ADDITIONAL MATHEMATICS 4038/02

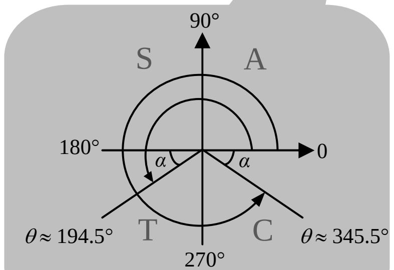
Paper 2Suggested Solutions — October/November 2010

1. **Topic: Trigonometric Functions**

$$3\cot^2\theta + 10\operatorname{cosec}\theta = 5,\ 0° \le \theta \le 360°$$
$$3(\operatorname{cosec}^2\theta - 1) + 10\operatorname{cosec}\theta = 5$$
$$3\operatorname{cosec}^2\theta + 10\operatorname{cosec}\theta - 8 = 0$$
$$(3\operatorname{cosec}\theta - 2)(\operatorname{cosec}\theta + 4) = 0$$
$$3\operatorname{cosec}\theta - 2 = 0 \text{ or } \operatorname{cosec}\theta + 4 = 0$$
$$\operatorname{cosec}\theta = \frac{2}{3} \qquad \operatorname{cosec}\theta = -4$$
$$\sin\theta = \frac{3}{2} \text{ (reject)} \qquad \sin\theta = -\frac{1}{4}$$
$$\text{Basic } \angle\alpha = 14.48°$$
$$\therefore \theta = 180° + 14.48°,\ 360° - 14.48°$$
$$= 197.47°,\ 345.52°$$
$$\approx \mathbf{194.5°,\ 345.5°\ (1\ d.p.)}$$

Given in formula sheet:
$\operatorname{cosec}^2 A = 1 + \cot^2 A$

$\operatorname{cosec} x = \dfrac{1}{\sin x}$



$\alpha = 14.48°$

2. **Topic: Applications of Differentiation (Maxima & Minima)**

(i) For ΔRVU and ΔRPQ,

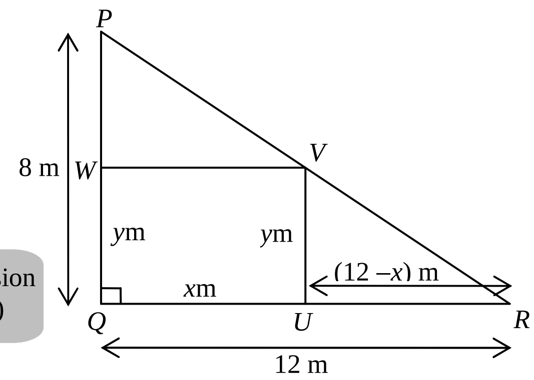
$\angle VRU = \angle PRQ$ (common)

$\angle UVR = \angle QPR$ (corresponding ∠s, VU // WQ since $QUVW$ is a rectangle)

$\Rightarrow \Delta RVU$ is similar to ΔRPQ.

Hence $\dfrac{RU}{RQ} = \dfrac{VU}{PQ}$

$$\frac{12-x}{12} = \frac{y}{8}$$
$$y = \frac{8}{12}(12-x)$$
$$\therefore y = 8 - \frac{2}{3}x \textbf{ (Shown)}$$



(ii) $A = xy$

$$= x\left(8 - \frac{2}{3}x\right)$$
$$= \mathbf{8x - \frac{2}{3}x^2}$$

Sub expression of y from (i)

(iii) $\dfrac{dA}{dx} = 8 - \dfrac{4}{3}x$

For a maximum value of A, $\dfrac{dA}{dx} = 0$,

$$8 - \frac{4}{3}x = 0$$
$$x = 6$$
$$\frac{d^2A}{dx^2} = -\frac{4}{3} < 0 \Rightarrow A \text{ is maximum when } x = 6 \text{ m}$$

When $x = 6$ m,

$$A = 8(6) - \frac{2}{3}(6)^2$$
$$= 24 \text{ m}^2$$

$\therefore$ **Maximum value of A is 24 m^2.**

2nd derivative test for stationary points of curve $y = f(x)$:

