1. $y = \frac{e^x}{x}$

$\frac{dy}{dx} = \frac{x(e^x) - e^x(1)}{x^2}$

$= \frac{e^x(x-1)}{x^2}$

Since y is a decreasing function, $\frac{dy}{dx} < 0$

$\Rightarrow \quad x - 1 < 0 \quad$ as $\frac{e^x}{x^2} > 0$

$x < 1$

$\therefore$ Range of values of $x = 0 < x < 1$ ※

2. Eq. of the line: $xy = -3y + c$

Sub $(2, 6)$,

$6 = -3(2) + c$

$c = 12.$

$\therefore \quad xy = -3y + 12$

$xy + 3y = 12$

$y(x+3) = 12$

$y = \frac{12}{x+3}$

$\therefore \; a = 12, \; b = 3$

3. $3^{x+2} = 12^{2-x}$

$3^{x+2} = (2^2 \times 3)^{2-x}$

$3^{x+2} = (2^2)^{2-x}(3)^{2-x}$

$\frac{3^{x+2}}{3^{2-x}} = 2^{4-2x}$

$3^{x+2-(2-x)} = 2^{4-2x}$

$3^{x+2-2+x} = \frac{2^4}{2^{2x}}$

$3^{2x} = \frac{2^4}{2^{2x}}$

$3^{2x}(2^{2x}) = 2^4$

$6^{2x} = 16$

$(6^x) = \sqrt{16}$

$\Rightarrow \quad 6^x = 4$ or $6^x = -4$ (rej.)

#

4. $2\cos 2\theta = 4 + 5\cos\theta$

$2(2\cos^2\theta - 1) - 5\cos\theta - 4 = 0$

$4\cos^2\theta - 5\cos\theta - 6 = 0$

$(4\cos\theta + 3)(\cos\theta - 2) = 0$

$4\cos\theta + 3 = 0$ or $\cos\theta - 2 = 0$

$\cos\theta = 2$ (N.A.)

$\cos\theta = -\frac{3}{4}$

Basic $\measuredangle = 0.72273$

$\theta = \pi - 0.72273;\ \pi + 0.72273$

$\approx 2.42,\ 3.86$ (2 d.p.) #

5. $y = k\sqrt{4x+1}$

$= k(4x+1)^{\frac{1}{2}}$

$\frac{dy}{dx} = k\left(\frac{1}{2}\right)(4x+1)^{-\frac{1}{2}}(4)$

$= \frac{2k}{\sqrt{4x+1}}$

Given $\frac{dx}{dt} = 2\frac{dy}{dt}$ when $x=2$,

Using the chain rule,

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\Rightarrow \quad \frac{dy}{dt} = \frac{2k}{\sqrt{4(2)+1}} \times 2\frac{dy}{dt}$

$1 = \frac{4k}{\sqrt{9}}$

$3 = 4k$

$k = \frac{3}{4}$ #

6. Total surface area of the block = 120 cm²

$\Rightarrow \quad 2x^2 + 4xy = 120$

$x^2 + 2xy = 60 \quad \text{——} (1)$

Total length of the 12 edges = 54 cm

$\Rightarrow \quad 8x + 4y = 54$

$2x + y = 13.5$

$y = 13.5 - 2x \quad \text{——} (2)$

Sub (2) into (1),

$x^2 + 2x(13.5 - 2x) = 60$

$x^2 + 27x - 4x^2 = 60$

$3x^2 - 27x + 60 = 0$

$\therefore x^2 - 9x + 20 = 0$ (shown)

