2012 AM P1. V1.0

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1 $y = \dfrac{2x+1}{x+3}$.

$\dfrac{dy}{dx} = \dfrac{(x+3)2 - (2x+1)}{(x+3)^2}$

$= \dfrac{2x+6-2x-1}{(x+3)^2}$

$= \dfrac{5}{(x+3)^2}$

Given $\frac{dx}{dt} = 0.4$ units/s. when $x = 2$.

$\therefore \dfrac{dy}{dt} = \dfrac{dy}{dx} \times \dfrac{dx}{dt}$

$= \dfrac{5}{(2+3)^2} \times 0.4$.

$= 0.08$ unit/s.

2. In $\triangle ACB$,

$\cos \angle CAB = \dfrac{AB}{AC}$.

$8 \cos \frac{\pi}{6} = AB$.

$AB = 8 \times (\frac{\sqrt{3}}{2}) = 4\sqrt{3}$ m.

$\therefore AM = MB = 2\sqrt{3}$ m (Given M is midpoint of AB).

$BC^2 = AC^2 - AB^2$.

$= 8^2 - (4\sqrt{3})^2$

$= 16$

$BC = 4$ m

In $\triangle MBC$,

$CM^2 = MB^2 + CB^2$.

$= (2\sqrt{3})^2 + 4^2$.

$= 28$.

$CM = \sqrt{28}$ m.

In $\triangle AMC$, using sine rule.

$\dfrac{\sin \angle ACM}{AM} = \dfrac{\sin \angle CAM}{CM}$.

$\sin \angle ACM = \dfrac{\sin \frac{\pi}{6}}{\sqrt{28}} \times 2\sqrt{3}$.

$= \dfrac{1}{2} \times \dfrac{\cancel{2}\sqrt{3}}{\cancel{2}\sqrt{7}}$

$= \dfrac{\sqrt{3}}{2\sqrt{7}} \times \dfrac{\sqrt{7}}{\sqrt{7}}$

$= \dfrac{\sqrt{21}}{14}$.

$\therefore \angle ACM = \sin^{-1}(\frac{\sqrt{21}}{14})$.

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$\therefore k = 21$ #

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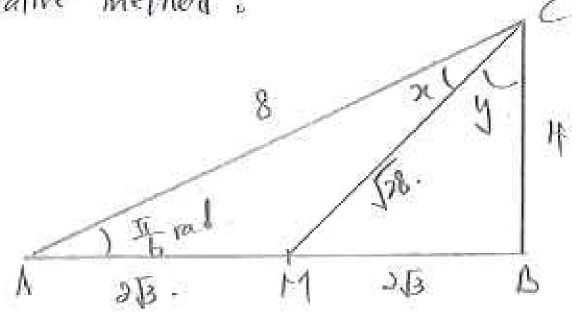
2 Alternative method:



Let $\measuredangle ACM = x$ and $\measuredangle MCB = y$.

In ΔACB,

$$\cos \measuredangle CAB = \frac{AB}{AC}.$$

$$AB = 8 \times \left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3} \text{ m}.$$

$\therefore AM = MB = 2\sqrt{3}$ m (Given M is mid-point of AB)

$$BC^2 = AC^2 - AB^2$$
$$= 8^2 - (4\sqrt{3})^2$$
$$= 16$$
$$BC = 4 \text{ m}.$$

In ΔMBC,

$$CM^2 = MB^2 + CB^2$$
$$= (2\sqrt{3})^2 + 4^2$$
$$= 28.$$
$$CM = \sqrt{28}.$$

Note $\measuredangle x = \frac{\pi}{3} - y.$

$$\sin x = \sin\left(\frac{\pi}{3} - y\right)$$
$$= \sin\frac{\pi}{3}\cos y - \cos\frac{\pi}{3}\sin y$$
$$= \frac{\sqrt{3}}{\cancel{2}}\left(\frac{\cancel{4}2}{\sqrt{28}}\right) - \frac{1}{\cancel{2}}\frac{\cancel{2}\sqrt{3}}{\sqrt{28}}.$$
$$= \frac{\sqrt{3}}{\sqrt{28}}.$$
$$= \frac{\sqrt{3}}{2\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}.$$
$$= \frac{\sqrt{21}}{14}.$$

$$\therefore x = \sin^{-1}\left(\frac{\sqrt{21}}{14}\right)$$

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$k = 21$ #

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3 (i) $\frac{d}{dx}(x^2 \ln x) = 2x \ln x + x^2\left(\frac{1}{x}\right)$.
$= 2x \ln x + x$.

