ELEMENTARY MATHEMATICS 4016/01
Paper 1 Suggested Solutions October/November 2010

1. **Topic: Arithmetic (Percentages& Fractions)**

(a) $\frac{3}{35} \times 100 \approx$ **8.57% (3 sig. fig.)**

(b) $17\frac{1}{2} \div 100 = \mathbf{\frac{7}{40}}$

Answer (a) 8.57 % [1]

(b) $\frac{7}{40}$ [1]

2. **Topic: Arithmetic (Approximation & Estimation)**

$\frac{3.93}{(7.47 + 3.02)\,5.67} = \mathbf{0.06607}$

$\approx$ **0.066 (2 sig. fig.)**

Answer 0.066 [2]

3. **Topic: Algebra**

$3x^3 - 12xy^2 = 3x(x^2 - 4y^2)$

$= \mathbf{3x\,(x + 2y)(x - 2y)}$

$a^2 - b^2 = (a - b)(a + b)$

Answer $3x\,(x + 2y)(x - 2y)$ [2]

4. **Topic: Algebra (Indices)**

(a) $3^{23} \div 27 = 3^k$

$3^{23} \div 3^3 = 3^k$

$3^k = 3^{23-3}$

$k = \mathbf{20}$

$\frac{a_m}{a_n} = a^{m-n}$

(b) $1 \div 2x^{-5} = 1 \div (2x^{-5})$

$= 1 \div \left(\frac{2}{x^5}\right)$

$= 1 \times \frac{x^5}{2}$

$= \mathbf{\frac{x^5}{2}}$

$a^{-n} = \frac{1}{a^n}$

Answer (a) 20 [1]

(b) $\frac{x^5}{2}$ [1]

5. **Topic: Arithmetic (Directed numbers in practical situations)**

(a) Difference in temperature between the first and third day $= t - (-3)$

$= \mathbf{(t + 3)}$°C

(b) Mean temperature for 3 days $= \frac{(-3) + 5 + t}{3}$

$= \mathbf{\frac{t + 2}{3}}$ °C

Answer (a) $(t + 3)$ °C [1]

(b) $\frac{t + 2}{3}$ °C [1]

6. **Topic: Arithmetic (Ratio & Proportion)**

Let \$(x) be the unit share of money.

$\Rightarrow$ A's share = \$($2x$)

Total sum of money = \$($2x + 3x + 4x$)= \$($9x$)

If the money was divided equally, A would have $\$\left(\frac{9x}{3}\right)$ =\$($3x$).

Given that A would have received an extra \$20,

$\$(3x - 2x) = \$20$

$x = 20$

$\therefore$ Total sum of money = \$($9x$)

= \$(9×20)

= **\$180**

Alternative Method:

	Before	After
A's share	2	3
B's share	3	3
C's share	4	3
Total sum	9	9

Extra shares of money$A$ receives = 3 – 2 = 1

$\therefore$ 1 share $\rightarrow$ \$20

9 shares $\rightarrow$ \$20 × 9 = **\$180**

Answer \$ 180 [2]

7. **Topic: Algebra (Solving Quadratic Equations by Factorisation)**

(a) $2x^2 - 5x - 3 = \mathbf{(2x + 1)(x - 3)}$

(b) $2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

$x = -\frac{1}{2}$ or $x = \mathbf{3}$

By inspection,

$2x$	1	x
x	-3	$-6x$
$2x^2$	-3	$-5x$

Hence question: use factorized expression from (a).

Answer (a) $(2x + 1)(x - 3)$ [1]

(b) $x = -\frac{1}{2}$ or 3 [1]

8. **Topic: Inequalities**

Given $-2 \le 2x + 4 < 18$

$-2 \le 2x + 4$ and $2x + 4 < 18$

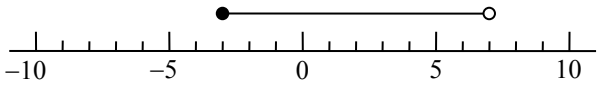
$-6 \le 2x$ $2x < 14$

$x \ge -3$ $x < 7$

$\therefore \mathbf{-3 \le x < 7.}$

Inequality sign reversed when multiplying both sides by –1

Answer

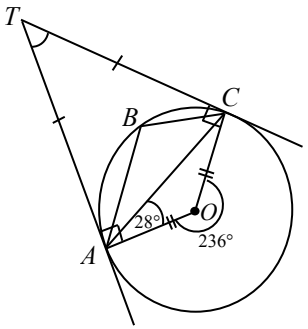


[2]