$(x - 5)(x - 4) = 0$

$x - 5 = 0$ or $x - 4 = 0$

$x = 5$ $\quad x = 4$

Sub $x = 5$ into (2), $y = 13.5 - 2(5)$

$= 3.5$

Sub $x = 4$ into (2), $y = 13.5 - 2(4)$

$= 5.5$

$\therefore x = 5, y = 3.5$ or $x = 4, y = 5.5$ #

7. $(1+ax)^6 = (1)^6 + {}^6C_1(1)^5(ax)^1 + {}^6C_2(1)^4(ax)^2 + \dots$

$= 1 + 6ax + 15a^2x^2 + \dots$

(i) $(2-3x)(1+ax)^6 = (2-3x)[1+6ax+15a^2x^2+\dots]$

$= \dots + 12ax - 3x + \dots$

Coefficient of $x = 12a-3$

$3 = 12a-3$

$12a = 6$

$a = \frac{1}{2}$ #

(ii) Sub $a=\frac{1}{2}$,

$(2-3x)(1+\frac{1}{2}x)^6 = (2-3x)[1+3x+\frac{15}{4}x^2+\dots]$

$= \dots + 2\left(\frac{15}{4}x^2\right) - 3x(3x) + \dots$

$= \dots + \frac{15}{2}x^2 - 9x^2 + \dots$

$= \dots - \frac{3}{2}x^2 + \dots$

Coefficient of $x^2 = -\frac{3}{2}$ #

8. (i) $y = 3 - |2x+1|$ ⟵ ①

when $y=0$, $3 - |2x+1| = 0$

$|2x+1| = 3$

$2x+1 = 3$ or $2x+1 = -3$

$2x = 2$ $\quad$ $2x = -4$

$x = 1$ $\quad$ $x = -2$

$\Rightarrow A(-2,0)$ & $C(1,0)$ #

x-coordinate of $B = \dfrac{-2+1}{2}$

$= -\frac{1}{2}$

Sub $x = -\frac{1}{2}$ into ①,

$y = 3 - |2(-\frac{1}{2})+1|$

$= 3$

$\Rightarrow B(-\frac{1}{2}, 3)$ #

(ii) $3 - |2x+1| = x$

$|2x+1| = 3 - x$

$2x+1 = 3-x$ or $2x+1 = -(3-x)$

$3x = 2$ $\quad$ $2x+1 = -3+x$

$x = \frac{2}{3}$ $\quad$ $x = -4$

#

OR

$3 - |2x+1| = x$

$|2x+1| = 3 - x$

$(2x+1)^2 = (3-x)^2$

$4x^2 + 4x + 1 = 9 - 6x + x^2$

$3x^2 + 10x - 8 = 0$

$(3x-2)(x+4) = 0 \Rightarrow x = \frac{2}{3}$ or $-4$ #

9. $y = ax^3 + b \quad — ①$

$\frac{dy}{dx} = 3ax^2$

Given equation of normal at $x = 1$

$\Rightarrow \quad 5y + 2x = 12$

$5y = -2x + 12$

$y = -\frac{2}{5}x + \frac{12}{5}$

$\therefore$ Gradient of tangent $= \frac{5}{2}$

$\Rightarrow \quad 3(a)(1)^2 = \frac{5}{2}$

$3a = \frac{5}{2}$

$a = \frac{5}{6}$

When $x = 1$, $5y + 2x = 12$

$5y + 2 = 12$

$5y = 10$

$y = 2.$

Sub $x = 1, y = 2$ and $a = \frac{5}{6}$ into ①,

$2 = \frac{5}{6}(1)^3 + b$

$b = 2 - \frac{5}{6}$

$= 1\frac{1}{6}.$

$\therefore a = \frac{5}{6}$ and $b = 1\frac{1}{6}$ #

10. (i) $AB \times BC = (9+\sqrt{6})\ cm^2$

$(2\sqrt{2}+\sqrt{3}) \times BC = 9+\sqrt{6}$

$BC = \dfrac{9+\sqrt{6}}{2\sqrt{2}+\sqrt{3}}$

$= \dfrac{9+\sqrt{6}}{2\sqrt{2}+\sqrt{3}} \times \dfrac{2\sqrt{2}-\sqrt{3}}{2\sqrt{2}-\sqrt{3}}$

$= \dfrac{18\sqrt{2}-9\sqrt{3}+2\sqrt{12}-\sqrt{18}}{(2\sqrt{2})^2-(\sqrt{3})^2}$

$= \dfrac{18\sqrt{2}-9\sqrt{3}+4\sqrt{3}-3\sqrt{2}}{8-3}$

$= \dfrac{15\sqrt{2}-5\sqrt{3}}{5}$

$= 3\sqrt{2}-\sqrt{3}$

$= \sqrt{3^2(2)}-\sqrt{3}$

$= (\sqrt{18}-\sqrt{3})\ cm$ #

(ii) $(AC)^2 = (AB)^2+(BC)^2$

$= (2\sqrt{2}+\sqrt{3})^2 + (\sqrt{18}-\sqrt{3})^2$

$= (2\sqrt{2})^2+2(2\sqrt{2})(\sqrt{3})+(\sqrt{3})^2+(\sqrt{18})^2-2(\sqrt{18})(\sqrt{3})+(\sqrt{3})^2$

$= 8+4\sqrt{6}+3+18-2\sqrt{54}+3$

$= 32+4\sqrt{6}-6\sqrt{6}$

$= 32-2\sqrt{6}$

$= 32-\sqrt{2^2(6)}$

$= (32-\sqrt{24})\ cm^2$ #

11.(i) Gradient of AC $= \frac{8-2}{10-(-2)}$

$= \frac{1}{2}$

Midpt of AC $= \left(\frac{-2+10}{2}, \frac{2+8}{2}\right)$

$= (4, 5)$

Gradient of BD $= -2$ ( $\because$ AC $\perp$ BD in Rhombus)

Equation of BD: $y-5 = -2(x-4)$

$y = -2x + 8 + 5$

$y = -2x + 13$

sub $x=0$, $y = 13$

$\Rightarrow$ B$(0, 13)$ #

let D be $(x, y)$,

Midpoint of BD $= (4, 5)$

$\left(\frac{x+0}{2}, \frac{y+13}{2}\right) = (4, 5)$

$\Rightarrow \frac{x}{2} = 4$ and $\frac{y+13}{2} = 5$

$x = 8$ $y = -3$

$\Rightarrow$ D$(8, -3)$ #

(ii) Area of rhombus $= \frac{1}{2}\begin{vmatrix} -2 & 8 & 10 & 0 & -2 \\ 2 & -3 & 8 & 13 & 2 \end{vmatrix}$

