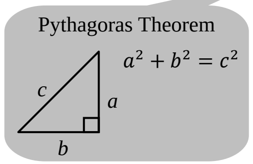
ELEMENTARY MATHEMATICS 4016/02

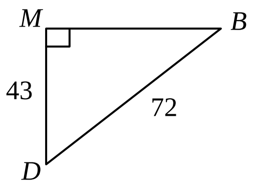
Paper 2Suggested Solutions October/November 2010

1. **Topics: Trigonometry**
(Trigonometric Ratios, Pythagoras' Theorem, Bearings)

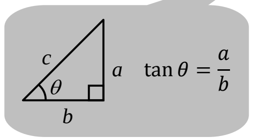
(a) (i)

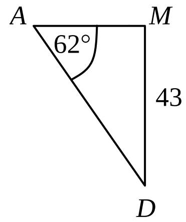
$BD^2 = DM^2 + MB^2$
$MB^2 = BD^2 - DM^2$
$= 72^2 - 43^2$
$MB = \sqrt{3335}$
$= 57.749$
$\approx$ **57.7 m (3 sig. fig.)**



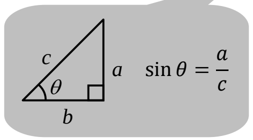


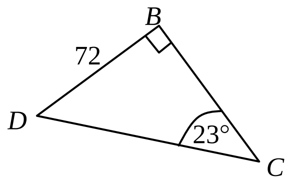
(ii) $\tan 62° = \frac{43}{AM}$
$AM = \frac{43}{\tan 62°}$
$= 22.863$
$\therefore AB = AM + MB$
$= 22.863 + \sqrt{3335}$
$\approx$ **80.6 m (3 sig. fig.)**





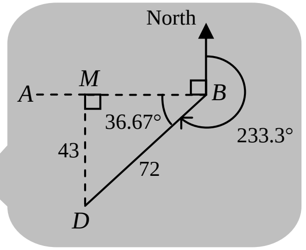
(iii) $\sin 23° = \frac{72}{CD}$
$CD = \frac{72}{\sin 23°}$
$= 184.269$
$\approx$ **184 m (3 sig. fig.)**





(b) $\sin M\hat{B}D = \frac{43}{72}$
$M\hat{B}D = \sin^{-1}\frac{43}{72}$
$= 36.67°$
$\therefore$ Bearing of D from B $= 270° - 36.67°$
$\approx$ **233.3° (1 d.p.)**

"B is due east of A" $\Rightarrow$ bearing of A from $B = 270°$



2. **Topic: Algebra (Solutions to Quadratic Equations, Formulae)**

(a)
$\frac{x}{8} = \frac{50}{x}$
$x^2 = 8(50)$
$x = \pm\sqrt{400}$
$=$ **20 or −20**

(b)
$\frac{t+p}{4} = \frac{q}{5}$
$5(t + p) = 4q$
$5t + 5p = 4q$
$5t = 4q - 5p$
$\boldsymbol{t = \frac{4q-5p}{5}}$

(c) $y = a + \frac{600}{x}$ — y: cost <u>per</u> copy; x: <u>total</u> no. of copies

(i) Sub $x = 50$, $y = 17$, $17 = a + \frac{600}{50}$
$a = 17 - 12$
$=$ **5**

(ii) Sub $x = 100$, $y = 5 + \frac{600}{100}$
$= 11$

∴ When 100 copies are printed, the cost of each copy is $11.

(iii) Sub $x = 300$, $y = 5 + \frac{600}{300} = 7$ — Cost per copy
∴ Total cost $= 7 \times 300$
$=$ **$2100**

(iv) Sub $y = 5.20$, $5.20 = 5 + \frac{600}{x}$
$\frac{600}{x} = 0.2$
$x = \frac{600}{0.2}$
$=$ **3000**

∴ 3000 copies were printed.

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3. **Topic: Arithmetic (Application of Mathematics in Practical Situations)**

(a) (i) Total amount Alan will pay for the computer

$= \frac{1}{3}(1299) + 24(40.30)$

$=$ **$1400.20**

Deposit: $\frac{1}{3}(1299)$; 24 monthly instalments: $24(40.30)$

(ii) Extra cost of computer (as % of cash price)

$= \frac{\$1400.20 - \$1299}{\$1299} \times 100\%$

$= 7.7906\%$

$\approx$ **7.79% (3 sig. fig.)**

"extra cost as % of cash price": $\frac{\text{hire purchase price} - \text{cash price}}{\text{cash price}} \times 100\%$

(b) Total amount Betty will pay

$= 1299\left(1 + \frac{6}{100}\right)^3$

$= \$1547.129$

Interest Betty will pay $= \$1547.129 - \$1299$

$= \$248.129$

$\approx$ **$248.13 (2 d. p.)**

Given in formula sheet (compound interest): **Total <u>amount</u>** $= P\left(1 + \frac{r}{100}\right)^n$

Total <u>interest</u> = Total amt. – Principal amt.