9. **Topics: Geometry**

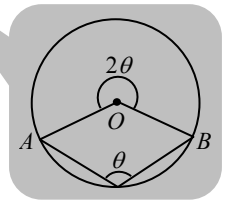
(a) $OA = OC$ = radius of circle
$\Rightarrow \Delta\, OAC$ is isosceles

$\therefore \angle AOC = 180° - 28° - 28°$
$= \mathbf{124°}$
(sum of interior $\angle$s of Δ)



(b) Reflex $\angle AOC = 360° - 124°$
($\angle$s at a point)
$= 236°$

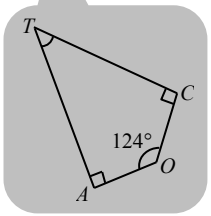
$\therefore \angle ABC = \frac{236°}{2}$ ($\angle$ at centre = 2 × $\angle$s at circumference)
$= \mathbf{118°}$

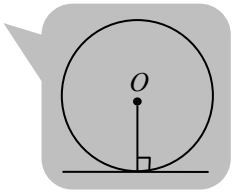


(c) $\angle OAT = \angle OCT = 90°$ (radius of circle⊥ tangent)

In quadrilateral $OATC$,

$\angle ATC = 360° - 90° - 90° - 124°$
$= \mathbf{56°}$





Answer (a) $A\hat{O}C =$ 124° [1]
(b) $A\hat{B}C =$ 118° [1]
(c) $A\hat{T}C =$ 56° [1]

10. **Topic: Factors & Multiples**

(a) $168 = \mathbf{2^3 \times 3 \times 7}$

2	168
2	84
2	42
3	21
7	7
	1

(b) (i) Given $4900 = 2^2 \times 5^2 \times 7^2$
$= 2 \times 2 \times 5 \times 5 \times 7 \times 7$
and $168 = 2 \times 2 \times 2 \times 3 \times 7$
LCM of 4900 and 168 $= 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 7 \times 7$
$= \mathbf{2^3 \times 3 \times 5^2 \times 7^2}$

(ii) HCF of 4900 and 168 $= 2 \times 2 \times 7$
$= \mathbf{28}$

Answer (a) $168 =$ $2^3 \times 3 \times 7$ [1]
(b) (i) $2^3 \times 3 \times 5^2 \times 7^2$ [1]
(ii) 28 [1]

11. **Topic: Probability**

(a) P(Total of the three numbers shown is 18) = P(6, 6, 6)
$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$
$= \mathbf{\frac{1}{216}}$

(b) P(Three dice show the same number)
=P(1, 1, 1) + P(2, 2, 2) + P(3, 3, 3) + P(4, 4, 4) + P(5, 5, 5) + P(6, 6, 6)
$= \frac{1}{216} \times 6$
$= \mathbf{\frac{1}{36}}$

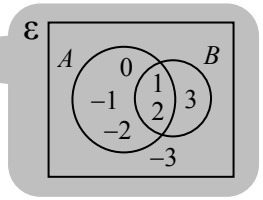
Note: There's only <u>one</u> chance to throw the three dice
⇒ the events above are mutually exclusive since if for e.g. (1, 1, 1) occurs, none of the rest can possibly occur. Hence it's a case of (1, 1, 1) OR (2, 2, 2) OR (3, 3, 3) OR …

(c) P(Total of the three numbers shown is 17)
= P(6, 6, 5) + P(6, 5, 6) + P(5, 6, 6)
$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times 3$
$= \mathbf{\frac{1}{72}}$

Answer (a) $\frac{1}{216}$ [1]
(b) $\frac{1}{36}$ [1]
(c) $\frac{1}{72}$ [1]

12. **Topic: Set Language & Notation**

ε = {–3, –2, –1, 0, 1, 2, 3}
A = {–2, –1, 0, 1, 2}
B = {1, 2, 3}



(a) A' = **{–3, 3}**
(b) $A \cap B$ = **{1, 2}**
(c) $A \cup B$ = **{–2, –1, 0, 1, 2, 3}**

Answer (a) {–3, 3} [1]
(b) {1, 2} [1]
(c) {–2, –1, 0, 1, 2, 3} [1]

13. **Topic: Variation**

(a) $V = \frac{k}{P}$, k is a constant.