Max. point: $\dfrac{dy}{dx} = 0, \dfrac{d^2y}{dx^2} < 0$

Min. point: $\dfrac{dy}{dx} = 0, \dfrac{d^2y}{dx^2} > 0$

3. **Topic: Indices, Simultaneous Equations**

$$32^x \times 2^y = 1$$
$$(2^5)^x \times 2^y = 1$$
$$2^{5x} \times 2^y = 2^0$$

$(a^m)^n = a^{mn}$; $a^0 = 1$

$$5x + y = 0$$
$$y = -5x \quad -(1)$$
$$3^{x-12} \div 27^y = 81^{\frac{1}{x}}$$
$$3^{x-12} \div (3^3)^y = (3^4)^{\frac{1}{x}}$$
$$3^{x-12-3y} = 3^{\frac{4}{x}}$$

$a^m \div a^n = a^{m-n}$

$$x - 12 - 3y = \frac{4}{x}$$
$$x^2 - 12x - 3xy = 4 \quad -(2)$$

Sub. (1) into (2),

$$x^2 - 12x - 3x(-5x) = 4$$
$$x^2 - 12x + 15x^2 = 4$$
$$16x^2 - 12x - 4 = 0$$
$$4x^2 - 3x - 1 = 0$$
$$(4x + 1)(x - 1) = 0$$
$$x = -\frac{1}{4} \quad \text{or} \quad x = 1$$

Sub. $x = -\frac{1}{4}$ into (1),

$$y = -5\left(-\frac{1}{4}\right) = \frac{5}{4}$$

Sub. $x = 1$ into (1),

$$y = -5(1) = -5$$

$\therefore$ **The solutions are $x = -\frac{1}{4}$, $y = \frac{5}{4}$ and $x = 1$, $y = -5$.**

4. **Topic: Binomial Expansions**

(i) $(r + 1)^{\text{th}}$ term of $\left(x - \frac{k}{x^3}\right)^8 = \binom{8}{r} x^{8-r}(-kx^{-3})^r$

$$= \binom{8}{r} x^{8-r}(-k)^r x^{-3r}$$
$$= \binom{8}{r} (-k)^r x^{8-4r} \quad -(1)$$

In expanding $(a + b)^n$, $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

In the constant term, $x^{8-4r} = x^0$

Constant term = term that contains x^0

$$8 - 4r = 0$$
$$r = 2$$

Given that the constant term of $\left(x - \frac{k}{x^3}\right)^8 = 7$, sub $r = 2$ into (1),

$$\binom{8}{2}(-k)^2 = 7$$
$$28k^2 = 7$$
$$k^2 = \frac{1}{4}$$
$$k = \pm\frac{1}{2}$$

Since k is a positive constant, $\boldsymbol{k = \frac{1}{2}}$.

(ii) For $k = \frac{1}{2}$,

Constant term in $\left(x - \frac{1}{2x^3}\right)^8 = 7$

$$(1 + x^4)\left(x - \frac{1}{2x^3}\right)^8 = (1 + x^4)\left[\ldots + 7 + \binom{8}{3}\left(-\frac{1}{2}\right)^3 x^{-4} + \cdots\right]$$
$$= (1 + x^4)[\ldots + 7 - 7x^{-4} + \ldots]$$

x^{-4} term in $\left(x - \frac{1}{2x^3}\right)^8 = \binom{8}{3}(x^{8-3})\left(-\frac{1}{2x^3}\right)^3 = \binom{8}{3}\left(-\frac{1}{2}\right)^3 x^{-4}$

$$\text{Constant term} = (1)(7) + (x^4)(-7x^{-4})$$
$$= 7 - 7$$
$$= 0$$

$\therefore$ **Since the constant term is zero, there is no constant term in the expansion. (Shown)**

5. **Topic: Exponential & Logarithmic functions**

$y = 5 - e^{2x}$

When $x = 0$, $y = 5 - e^0 = 4$

Curve intersects y-axis when $x = 0$.