(c) $115\% \rightarrow \$759$

$1\% \rightarrow \$\frac{759}{115}$

$100\% \rightarrow \$\frac{759}{115} \times 100 = \$660$

$\therefore$ **The trader paid $660 for the camera**.

Selling price
= Cost price (100%) + Profit (15%)
= 115% × Cost price

4. **Topic: Coordinate Geometry; Vectors in Two Dimensions**

(a) Equation of AB:

$\frac{y-4}{x-(-5)} = \frac{4}{3}$

$y - 4 = \frac{4}{3}(x+5)$

$y - 4 = \frac{4}{3}x + \frac{20}{3}$

$y = \frac{4}{3}x + \frac{20}{3} + 4$

$y = \frac{4}{3}x + 10\frac{2}{3}$

$\mathbf{3y = 4x + 32}$

Equation of straight line passing through $A(x_1, y_1)$ and $B(x_2, y_2)$:
$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = \text{gradient } m$

<u>Alternative Method</u>

Equation of straight line with gradient m & y-intercept c:

$y = mx + c$

$y = \frac{4}{3}x + c$

Sub (–5, 4),

$4 = \frac{4}{3}(-5) + c$

$c = \frac{32}{3}$

$\therefore y = \frac{4}{3}x + 10\frac{2}{3}$

$\mathbf{3y = 4x + 32}$

(b) From (a), $3y = 4x + 32$ – (1)

Given $2x + 9y = 68$,

$x = \frac{68 - 9y}{2}$ – (2)

Sub (2) into (1), $3y = 4\left(\frac{68-9y}{2}\right) + 32$

$3y = 136 - 18y + 32$

$21y = 168$

$y = 8$

Sub $y = 8$ into (2), $x = \frac{68 - 9(8)}{2} = -2$

$\therefore$ **Coordinates of B = (–2, 8)**

(c) (i) $|\overrightarrow{AE}| = \sqrt{6^2 + 1^2}$

$= \sqrt{37}$

$\approx$ **6.08 (3 sig.fig.)**

Magnitude of $\begin{pmatrix} u \\ v \end{pmatrix}$: $\left|\begin{pmatrix} u \\ v \end{pmatrix}\right| = \sqrt{u^2 + v^2}$

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(ii) $\overrightarrow{AE} = \overrightarrow{OE} - \overrightarrow{OA}$

$\overrightarrow{OE} = \overrightarrow{AE} + \overrightarrow{OA}$

$= \begin{pmatrix}6\\1\end{pmatrix} + \begin{pmatrix}-5\\4\end{pmatrix}$

$= \begin{pmatrix}1\\5\end{pmatrix}$

∴ **Coordinates of *E* = (1, 5)**

(iii) $\overrightarrow{OD} = \begin{pmatrix}4\\2\end{pmatrix}$

(a) $\overrightarrow{DE} = \overrightarrow{OE} - \overrightarrow{OD}$

$= \begin{pmatrix}1\\5\end{pmatrix} - \begin{pmatrix}4\\2\end{pmatrix}$

$= \begin{pmatrix}-3\\3\end{pmatrix}$

From (c)(ii), $\overrightarrow{OE} = \begin{pmatrix}1\\5\end{pmatrix}$

(b) $\overrightarrow{DB} = \overrightarrow{OB} - \overrightarrow{OD}$

$= \begin{pmatrix}-2\\8\end{pmatrix} - \begin{pmatrix}4\\2\end{pmatrix}$

$= \begin{pmatrix}-6\\6\end{pmatrix}$

From (b), $\overrightarrow{OB} = \begin{pmatrix}-2\\8\end{pmatrix}$

(iv) $\overrightarrow{DE} = 3\begin{pmatrix}-1\\1\end{pmatrix}$

$\overrightarrow{DB} = 6\begin{pmatrix}-1\\1\end{pmatrix}$

Since $\overrightarrow{DB} = 2\overrightarrow{DE} = k\overrightarrow{DE}$

⇒ ***D*, *E* and *B* are collinear** (i.e. they all lie on a straight line)

and since $\overrightarrow{DB} = 2\overrightarrow{DE}$

⇒ ***E* is the mid-point of *D* and *B*.**

$\overrightarrow{AB} = k\overrightarrow{BC}$
⇒ *A*, *B*, *C* are collinear (straight line)