Inverse variation:
$y \propto \frac{1}{x} \Rightarrow y = \frac{k}{x}$

Given V = 3 m^3, P = 200 N/m^2.
$\Rightarrow 3 = \frac{k}{200}$
$k = 600$
$\therefore V = \frac{600}{P}$
When P = 150 N/m^2, $V = \frac{600}{150} =$ **4 m^3**

(b) When V = 5 m^3, $5 = \frac{600}{P}$
$P = \frac{600}{5} =$ **120 N/m^2**

Answer (a) 4 m^3 [2]
(b) 120 N/m^2 [1]

14. **Topic: Standard form**

(a) 3×10^5 km/s
$= 3 \times 10^5 \times 1000$ m/s
$=$ **3×10^8 m/s**

1 km/s = 1000m/s

(b) Distance = 1m; Speed = 3×10^8 m/s
$\therefore$ Time taken $= \frac{1}{3 \times 10^8}$
$= 3\frac{1}{3} \times 10^{-9}$ s
$=$ **$3\frac{1}{3}$ nanoseconds**

1 nano = 1.0×10^{-9}

Answer (a) 3×10^8 m/s [1]
(b) $3\frac{1}{3}$ nanoseconds [2]

15. **Topic: Matrices**

(a) $\begin{pmatrix}7 & 2 & 3\\ 6 & 6 & 2\end{pmatrix}\begin{pmatrix}3\\1\\0\end{pmatrix} = \begin{pmatrix}7\times3+2\times1+3\times0\\6\times3+6\times1+2\times0\end{pmatrix}$

$= \begin{pmatrix}\mathbf{23}\\\mathbf{24}\end{pmatrix}$

$\begin{pmatrix}a & b & c\\ d & e & f\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}ax+by+cz\\dx+ey+fz\end{pmatrix}$

Answer (a) $\begin{pmatrix}23\\24\end{pmatrix}$ [2]

(b) **The matrix represents the total number of points awarded toCity (23 points)and United (24 points).** [1]

16. **Topic: Algebra (Algebraic manipulation)**

(a) $\frac{3a^2}{7bc} \div \frac{9a}{14b} = \frac{3a^2}{7bc} \times \frac{14b}{9a}$

$= \mathbf{\frac{2a}{3c}}$

(b) $\frac{2x}{(2x-3)^2} - \frac{1}{2x-3} = \frac{2x-(2x-3)}{(2x-3)^2}$

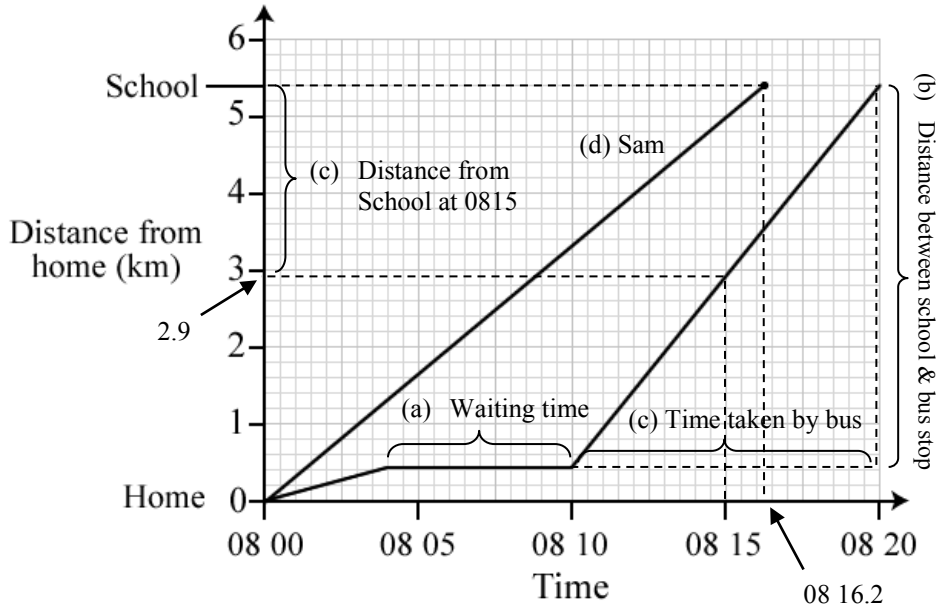
$= \frac{2x-2x+3}{(2x-3)^2}$

$= \mathbf{\frac{3}{(2x-3)^2}}$

Answer (a) $\frac{2a}{3c}$ [2]

(b) $\frac{3}{(2x-3)^2}$ [2]

17. **Topic: Graphs in Practical Situations (Distance-Time Graphs)**



(a) Time between 0804 and 0810 is **6 minutes**.