(ii) $\int x \ln x \, dx = \frac{1}{2}\int 2x \ln x \, dx$.
$= \frac{1}{2}\left[\int (2x \ln x + x - x)\, dx\right]$.
$= \frac{1}{2}\int (2x \ln x + x)\, dx - \frac{1}{2}\int x \, dx$.
$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\left(\frac{x^2}{2}\right) + c$.
$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$, where c is constant.

4. Let $A = \begin{pmatrix} 3 & -5 \\ 7 & 1 \end{pmatrix}$.

$A^{-1} = \frac{1}{3-(-35)}\begin{pmatrix} 1 & 5 \\ -7 & 3 \end{pmatrix}$.
$= \frac{1}{38}\begin{pmatrix} 1 & 5 \\ -7 & 3 \end{pmatrix}$.

Given $a + 7b - 11 = 0$. —— (1)
$5a - 3b + 2 = 0$. —— (2).

From (2), $3b - 5a = 2$. —— (3).
From (1). $7b + a = 11$ —— (4).
Using (3) & (4).

$$\begin{pmatrix} 3 & -5 \\ 7 & 1 \end{pmatrix}\begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} 2 \\ 11 \end{pmatrix}.$$

$$\begin{pmatrix} b \\ a \end{pmatrix} = \frac{1}{38}\begin{pmatrix} 1 & 5 \\ -7 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 11 \end{pmatrix}$$
$$= \frac{1}{38}\begin{pmatrix} 2 + 55 \\ -14 + 33 \end{pmatrix}$$
$$= \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}.$$

$\therefore b = 1.5$ #
$a = 0.5$ #

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5 (i) Given $x = Me^{-kt}$

$\ln x = \ln(Me^{-kt})$

$\ln x = \ln M + \ln e^{-kt}$

$\ln x = -kt + \ln M$.

$\therefore$ Let $Y = \ln x$.

$X = t$.

Gradient $= -k$.

Y-intercept $= \ln M$.

$X =$ t (minutes)	1	2	3	4	5
x (grams)	6.55	5.36	4.40	3.60	2.94
$Y = \ln x$	1.879	1.679	1.482	1.281	1.078

(ii) From Graph,

y-intercept $= 2.08$.

$\ln M = 2.08$

$M = e^{2.08} \approx 8.00$ ((cor to 3 sig fig).

(iii) Given $x = \frac{1}{2}M$.

$\ln x = \ln(\frac{1}{2}M) = \ln(\frac{1}{2}e^{2.08}) = 1.387$

From graph when $\ln x = 1.387$,

we have $t = 3.45$ mins.