When $y = 0$, $0 = 5 - e^{2x}$

Curve intersects x-axis when $y = 0$.

$$e^{2x} = 5$$
$$2x = \ln 5$$
$$x = \frac{1}{2}\ln 5$$
$$\Rightarrow A\,(0, 4) \text{ and } B\left(\frac{1}{2}\ln 5\,, 0\right)$$

(i) Gradient of $AB = \frac{4-0}{0-\frac{1}{2}\ln 5} = -\frac{8}{\ln 5}$

For points $A(x_1, y_1)$&$B(x_2, y_2)$, gradient of $AB = \frac{y_1 - y_2}{x_1 - x_2}$

Equation of line AB:

$$y = -\frac{8}{\ln 5}x + 4$$

$A(0, 4) \Rightarrow y$-intercept is 4

Sub $x = \ln 5$, $y = k$,

$$\therefore k = -\frac{8}{\ln 5}(\ln 5) + 4$$
$$= -8 + 4$$
$$= \mathbf{-4}$$

Alternative Method:
Let P denote the point $(\ln 5, k)$.
Since P lies on AB
$\Rightarrow$ gradient of AP = gradient of BP

$$\frac{k-4}{\ln 5 - 0} = \frac{0-k}{\frac{1}{2}\ln 5 - \ln 5}$$
$$\frac{k-4}{\ln 5} = \frac{-k}{-\frac{1}{2}\ln 5}$$
$$k - 4 = 2k$$
$$\therefore k = \mathbf{-4}$$

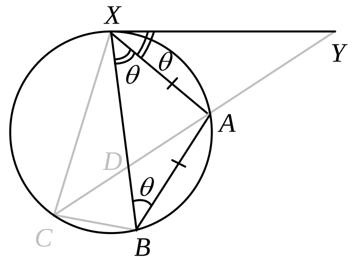
(ii)
$$x = \ln\sqrt{9-x}$$
$$x = \ln(9-x)^{\frac{1}{2}}$$
$$x = \frac{1}{2}\ln(9-x)$$

$\ln(x)^r = r\ln x$

$$2x = \ln(9-x)$$
$$e^{2x} = 9 - x$$
$$-e^{2x} = x - 9$$
$$5 - e^{2x} = x - 4$$
$$y = x - 4$$

$\therefore$ **Equation of the required line: $y = x - 4$**

6. **Topic: Geometric Proofs**



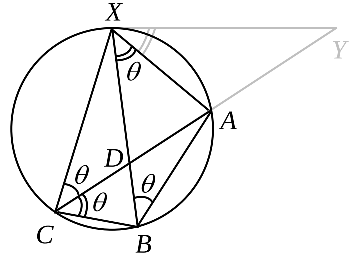
(i) $\angle AXB = \angle AXY = \theta$ (XA bisects $\angle YXB$)

$\angle ABX = \angle AXY = \theta$
(Alternate segment theorem)

$\therefore \angle AXB = \angle ABX = \theta$

$\Rightarrow AXB$ is isosceles.

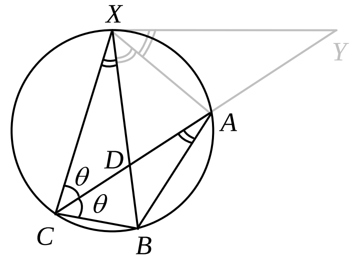
Hence $AX = XB$ (Proved)



(ii) $\angle ACB = \angle AXB = \theta$ ($\angle$s in the same segment)
$\angle ACX = \angle ABX = \theta$ ($\angle$s in the same segment)
$\therefore \angle ACB = \angle ACX = \theta$

Since D lies on AC, CD bisects $\angle XCB$. (Proved)

From (i),
$\angle AXB = \angle ABX = \theta$

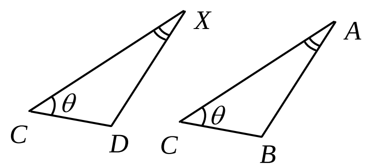


(iii) In ΔCDX and ΔCBA,

$\angle XCD = \angle ACB = \theta$ (Proven in (i))