(b) At 0815, distance from house = 2.9 km
Distance from house to school = 5.4 km
∴ Distance she was from school = 5.4 km – 2.9 km
= **2.5 km**

Since the sets of marks are not skewed by extreme values, it's more accurate to compare their mean marks (instead of median or mode) so that all marks are taken into account.

(c) Distance between school and bus stop = 5.4 km– 0.4km = 5 km
Time takenby bus to reach school = 10 mins $=\frac{10}{60}$ hr
∴ Speedof bus = $\frac{5}{\frac{10}{60}}$
= **30 km/h**

Speed = $\frac{\text{Distance}}{\text{Time}}$

(d) Time taken by Sam = $\frac{\text{Distance}}{\text{Speed}}$
= $\frac{5.4}{20}$
= 0.27 hr
= 0.27 × 60 minutes
= **16.2 minutes**

Answer (a) 6 minutes [1]
(b) 2.5 km [1]
(c) 30 km/h [1]
(d) See graph [1]

18. **Topic: Statistics (Mean, Median & Mode)**

(a) Mode = **75 marks**

Mode = the number which occurs most frequently.

(b) Middle position = 8th position
Median = **69 marks**

38, 58, 59, 59, 64, 67, 68, **69**, 71, 73, 75, 76, 78, 79, 80

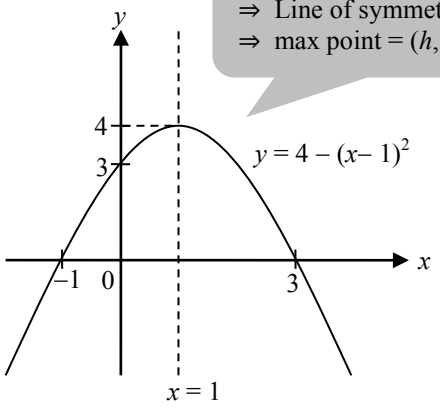
(c) Mean mark of boys = 62 marks
Mean mark of girls = 67.6 marks
∴**The girls performed better.**

Answer (a) 75 marks [1]
(b) 69 marks [1]
(c) The **girls** performed better because their mean mark (67.6) is higher than the mean mark (62) of the boys . [1]

19. **Topic: Graphs (Graphs of Quadratic Functions)**

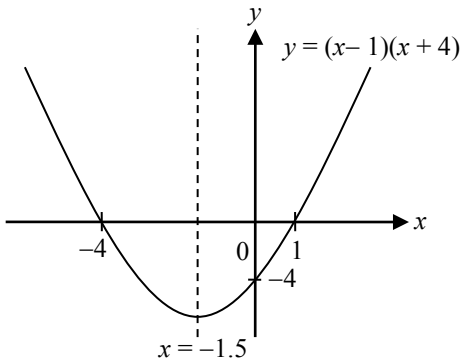
Answer (a)

$y = -(x-h)^2 + k$
⇒ Line of symmetry $x = h$
⇒ max point $= (h, k)$



[2]

(b) (i)



[2]

(b) (ii) $x = -1.5$ [1]

Line of symmetry of quadratic graph passes through max/min point.
$\therefore x = \frac{-4+1}{2} = -1.5$

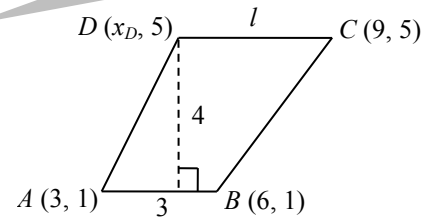
20. **Topic: Coordinate Geometry**

(a) Area of $\Delta ABC = \frac{1}{2}(6-3)(5-1)$
$= \mathbf{6\ units^2}$

Area of trapezium
$= \frac{1}{2} \times$ (sum of parallel sides) × height

(b) Area of trapezium $= \frac{1}{2}(3+l)(4)$
$14 = 2(3+l)$
$3 + l = 7$
$l = 4$
$\Rightarrow x_D = 9 - 4 = 5$