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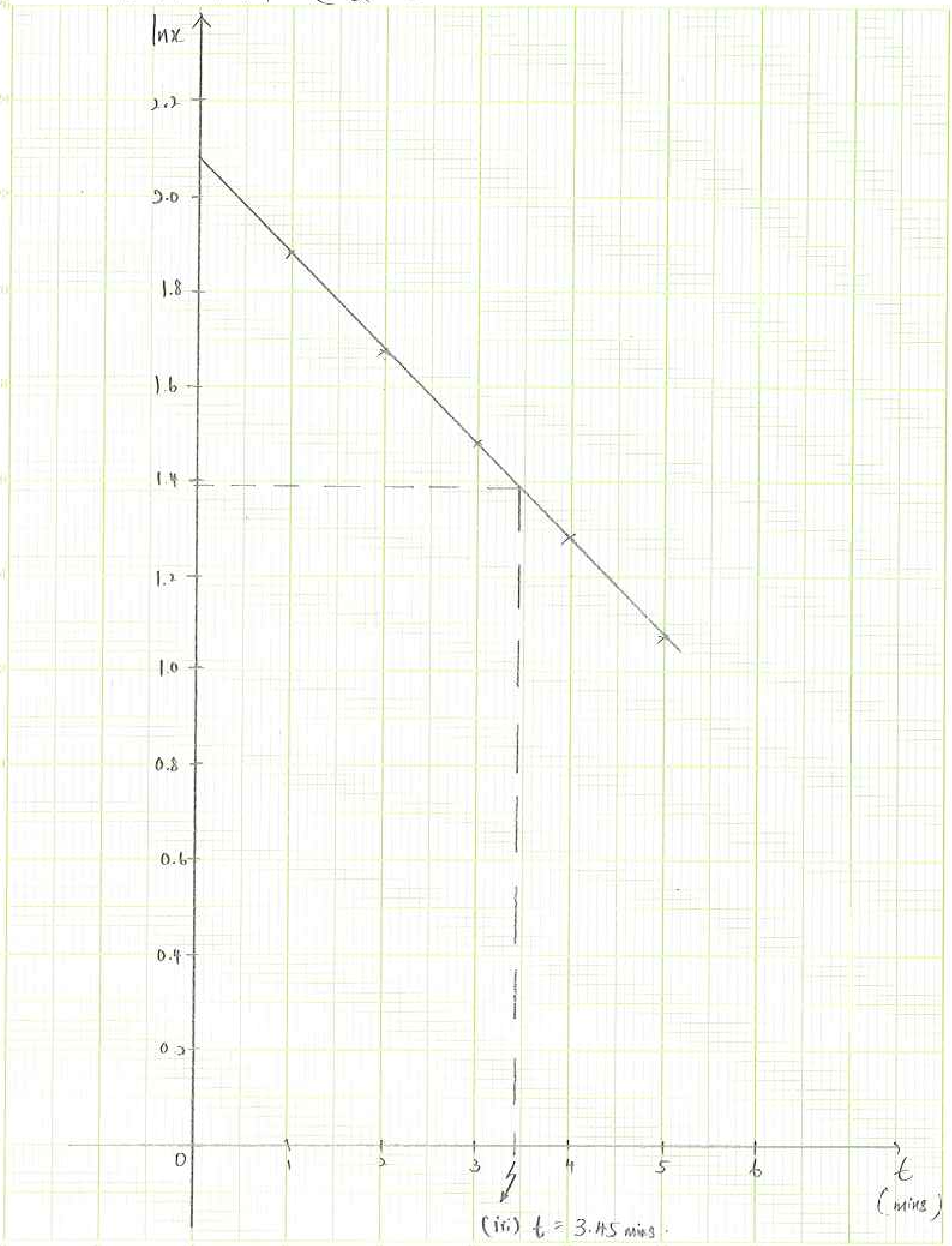
2012 AM P1 (Q5).



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6 (i) $2\sin 2\theta\,(\sec\theta - \tan\theta)$

$= 4\sin\theta\cos\theta\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)$

$= 4\sin\theta - 4\sin^2\theta$ #

(ii) Given $2\sin 2\theta\,(\sec\theta - \tan\theta) + 3 = 0$

$4\sin\theta - 4\sin^2\theta + 3 = 0$ {use result (i)}

$4\sin^2\theta - 4\sin\theta - 3 = 0.$

$(2\sin\theta + 1)(2\sin\theta - 3.) = 0.$

$\therefore \sin\theta = -\frac{1}{2}$ or $\sin\theta = \frac{3}{2}$

basic $\angle = \frac{\pi}{6}.$ (No solution, since $-1 < \sin\theta < 1$)

$\therefore \theta = \pi + \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$

$\theta = \frac{7\pi}{6},\ \frac{11\pi}{6}$ #.

7 $(2x+q)(x+p)^6.$

$= (2x+q)\left[x^6 + \binom{6}{1}x^5p^1 + \binom{6}{2}x^4p^2 + \dots\right].$

$= (2x+q)\left[x^6 + 6px^5 + 15p^2x^4 + \dots\right]$

$= 2x^7 + 12px^6 + 30p^2x^5 + qx^6 + 6pqx^5 + \dots$

$= 2x^7 + (12p+q)x^6 + (30p^2 + 6pq)x^5 + \dots$

Given coeff of $x^6 = -7$, $12p + q = -7.$

$q = -7 - 12p$ —①.