$\angle CXD = \angle CAB$ ($\angle$s in the same segment)

$\therefore \Delta CDX$ and ΔCBA are similar. (Proved)



7. **Topic: Applications of Differentiation (Gradients, Tangents & Normals) & Integration (Area of aRegion)**

(i)

$$y = \sqrt{2x+5} \qquad -(1)$$

$$\frac{dy}{dx} = \frac{1}{2}(2x+5)^{-\frac{1}{2}}(2)$$

$$= \frac{1}{\sqrt{2x+5}}$$

$$\frac{d}{dx}(\sqrt{u}) = \frac{1}{2\sqrt{u}}\frac{du}{dx}$$

Gradient of tangent at P, sub $x = 2$:

$$\frac{dy}{dx} = \frac{1}{\sqrt{2(2)+5}} = \frac{1}{3}$$

⇒ Gradient of $QR = \frac{1}{3}$ (since QR is parallel to the tangent to the curve at P)

At Q, sub $y = 0$ into (1):

$$0 = \sqrt{2x+5}$$

$$2x + 5 = 0$$

$$x = -\frac{5}{2}$$

$$\Rightarrow Q\left(-\frac{5}{2},\ 0\right)$$

Equation of straight line passing through (x_1, y_1):

$$\frac{y-y_1}{x-x_1} = \text{gradient } m$$

∴ Equation of QR: $y - 0 = \frac{1}{3}\left[x - \left(-\frac{5}{2}\right)\right]$

$$\boldsymbol{y = \frac{1}{3}x + \frac{5}{6}} \qquad -(2)$$

(ii) At R, sub $x = 2$ into (2):

$$y = \frac{1}{3}(2) + \frac{5}{6} = \frac{3}{2}$$

$$\Rightarrow R\left(2, \frac{3}{2}\right)$$

Length of ST = Length of RS = $\frac{3}{2}$ units

Length of QS = $2 - \left(-\frac{5}{2}\right)$ = $\frac{9}{2}$ units

Area of $\Delta QST = \frac{1}{2}(QS)(ST)$

$$= \frac{1}{2}\left(\frac{9}{2}\right)\left(\frac{3}{2}\right)$$

$$= \frac{27}{8} \text{ units}^2$$

$$\int (ax+b)^n\, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

Area of region $QPS = \int_{-\frac{5}{2}}^{2} y\ dx$

$$= \int_{-\frac{5}{2}}^{2} (2x+5)^{\frac{1}{2}}\, dx$$

$$= \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left[(2x+5)^{\frac{3}{2}}\right]_{-\frac{5}{2}}^{2}$$

$$= \frac{1}{3}\left\{[2(2)+5]^{\frac{3}{2}} - \left[2\left(-\frac{5}{2}\right)+5\right]^{\frac{3}{2}}\right\}$$

$$= \frac{1}{3}\left[(9)^{\frac{3}{2}} - 0\right]$$

$$= 9 \text{ units}^2$$

∴ Area of shaded region = Area of ΔQST + Area of QPS

$$= \frac{27}{8} + 9$$

$$= \boldsymbol{12\frac{3}{8} \textbf{ units}^2}$$

8. **Topic: Applications of Differentiation & Integration (Kinematics)**

(i) $a_P = 1.5$ m/s^2

"Particle P … moves with a constant acceleration of 1.5 m/s^2 …"

$v = \int a\,\mathrm{d}t$

$v_P = \int 1.5\,\mathrm{d}t$

$= 1.5t + c_1$

When $t = 0$, $v_P = 9$ m/s:

"Particle P starts with a velocity of 9 m/s …"

$9 = 1.5(0) + c_1$

$c_1 = 9$

$\therefore$ **Velocity of P, $v_P = 1.5t + 9$** – (1)

$a_Q = 1 + \frac{t}{2}$

$v_Q = \int \left(1 + \frac{t}{2}\right) \mathrm{d}t$

$v = \int a\,\mathrm{d}t$

$= t + \frac{1}{4}t^2 + c_2$

When $t = 0$, $v_Q = 0$:

"Particle Q starts from rest …"

$0 = 0 + \frac{1}{4}(0)^2 + c_2$

$c_2 = 0$

$\therefore$ **Velocity of Q, $v_Q = t + \frac{1}{4}t^2$** – (2)

(ii) From (i), $v_P = 1.5t + 9$

$s = \int v\,\mathrm{d}t$

$s_P = \int (1.5t + 9)\,\mathrm{d}t$

$= \frac{3}{4}t^2 + 9t + c_3$

When $t = 0$, $s_P = 0$:

"P and Q, leave a point O at the same time …"
"t seconds is the time since leaving O …"

$0 = \frac{3}{4}(0)^2 + 9(0) + c_3$

$c_3 = 0$

$\therefore$ **Distance traveled by P, $s_P = \frac{3}{4}t^2 + 9t$** –(3)

Note: s_P & s_Q are the displacements of P and Q from O but since $t \geq 0$ and there are no negative components in s_P & s_Q
$\Rightarrow s_P$ & s_Q are the distances travelled by P and Q.

From (i), $v_Q = t + \frac{1}{4}t^2$

$s = \int v\,\mathrm{d}t$

$s_Q = \int \left(t + \frac{1}{4}t^2\right) \mathrm{d}t$

$= \frac{1}{2}t^2 + \frac{1}{12}t^3 + c_4$

When $t = 0$, $s_Q = 0$:

"P and Q, leave a point O at the same time …"
"t seconds is the time since leaving O …"

$0 = \frac{1}{2}(0)^2 + \frac{1}{12}(0)^3 + c_4$

$c_4 = 0$

$\therefore$ **Distance traveled by Q, $s_Q = \frac{1}{2}t^2 + \frac{1}{12}t^3$**

(iii) When Q collides with P,

P and Q are at the same distance from O.

$s_Q = s_P$

$\frac{1}{2}t^2 + \frac{1}{12}t^3 = \frac{3}{4}t^2 + 9t$

$12\left(\frac{1}{2}t^2 + \frac{1}{12}t^3\right) = 12\left(\frac{3}{4}t^2 + 9t\right)$

$6t^2 + t^3 = 9t^2 + 108t$

$t^3 - 3t^2 - 108t = 0$

$t(t^2 - 3t - 108) = 0$

$t(t - 12)(t + 9) = 0$

$t = 0$ (reject), $t = 12$, $t = -9$ (reject).

Reject: $t = 0$ (P&Q at point O)
$t = -9$ (t is positive)

$\therefore$ Distance from O when Q collides with P $= \frac{3}{4}(12)^2 + 9(12)$

$=$ **216 m**

Sub $t = 12$ into (3)

(iv) When $t = 12$,

$v_P = 1.5(12) + 9$

$=$ **27 m/s**

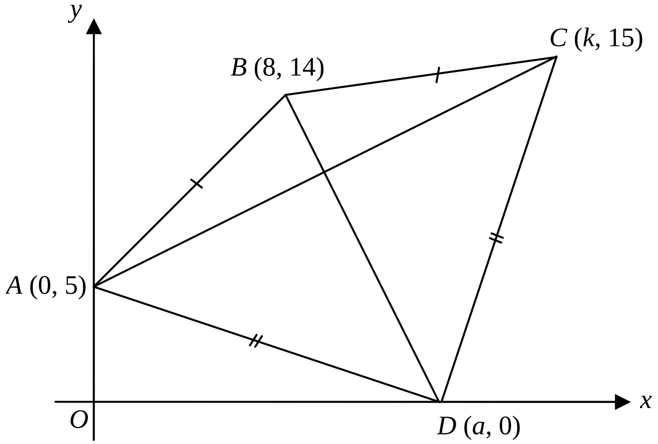
Sub $t = 12$ into (1)

$v_Q = 12 + \frac{1}{4}(12)^2$

$=$ **48 m/s**

Sub $t = 12$ into (2)

9. **Topic: Coordinate Geometry**



(i)

$$AB = BC$$
$$\sqrt{(8-0)^2+(14-5)^2} = \sqrt{(k-8)^2+(15-14)^2}$$
$$64 + 81 = (k-8)^2 + 1$$
$$(k-8)^2 = 144$$
$$k-8 = 12 \quad \text{or} \quad k-8 = -12$$
$$k = 20 \quad \text{or} \quad k = -4$$

Length of Line Segment $=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Since *k* is positive, *k* = 20.