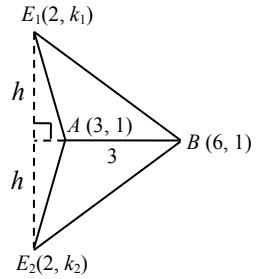
$\therefore$ **Coordinates of D = (5 , 5)**



(c) Area of $\Delta ABE = 9$ units^2
$\frac{1}{2}(3)(h) = 9$
$h = 6$

From the diagram,
$k_1 = 1 + 6 = 7$
$k_2 = 1 - 6 = -5$
$\Rightarrow E(2\ ,\ 7)$ or $E(2\ ,\ -5)$

$\therefore \boldsymbol{k = 7}$ **or** $\boldsymbol{k = -5}$.



Answer (a) 6 units^2 [1]
(b) (5 , 5) [2]
(c) $k =$ 7 or –5 [2]

21. **Topic: Arithmetic**

(a) Walking distance = 1000m = 1 km

$\therefore$ Walking time = $\frac{1}{4}$ h

= **15 minutes**

Time = $\frac{\text{Distance}}{\text{Speed}}$

(b) Running distance = 1.3 km

Running time = 5 minutes = $\frac{5}{60}$ hour

$\therefore$ Running speed = $1.3 \div \frac{5}{60}$

= **15.6 km/h**

Speed = $\frac{\text{Distance}}{\text{Time}}$

(c) Total distance = 1 + 1.3 = 2.3 km

Total time = 15 + 5 = 20 minutes = $\frac{20}{60}$ hour

$\therefore$ Average speed = $2.3 \div \frac{20}{60}$

= **6.9 km/h**

Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

Answer (a) 15 minutes [1]

(b) 15.6 km/h [2]

(c) 6.9 km/h [2]

22. **Topics: Algebra**

(a) Next two odd numbers after $2n-1$ are

$(2n-1)+2 = \mathbf{2n+1}$ and

$(2n-1)+2+2 = \mathbf{2n+3}$.

Note: $2n-1$ is odd.

(b) (i) Sum of three odd numbers = $(2n-1)+(2n+1)+(2n+3)$

$= 6n+3$

$= \mathbf{3(2n+1)}$

(ii) **Since $3(2n+1)$ has 3 as a factor, we conclude the sum is a multiple of 3.**

(c) Sum = $(2n-1)^2+(2n+1)^2+(2n+3)^2$

$= 4n^2-4n+1+4n^2+4n+1+4n^2+12n+9$

$= \mathbf{12n^2+12n+11}$

$(a+b)^2 = a^2+2ab+b^2$

$(a-b)^2 = a^2-2ab+b^2$

Answer

(a) $2n+1$ and $2n+3$ [1]

(b) (i) $3(2n+1)$ [1]

(b)(ii) **Since $3(2n+1)$ has 3 as a factor, we conclude the sum is a multiple of 3.** [1]

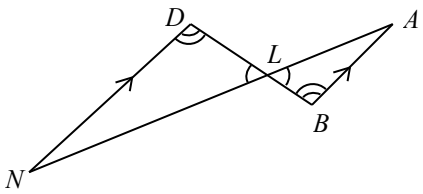
(c) $12n^2+12n+11$ [2]

23. **Topics: Similarity & Congruency**

(a) In ΔALB and ΔNLD,

$\angle ALB = \angle NLD$ (vert. opp. $\angle$s)

$\angle LBA = \angle LDN$ (alt. $\angle$s, AB // DC since $ABCD$ is a parallelogram)

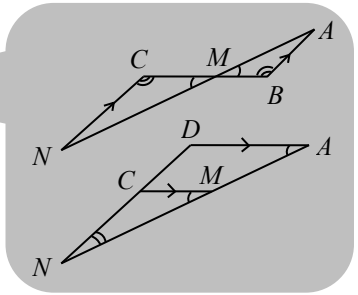


$\therefore \Delta ALB$ is similar to ΔNLD

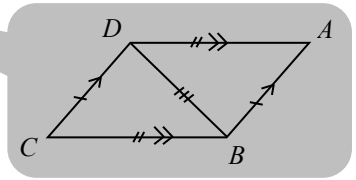
(b) Choose any one:

ΔNCM is similar to ΔABM.

ΔNCM is similar to ΔNDA.