Given no term in x^5, $30p^2 + 6pq = 0.$ —②.

Sub ① into ②, $30p^2 + 6p(-7-12p) = 0.$

$30p^2 - 42p - 72p^2 = 0.$

$-42p - 42p^2 = 0.$

$p(p+1) = 0.$

$p = 0$ or $p = -1$

Since $p \neq 0$, $\therefore p = -1$ #

Put $p = -1$ into ①, $q = 5$ #

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8 $f(x) = 2\cos\left(\frac{x}{2}\right) + c.$

(i) Given $\left(\frac{2\pi}{3}, 2\right)$ passes thru the curve.

$2 = 2\cos\left(\frac{1}{2} \times \frac{2\pi}{3}\right) + c.$

$2 = 2\cos\left(\frac{\pi}{3}\right) + c.$

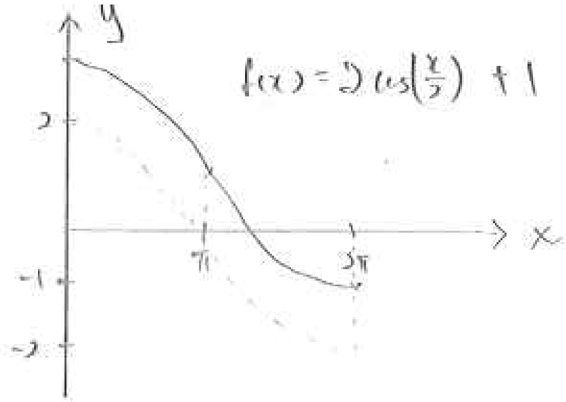
$2 = 2 \times \frac{1}{2} + c.$

$c = 1$ #.

(ii) Amplitude $= 2$ #.

Period $= \frac{2\pi}{1/2} = 4\pi$ #

(iii)



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9 Given $v = \frac{12}{(t+1)^2} - 3$.

(i) $a = \frac{dv}{dt} = -\frac{24}{(t+1)^3}$ m/s^2.

(ii). When $v = 0$,

$$0 = \frac{12}{(t+1)^2} - 3$$

$$(t+1)^2 = 4$$

$$t+1 = 2 \quad \text{or} \quad t+1 = -2.$$

$$t = 1s \qquad t = -3$$

$\therefore t = 1s$ #

Since $v = \frac{12}{(t+1)^2} - 3$.

$$\frac{ds}{dt} = 12(t+1)^{-2} - 3.$$

$$S = \int (12(t+1)^{-2} - 3)\, dt.$$

$$= -12(t+1)^{-1} - 3t + C.$$

$s = 0$, when $t = 0$,

$$0 = -12 + C.$$

$$C = 12.$$

$\therefore S = -12(t+1)^{-1} - 3t + 12.$

$\therefore$ When it comes to rest, $t = 1$.

$$S = -12(1+1)^{-1} - 3(1) + 12.$$

$$= -\frac{12}{2} - 3 + 12.$$

$$= -6 - 3 + 12.$$

$$= 3 \text{ m} \#.$$

∴ Distance travelled by the particle before it comes to instaneous rest $= 3$ m #.

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10 (i) $y = x^3 - 3x^2 - 24x + c$. —— ①.

$\frac{dy}{dx} = 3x^2 - 6x - 24$

For $\frac{dy}{dx} = 0$, $3x^2 - 6x - 24 = 0$.

$x^2 - 2x - 8 = 0$.

$(x-4)(x+2) = 0$.

$\therefore x = 4$ or $x = -2$.

x-coordinate of the two stationary pts is $x = 4$, $x = -2$ #.

(ii). $\frac{d^2y}{dx^2} = 6x - 6$

For $x = 4$, $\frac{d^2y}{dx^2} = 6(4) - 6 = 24 - 6 = 18 > 0$.

For $x = -2$, $\frac{d^2y}{dx^2} = 6(-2) - 6 = -18 < 0$.

$\therefore x = 4$ gives a min. pt.

Since min pt lies on x-axis, $\Rightarrow y = 0$.