(ii) Let $D(a, 0)$.

D lies on x-axis ⇒ y-coordinate is 0

$$AD = CD$$
$$\sqrt{(a-0)^2+(0-5)^2} = \sqrt{(20-a)^2+(15-0)^2}$$
$$a^2 + 25 = 400 - 40a + a^2 + 225$$
$$40a = 600$$
$$a = 15$$

∴ Coordinates of *D* = (15, 0)

Equation of *BD*:

$$\frac{y-0}{x-15} = \frac{14-0}{8-15}$$
$$y - 0 = -2(x-15)$$
$$\mathbf{y = -2x + 30}$$

Equation of straight line passing through $A(x_1, y_1)$ and $B(x_2, y_2)$:
$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} = \text{gradient } m$$

(iii) Area of ΔABC

$$= \frac{1}{2}\begin{vmatrix} 0 & 20 & 8 & 0 \\ 5 & 15 & 14 & 5 \end{vmatrix}$$
$$= \frac{1}{2}\left|[0 + (20)(14) + (8)(5)] - [(20)(5) + (8)(15) + 0]\right|$$
$$= 50 \text{ units}^2$$

Area of quadrilateral *ABCD*

$$= \text{Area of } \Delta ABC + \text{Area of } \Delta ADC$$
$$= 50 + \frac{1}{2}\begin{vmatrix} 0 & 15 & 20 & 0 \\ 5 & 0 & 15 & 5 \end{vmatrix}$$
$$= 50 + \frac{1}{2}\left|[0 + (15)(15) + (20)(5)] - [(15)(5) + 0 + 0]\right|$$
$$= 175 \text{ units}^2$$

$$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of quadrilateral } ABCD} = \frac{50}{175} = \mathbf{\frac{2}{7}} \textbf{ (Shown)}$$

Area of triangle *ABC* ('Shoelace Method')
$$= \frac{1}{2}\begin{vmatrix} x_A & x_C & x_B & x_A \\ y_A & y_C & y_B & y_A \end{vmatrix}$$
$$= \frac{1}{2}(x_Ay_c + x_Cy_B + x_By_A - x_Cy_A - x_By_C - x_Ay_B)$$
Note: Coordinates must be taken in an anticlockwise direction.

10. **Topic: Partial Fractions, Differentiation& Integration**

(i) $\frac{3x^2+4x-20}{(2x+1)(x^2+4)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+4}$

$3x^2 + 4x - 20 = A(x^2 + 4) + (Bx + C)(2x + 1)$

Sub $x = -\frac{1}{2}$,

$3\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) - 20 = A\left[\left(-\frac{1}{2}\right)^2 + 4\right] + 0$

$\boldsymbol{A = -5}$

Sub $x = 0$,

$-20 = 4A + C$

$-20 = 4(-5) + C$

$\boldsymbol{C = 0}$ **(Shown)**

$3x^2 + 4x - 20 = -5(x^2 + 4) + Bx(2x + 1)$

$3x^2 + 4x - 20 = (2B - 5)x^2 + Bx - 20$

Comparing coefficients of x,

$\boldsymbol{B = 4}$

(ii) $\frac{\mathrm{d}}{\mathrm{d}x}\ln(x^2+4) = \frac{1}{x^2+4}(2x)$

$= \boldsymbol{\frac{2x}{x^2+4}}$

$\frac{\mathrm{d}}{\mathrm{d}x}[\ln\ \mathrm{f}(x)] = \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}$