5. **Topic: Angles of Polygon**

(a) (i) $\angle XCD = \frac{360°}{15}$

$= \mathbf{24°}$

Each exterior ∠ of a regular n-sided polygon $= \frac{360°}{n}$
NOTE: *X* is not part of polygon *ABCDEF* …! ⇒∠*XCD* is an exterior ∠

(ii) Since *ABCDEF* … is a regular polygon and ∠*XCD* & ∠*XDC* are exterior angles,

⇒ ∠*XCD* = ∠*XDC* = 24°

⇒ Δ*XCD* is isosceles

⇒ ∠*CXD* = 180° – ∠*XCD* – ∠*XDC* (sum of ∠*s* in Δ)

= 180° – 24° – 24°

= **132°**

(b) Given *BC* = *DE* = *a*

From (a)(ii), Δ *XCD* is a isosceles

⇒ *XC* = *XD* = *b*

XB = *BC* + *CX* = *a* + *b*

XE = *DE* + *XD* = *a* + *b*

Hence *XB* = *XE*.

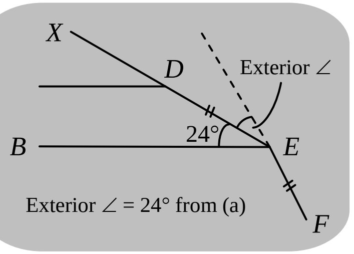
(c) From (b), Δ*XBE* is isosceles,

⇒ ∠*XBE* = ∠*XEB* = $\frac{180° - \angle CXD}{2}$ = 24°

∠*BEF* = 180° – exterior ∠ – ∠*XEB*

= 180° – 24° – 24°

= **132°**



<u>**Alternative Method**</u>

∠*BEF* = ∠*DEF* – ∠*XEB*

$= \frac{(15-2)\times 180°}{15} - 24°$

= **132°**

Each interior ∠ of a regular n-sided polygon $= \frac{(n-2)\times 180°}{n}$

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6. **Topics: Solutions to Quadratic Equations**

(a) Number of hours. John took $= \frac{42}{x}$

(b) Number of hours Peter took $= \frac{42}{x - \frac{1}{2}}$

> Time taken = $\frac{\text{Distance}}{\text{Speed}}$

(c)
$$\frac{42}{x-\frac{1}{2}} - \frac{42}{x} = \frac{10}{60}$$
$$\frac{84}{2x-1} - \frac{42}{x} = \frac{1}{6}$$
$$\frac{84x - 42(2x-1)}{(2x-1)x} = \frac{1}{6}$$
$$\frac{84x - 84x + 42}{2x^2 - x} = \frac{1}{6}$$
$$42\,(6) = 2x^2 - x$$
$$2x^2 - x - 252 = 0 \ \textbf{(Shown)}$$

> General solution to a quadratic equation ax^2+bx+c:
> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(d)
$$2x^2 - x - 252 = 0$$
$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(-252)}}{2(2)}$$
$$= \frac{1 \pm \sqrt{2017}}{4}$$
$$= 11.4777 \text{ or } -10.9777$$
$$\approx \mathbf{11.478} \text{ or } \mathbf{-10.978 \ (3\ d.p.)}$$

> Question simply asks to solve the equation. Do NOT reject the negative value of x here!

(e) Taking $x = 11.4777$ from (d), time that John took to complete the race

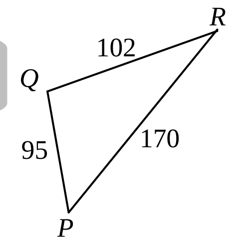
$= \frac{42}{11.4777}$

= 3.65925 hours

= 3 hrs 39.555 min

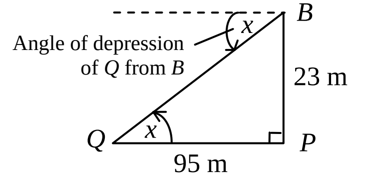
≈ **3 hrs 39 min 33 seconds**

7. **Topic: Trigonometry**



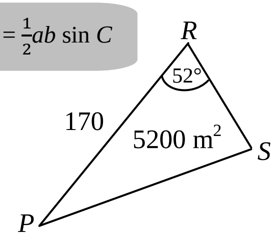
(a)
$$\cos P\hat{Q}R = \frac{95^2 + 102^2 - 170^2}{2(95)(102)}$$
$$= -0.48869$$
$$P\hat{Q}R = \cos^{-1}(-0.48869)$$
$$= 119.255$$
$$\approx \mathbf{119.3° \ (1\ d.p.)}$$