$= \frac{1}{2}\left| 6 + 64 + 130 + 0 - 16 + 30 - 0 + 26 \right|$

$= 120$ units$^2$ #

12. $s = t\ln(t+1) - t$

when $t = 20$,

(i) displacement of P from O

$= 20\ln(21) - 20$

$= 40.890$

$\simeq 40.9$ m (3 s.f.) #

(ii) $v = \frac{ds}{dt}$

$= t\left(\frac{1}{t+1}\right) + \ln(t+1) - 1$

sub $t=20$, $= \frac{20}{21} + \ln 21 - 1$

$= 2.9969$

$\simeq 3.00$ m/s (3 s.f.) #

(iii) $a = \frac{dv}{dt}$

$= \frac{(t+1)(1) - t(1)}{(t+1)^2} + \frac{1}{t+1}$

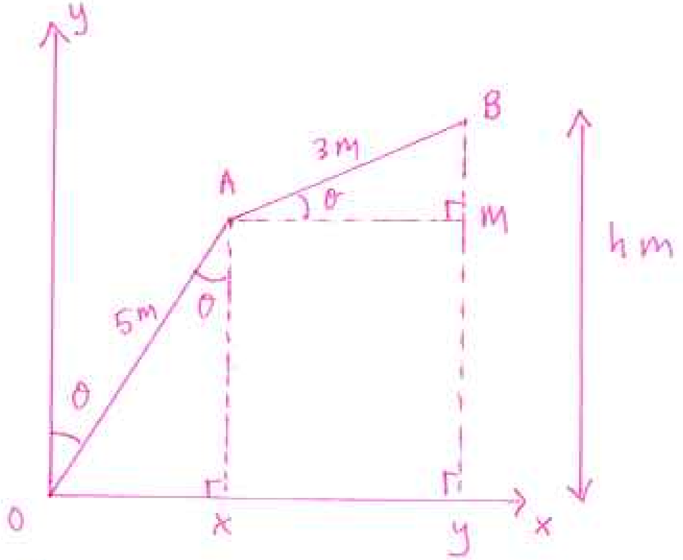
$= \frac{1}{(t+1)^2} + \frac{1}{t+1}$

sub $t=20$, $= \frac{1}{(21)^2} + \frac{1}{21}$

$= 0.049886$

$\simeq 0.0499$ m/s$^2$ (3 s.f.) #

13.




(i) $O\hat{A}X = \theta$

In $\triangle OAX$, $\cos\theta = \frac{AX}{5}$

$AX = 5\cos\theta$.

In $\triangle ABM$, $\sin\theta = \frac{BM}{3}$

$BM = 3\sin\theta$

$\Rightarrow h = AX + BM$

$= 5\cos\theta + 3\sin\theta$

$\therefore a = 5, \ b = 3$ #

13 (ii) $3\sin\theta + 5\cos\theta = R\sin(\theta + \alpha)$

$= R[\sin\theta\cos\alpha + \cos\theta\sin\alpha]$

$= R\cos\alpha\sin\theta + R\sin\alpha\cos\theta$

Comparing coefficients,

$3 = R\cos\alpha$ —①

$5 = R\sin\alpha$ —②

$\frac{②}{①}$ : $\frac{R\sin\alpha}{R\cos\alpha} = \frac{5}{3}$

$\tan\alpha = \frac{5}{3}$

$\alpha = 59.036°$

$\approx 59.0°$ (1 d.p.)

$(1)^2 + (2)^2$ : $R^2\cos^2\alpha + R^2\sin^2\alpha = 5^2 + 3^2$

$R^2(\cos^2\alpha + \sin^2\alpha) = 34$

$R^2 = 34$

$R = \sqrt{34}$ or $-\sqrt{34}$ (rej.)

$\Rightarrow h = \sqrt{34}\sin(\theta + 59.0°)$ #

13(iii) Maximum value of $h = \sqrt{34}$

when $\sqrt{34}\sin(\theta + 59.036^\circ) = \sqrt{34}$

$\Rightarrow \sin(\theta + 59.036^\circ) = 1$

$\theta + 59.036^\circ = 90^\circ$

$\theta = 30.964^\circ$

$\approx 31.0^\circ$ (1 d.p) #

(iv) when $h = \sqrt{17}$,

$\sqrt{34}\sin(\theta + 59.036^\circ) = \sqrt{17}$

$\sin(\theta + 59.036^\circ) = \dfrac{\sqrt{17}}{\sqrt{34}}$

$\sin(\theta + 59.036^\circ) = \dfrac{1}{\sqrt{2}}$

Basic $\angle = 45^\circ$

$\theta + 59.036^\circ = 45^\circ, 135^\circ$

$\theta = 45^\circ - 59.036^\circ, 135 - 59.036^\circ$

$= -14.036^\circ, 75.964^\circ$

$\approx 76.0^\circ, 346.0^\circ$ (1 d.p.)
(rej. $\because 0^\circ \leq \theta \leq 90^\circ$)

$\therefore$ The value of $\theta$ is $76.0^\circ$ #