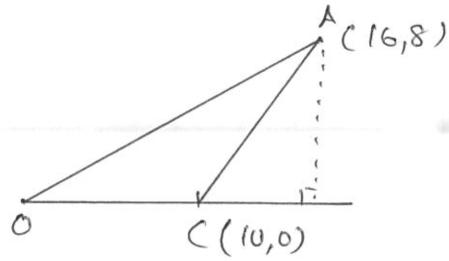
10 (ii). From ①, when $x = 0$, $y = 1$.

$B(0, 12)$.



Let Area of $\triangle OAB = \frac{1}{2}(12)(16)$

$= 96$ units^2.



Let Area of $\triangle OAC = \frac{1}{2}(10)(8)$

$= 40$ units^2.

$\therefore \dfrac{\text{Area of } \triangle OAB}{\text{Area of } \triangle OAC} = \dfrac{96}{40} = \dfrac{12}{5}$

$\therefore$ Ratio of Area of $\triangle OAB$ to the area of $\triangle OAC = 12 : 5$

2013 AM Paper 1 v1.0

11 $$y = \frac{x^2}{x+2} \quad \text{—— (A)}$$

(i) $$\frac{dy}{dx} = \frac{(x+2)(2x) - x^2(1)}{(x+2)^2} = \frac{2x^2+4x-x^2}{(x+2)^2} = \frac{x^2+4x}{(x+2)^2} \quad \#$$

$$\frac{d^2y}{dx^2} = \frac{(x+2)^2(2x+4) - (x^2+4x)2(x+2)}{(x+2)^4}$$

$$= \frac{(x+2)[(x+2)(2x+4) - 2(x^2+4x)]}{(x+2)^4}$$

$$= \frac{2x^2+8x+8-2x^2-8x}{(x+2)^3}$$

$$= \frac{8}{(x+2)^3}$$

(ii) When $\frac{dy}{dx} = 0$.

$$\frac{x^2+4x}{(x+2)^2} = 0$$

$$x(x+4) = 0$$

$x = 0$, $x = -4$.

using (A) $y = 0$ $y = -8$.

Stationary pts are $(0,0)$ and $(-4,-8)$.

When $x = 0$, $\frac{d^2y}{dx^2} = \frac{8}{(0+2)^3} > 0$

when $x = -4$, $\frac{d^2y}{dx^2} = \frac{8}{(-4+2)^3} < 0$

$\therefore$ $(0,0)$ is minimum pt and $(-4,-8)$ is maximum pt

2013 AM Paper 1 v1.0

12 (i). $\frac{2x}{2x+3} = \frac{2x+3-3}{2x+3}$

$= \frac{2x+3}{2x+3} + \frac{-3}{2x+3}$

$= 1 + \frac{-3}{2x+3}$

$= 1 - \frac{3}{2x+3}$ #.

(ii). Let $y = x\ln(2x+3)$.

$\frac{dy}{dx} = \ln(2x+3) + \frac{2x}{2x+3}$ —— (A).

(iii). Integrate from (A):

$\int \frac{dy}{dx}\,dx = \int \ln(2x+3)\,dx + \int \frac{2x}{2x+3}\,dx$.

$x\ln(2x+3) = \int \ln(2x+3)\,dx + \int \left(1 - \frac{3}{2x+3}\right) dx$.

$x\ln(2x+3) = \int \ln(2x+3)\,dx + x - \frac{3}{2}\ln(2x+3)$

$\therefore \int \ln(2x+3)\,dx = x\ln(2x+3) - x + \frac{3}{2}\ln(2x+3) + C$ #

where C is a constant

OR

$\int \ln(2x+3)\,dx$

$= \int \left[\ln(2x+3) + 1 + \frac{-3}{2x+3} - 1 - \frac{-3}{2x+3}\right] dx$

$= \int \left[\ln(2x+3) + 1 + \frac{-3}{2x+3}\right] dx - \int 1\,dx + 3\int \frac{1}{2x+3}\,dx$.

$= x\ln(2x+3) - x + \frac{3}{2}\ln(2x+3) + C$,

where C is a constant.

2013 AM Paper 1 v1.0

13. Given $V = ae^{kt}$

$\ln V = \ln a + \ln e^{kt}$

$\ln V = kt + \ln a$. —— (1).

(i) Plot $\ln V$ against t.

Year	1980	1990	2000	2010.
t (Years)	10	20	30	40
V ($)	8000	9200	10600	12200
$\ln V$	8.987	9.127	9.269	9.409

(ii) From (1), Gradient $= k = \dfrac{9.269 - 8.987}{30 - 10}$

≈ 0.0141

. $\ln V$-intercept $= \ln a = 8.85$

$a = e^{8.85}$

$= 6974.3$

$\approx \$6970$ (corr to 3 sig fig).

(iii) In 2014, $t = 44$

from graph, $\ln V = 9.465$

$V = e^{9.465}$

$= 12900.22$

$\approx \$12900$ (Corr to 3 sig fig).

Value of the diamond in 2014 is $12900 #

13 2013 AM Paper 1 V1.0