> Cosine rule:
> $a^2 = b^2 + c^2 - 2bc\cos A$



(b) In ΔBPQ,
$$\tan x = \frac{23}{95}$$
$$x = 13.609°$$
$$\approx 13.6° \text{ (1d.p.)}$$

Angle of depression of Q from B = **13.6°**



(c)
$$\text{Area of } \Delta PRS = 5200 \text{ m}^2$$
$$\tfrac{1}{2}(PR)(RS)\sin 52° = 5200$$
$$\tfrac{1}{2}(170)(RS)\sin 52° = 5200$$
$$RS = 77.634$$
$$\approx \mathbf{77.6 \text{ m} \ (3\ s.f.)}$$

> Area of Δ = $\frac{1}{2}ab\sin C$

(d) (i) $\frac{77.634}{3} = 25.87$

∴ Number of panels that needs to be bought = **26**

> Rounded up to 26 ∵ need to buy 26 × panels (of unit length 3 m) to fence up the full distance of RS.

(ii) Number of posts required = 27

∴ Total cost of the panels and post

= 26 × 28.50 + 27 × 14.95

= **$1144.65**

> 26 posts for each of the panels + 1 extra post at the end.

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8. **Topics: Trigonometry, Mensuration**

(a) (i) Perimeter of sector = 44 m

$\Rightarrow$ Length of major arc *PRQ* + 2 × radius (r) = 44 m

$r\theta + 2r = 44$

$8\theta + 2(8) = 44$

$\theta = \frac{28}{8}$

= **3.5 radians**

Arc length = $r\theta$
Note: θ must be in radians and can be reflex.

(ii) Obtuse ∠ *POQ* (α) = 2π – reflex ∠ *POQ* (θ) (∠s at a pt.)

$\Rightarrow$ α = 2π – 3.5 radians

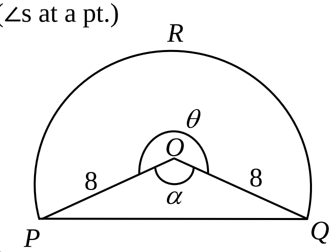
Area of Δ*POQ* = $\frac{1}{2}(OP)(OQ)\sin\alpha$

= $\frac{1}{2}(8)^2\sin(2\pi - 3.5)$

= 11.225

≈ **11.2 m² (3 s.f.)**

Area of Δ = $\frac{1}{2}ab\sin C$



Calculator must be in RAD mode to perform this sin operation!

(iii) Area of major sector = $\frac{1}{2}r^2\theta$

= $\frac{1}{2}(8)^2(3.5)$

= 112 m²

Total area of the cross-section of the tunnel

= Area of major sector + Area of Δ*POQ*

= 112 + 11.225

= 123.225

≈ **123 m² (3 s.f.)**

(b) (i) Volume of the bollard

= Volume of pyramid + Volume of cuboid

= $\frac{1}{3}(10)(10)(12) + (10)(10)(30)$

= **3400 cm³**

Volume of pyramid
= $\frac{1}{3}$ × base area × height

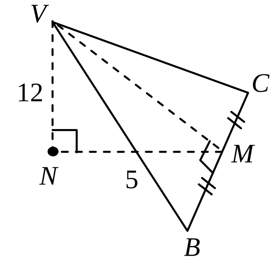
(ii) Let *M* be the midpoint of *BC*.

Using Pythagoras' Theorem in Δ*VNM*,

$VM = \sqrt{VN^2 + NM^2}$

$= \sqrt{12^2 + 5^2}$

= 13 cm



∴ Surface area of the bollard

= [4 × Area of Δ*VBC*] + [Perimeter of *ABCD* × *AE*]

= $4 \times \left(\frac{1}{2} \times 10 \times 13\right) + (10 + 10 + 10 + 10) \times 30$

= **1460 cm²**

Surface area of pyramid (excl. base)

Surface area of cuboid (excl. top &base)