(c) **ΔDAB is congruent to ΔBCD.**



(d) Given $DL = 3LB \Rightarrow \frac{LB}{DL} = \frac{1}{3}$

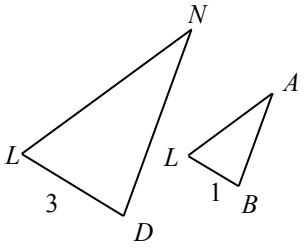
(i) Since ΔALB & ΔNLD are similar, (from (a))

$$\frac{AB}{ND} = \frac{LB}{LD} = \frac{1}{3}$$

$$\frac{AB}{CN} = \frac{AB}{\frac{2}{3}ND}$$

$$= \frac{1}{3} \div \frac{2}{3} = \mathbf{\frac{1}{2}}$$

$$\begin{aligned} ND &= CN + DC \\ &= CN + AB \\ &= CN + \tfrac{1}{3}ND \\ \Rightarrow CN &= \tfrac{2}{3}ND \end{aligned}$$

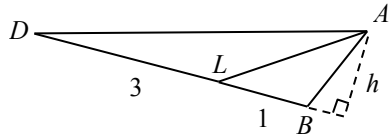


$$\frac{LB}{LD} = \frac{AB}{ND} = \frac{AL}{NL} = \frac{1}{3}$$

(ii) $$\frac{\text{Area of } \Delta ABL}{\text{Area of } \Delta ADL} = \frac{\frac{1}{2}(LB)(h)}{\frac{1}{2}(LD)(h)}$$

$$= \frac{LB}{LD}$$

$$= \mathbf{\frac{1}{3}}$$



(iii) Since ΔLMB & ΔLAD are similar,

$$\frac{\text{Area of } \Delta MLB}{\text{Area of } \Delta ALD} = \left(\frac{LB}{LD}\right)^2$$

$$= \left(\frac{1}{3}\right)^2$$

$$= \mathbf{\frac{1}{9}}$$

Area of similar figures $\Rightarrow \frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

Answer

(a) In Δs ALB and NLD $\angle ALB = \angle NLD$ (vert. opp. $\angle$s)
$\angle LBA = \angle LDN$ (alt. $\angle$s, AB // DC since $ABCD$ is a //gram)
$\therefore \Delta ALB$ is similar to ΔNLD. [2]

(b) ΔABM or ΔNDA [1]

(c) ΔDAB and ΔBCD [1]

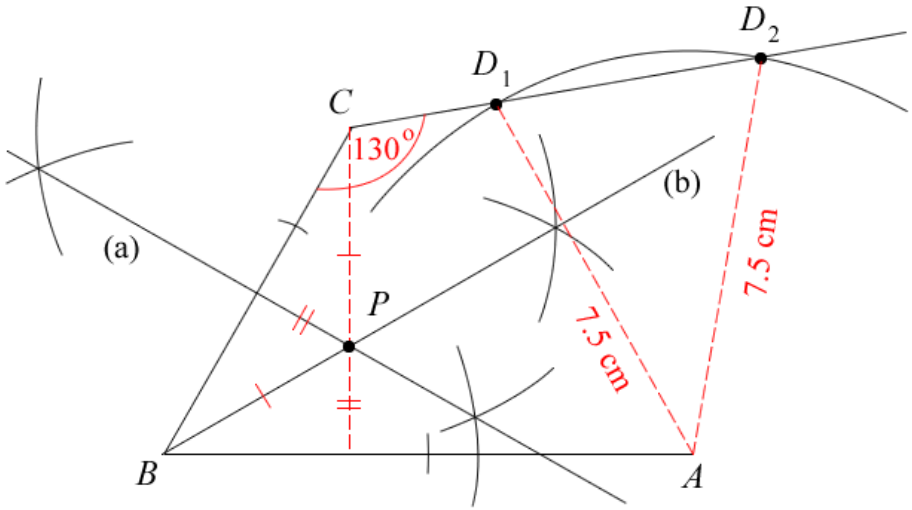
(d)(i) $\frac{1}{2}$ [1]

(d)(ii) $\frac{1}{3}$ [1]

(d)(iii) $\frac{1}{9}$ [1]

24. **Topic: Geometrical Constructions**

Answer (a), (b) and (d) [1], [1], [2]



Answer (c) The point P is equidistant from the lines ___BC___ and ___AB___ and equidistant from the points ___B___ and ___C___ [1]