Hence, sub. $x = 4$, $y = 0$ into ①.

$0 = 4^3 - 3(4)^2 - 24(4) + c$.

$c = 80$ #

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11 Given $x^2 - 16x + 52 = 0$, where $\alpha + 3\beta$ and $3\alpha + \beta$ are roots.

$$\alpha + 3\beta + 3\alpha + \beta = -\frac{(-16)}{1} = 16.$$
$$4\alpha + 4\beta = 16$$
$$\alpha + \beta = 4.$$

$$(\alpha + 3\beta)(3\alpha + \beta) = \frac{52}{1}$$
$$3\alpha^2 + \alpha\beta + 9\alpha\beta + 3\beta^2 = 52.$$
$$3\alpha^2 + 10\alpha\beta + 3\beta^2 = 52.$$
$$3\alpha^2 + 6\alpha\beta + 3\beta^2 + 4\alpha\beta = 52.$$
$$3(\alpha^2 + 2\alpha\beta + \beta^2) + 4\alpha\beta = 52.$$
$$3(\alpha + \beta)^2 + 4\alpha\beta = 52.$$
$$3(4)^2 + 4\alpha\beta = 52 \quad (\text{use } \alpha + \beta = 4)$$
$$4\alpha\beta = 52 - 48.$$
$$4\alpha\beta = 4$$
$$\alpha\beta = 1.$$

$\therefore$ Quadratic Eqn with roots α & β:
$$x^2 - (\alpha + \beta)x + \alpha\beta = 0.$$
$$x^2 - 4x + 1 = 0. \#$$

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12 (i).

$$9(3^x)^3 = \sqrt{\frac{1}{27^x}}$$
$$3^2(3^{3x}) = (3^{-3x})^{1/2}$$
$$3^{2+3x} = 3^{-\frac{3x}{2}}$$
$$2+3x = -\frac{3x}{2}.$$
$$4+6x = -3x.$$
$$9x = -4$$
$$x = -\frac{4}{9} \#.$$

(ii) Given $\dfrac{a+b\sqrt{3}}{5+2\sqrt{3}} = \dfrac{5+2\sqrt{3}}{2+\sqrt{3}}.$

$$a+b\sqrt{3} = \frac{(5+2\sqrt{3})^2}{2+\sqrt{3}}$$
$$= \frac{25+20\sqrt{3}+12}{(2+\sqrt{3})} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$$
$$= \frac{(37+20\sqrt{3})(2-\sqrt{3})}{4-3.}$$
$$= 74 - 37\sqrt{3} + 40\sqrt{3} - 60.$$
$$= 14 + 3\sqrt{3}.$$

$\therefore$ $a = 14$ and $b = 3$.

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13 (i) Given $x^2 + y^2 - 6x + 8y - 375 = 0$.

$$x^2 - 6x + y^2 + 8y = 375.$$
$$(x-3)^2 + (y+4)^2 = 375 + 9 + 16.$$

$$\therefore (x-3)^2 + (y+4)^2 = 20^2.$$

$\therefore$ Center of Circle C $= (3, -4)$
radius $r = 20$.

Given $P(-9, 12)$ lies on circle.

(ii) Gradient $CP = \frac{12+4}{-9-3} = \frac{16}{-12} = -\frac{4}{3}$.

$\therefore$ Equation of line CP:

$$(y+4) = -\frac{4}{3}(x-3).$$
$$y+4) = -\frac{4}{3}x + 4.$$

$$y = -\frac{4}{3}x$$

$\therefore$ Line CP passes through origin since y-intercept $= 0$.

(iii). Given OP is a diameter.

Midpoint of $OP = \left(\frac{0+(-9)}{2}, \frac{0+12}{2}\right) = (-4.5, 6)$.

Radius $= \sqrt{(-4.5-0)^2 + (6-0)^2} = 7.5$.

$\therefore$ Equation of the circle for which OP is a diameter.

$$(x+4.5)^2 + (y-6)^2 = 7.5^2.$$
$$x^2 + 9x + 20.25 + y^2 - 12y + 36 = 56.25.$$

$$x^2 + 9x + y^2 - 12y = 0.$$

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