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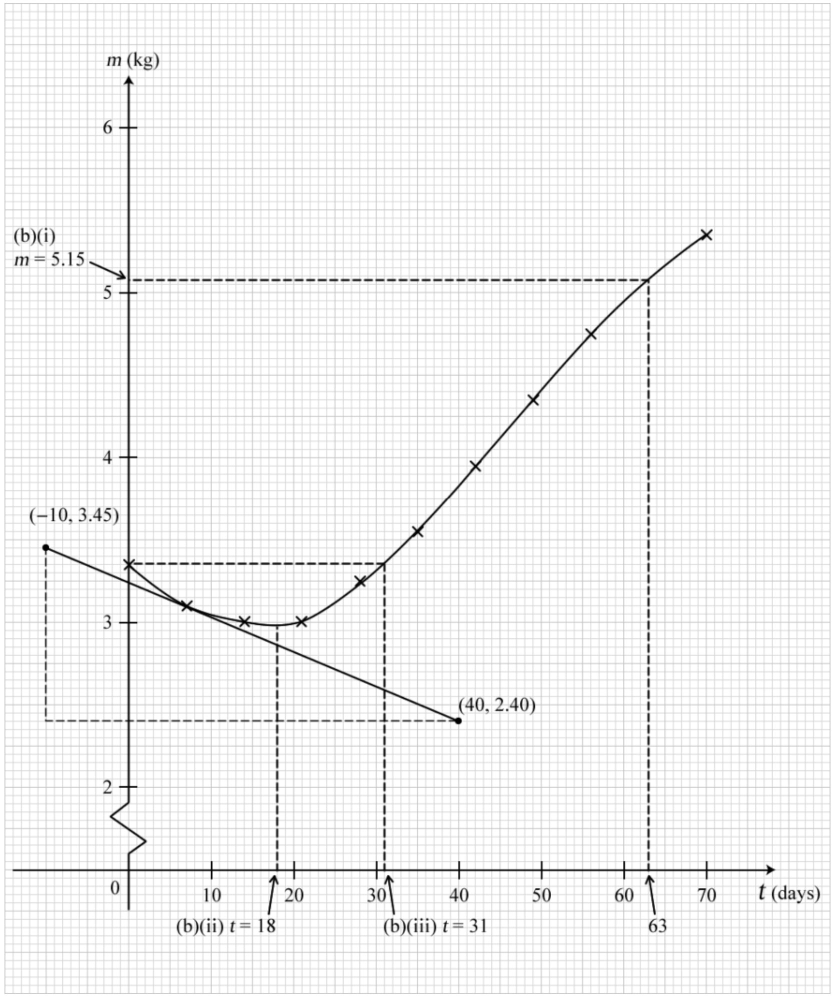
9. **Topic: Graphical Solution of Equations**

(a)



(b) From the graph

(i) Mass of the baby after 63 days = **5.15 kg**

(ii) Days since birth when the baby's mass was least = **18 days**

(iii) Days since birth when the baby regained its birth mass = **31 days**

(c) (i) From the tangent drawn in the graph, gradient of the curve at (7 , 3.10)

$= \frac{3.45 - 2.40}{-10-40}$

$=$ **−0.0210 (3 s.f.)**

(ii) **This gradient represents the rate of change of the baby's mass at seven days since birth (i.e. *t* = 7).**

(d) **As the graph is non-linear, it is not appropriate to estimate the mass of the baby when it is 1 year old by extending the graph linearly up to *t* = 365.**

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10. **Topics: Statistics, Simple Probability**

(a) (i) $a = 28 \div 4 = \mathbf{7}$

$b = 60 - (12 + 15 + 10 + 7 + 4 + 0 + 2 + 1) = \mathbf{9}$

$c = 0 \times 12 = \mathbf{0}$

$d = 3 \times 9 = \mathbf{27}$

$e = 0 + 15 + 20 + 27 + 28 + 20 + 0 + 14 + 8 = \mathbf{132}$

(ii) Mean $= \frac{\Sigma fx}{\Sigma f}$

$= \frac{132}{60}$

$= \mathbf{2.2}$

Standard deviation $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$

$= \sqrt{\frac{510}{60} - (2.2)^2}$

$= 1.9131$

$\approx$ **1.91 (3 sig. fig.)**

$\frac{\Sigma fx}{\Sigma f}$ from (a)(ii)

(b) P(One pupil read exactly 6 books) $= \frac{0}{60} = \mathbf{0}$

$\frac{\text{no. of pupils who had read exactly 6 books}}{\text{total no. of pupils in group}}$

(c) P(Both had read more than 4 books) $= \left(\frac{7}{60}\right)\left(\frac{6}{59}\right) = \mathbf{\frac{7}{590}}$

P[1st pupil (chosen from the 60) had read > 4 books] AND
P[2nd pupil (chosen from the remaining 59) had read > 4 books]

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