(iii) From (i), we have

$\frac{3x^2+4x-20}{(2x+1)(x^2+4)} = -\frac{5}{2x+1} + \frac{4x}{x^2+4}$ Using resultfrom (i)

$\int \frac{3x^2+4x-20}{(2x+1)(x^2+4)}\,dx = \int\left(-\frac{5}{2x+1} + \frac{4x}{x^2+4}\right)\mathrm{d}x$ Using result from (ii)

$= -5\int\frac{1}{2x+1}\,\mathrm{d}x + 2\int\left(\frac{2x}{x^2+4}\right)\mathrm{d}x$

$= -5\left[\frac{1}{2}\ln(2x+1)\right] + 2[\ln(x^2+4)] + c$

$= \boldsymbol{-\frac{5}{2}\ln(2x+1) + 2\ln(x^2+4) + c,\ x > -\frac{1}{2}}$

11. **Topic: Trigonometric Functions, Further Trigonometric Identities (*R*-Formula)**

(i) At A, $2 + 3\sin x$ is maximum:

$\Rightarrow \sin x = 1$

$x = \frac{\pi}{2}$

$\Rightarrow y_{\max} = 2 + 3(1)$

$= 5$

$\therefore$ **Coordinates of** $\boldsymbol{A = \left(\frac{\pi}{2},\ 5\right)}$

At B, $2 + 3\sin x$ is minimum:

$\Rightarrow \sin x = -1$

$x = \frac{3\pi}{2}$

$\Rightarrow y_{\min} = 2 + 3(-1)$

$= -1$

$\therefore$ **Coordinates of** $\boldsymbol{B = \left(\frac{3\pi}{2}, -1\right)}$

At C, $4\cos x$ is minimum:

$\Rightarrow \cos x = -1$

$x = \pi$

$\Rightarrow y_{\min} = 4(-1)$

$= -4$

$\therefore$ **Coordinates of** $\boldsymbol{C = (\pi, -4)}$

(ii)

$$4\cos x = 2 + 3\sin x$$
$$4\cos x - 3\sin x = 2 \qquad -(1)$$

R-Formula:
$a\cos\theta \pm b\sin\theta = R\cos(\theta \mp \alpha)$
where
$\tan\alpha = \dfrac{b}{a}$
$R = \sqrt{a^2 + b^2}$

Using *R*-formula,

$$4\cos x - 3\sin x = R\cos(x + \alpha)$$
$$= R\cos\alpha\cos x - R\sin\alpha\sin x$$

Comparing coefficients:

$$R\cos\alpha = 4 \qquad -(2)$$
$$R\sin\alpha = 3 \qquad -(3)$$

$(2)^2 + (3)^2$:

$$R^2(\cos^2\alpha + \sin^2\alpha) = 4^2 + 3^2$$
$$R^2 = 25$$
$$R = 5 \text{ or } -5 \text{(rejected)}$$

$\cos^2\theta + \sin^2\theta = 1$

$\frac{(3)}{(2)}$:

$$\tan\alpha = \frac{3}{4}$$
$$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$$
$$= 0.6435$$
$$\approx 0.644 \text{ (3 sig. fig.)}$$

$$\therefore \quad 4\cos x - 3\sin x = 5\cos(x + 0.644)$$

From (1),

$$4\cos x - 3\sin x = 2$$
$$\Rightarrow 5\cos(x + 0.644) = 2$$

$\cos(x + 0.644) = \frac{2}{5}$ where $\alpha = 0.644$, $k = \frac{2}{5}$.

(iii)

$$4\cos x = 2 + 3\sin x$$

From (ii),

***Hence question**: use result from (ii)

$$\cos(x + 0.6435) = \frac{2}{5}$$
$$\text{Basic } \angle\phi = 1.159$$
$$\therefore x + 0.6435 = 1.159,\ 2\pi - 1.159$$
$$x = 0.5155,\ 4.481$$
$$\approx 0.516,\ 4.48 \text{ (3 sig. fig.)}$$

$\therefore$ *x*-coordinate of *D* = 0.516, *x*-coordinate of *E* = 4.48.