

'O' Level | 2009 EM 4016 paper 2

• Suggested solutions

$$\begin{aligned} 1(a) \quad 1\frac{1}{2} \text{ hours} &= 1\frac{1}{2} \times 60 \\ &= 90 \text{ minutes.} \end{aligned}$$

$$\begin{aligned} (i) \quad \text{Time he spends warming up} &= \frac{90}{12} \times 3 \\ &= 22.5 \text{ minutes} \# \end{aligned}$$

$$\begin{aligned} (iii) \quad \% \text{ of the } 1\frac{1}{2} \text{ hours at the sport centre he spends running} \\ &= \frac{7}{2+3+7} \times 100\% \\ &= 58\frac{1}{3} \% \# \end{aligned}$$

$$\begin{aligned} (b) \quad (i) \quad \text{Speed} &= \frac{3000 \text{ m}}{9\frac{1}{2} \text{ minutes}} \\ &= \frac{3}{\frac{9\frac{1}{2}}{60}} \text{ km/h} \\ &= 18\frac{18}{19} \text{ km/h} \# \end{aligned}$$

$$\begin{aligned} (ii) \quad \text{His best time in 2009} &= 90\% \times 9\frac{1}{2} \text{ minutes} \\ &= 8.55 \text{ minutes} \\ &= 8 \text{ minutes } 33 \text{ seconds} \# \end{aligned}$$

$$\begin{aligned} (iii) \quad \text{His best time in 2007} &= \frac{9.5}{95} \times 100' \\ &= 10 \text{ minutes } 0 \text{ seconds} \# \end{aligned}$$

$$2. (a) (i) \quad 25 - p^2 = 5^2 - p^2 \\ = (5+p)(5-p) \quad \#$$

$$(ii) \quad \frac{25 - p^2}{15 + 3p} = \frac{(5+p)(5-p)}{3(5+p)} \\ = \frac{5-p}{3} \quad \#$$

$$(b) \quad \frac{3}{(x+2)^2} - \frac{4}{x+2} = \frac{3 - 4(x+2)}{(x+2)^2} \\ = \frac{3 - 4x - 8}{(x+2)^2} \\ = \frac{-4x - 5}{(x+2)^2} \quad \#$$

$$(c) \quad v^2 = u^2 - 2gh$$

(i) when $u = 30$, $g = 9.8$ and $h = 24$,

$$v^2 = (30)^2 - 2(9.8)(24)$$

$$= 900 - 470.4$$

$$v^2 = 429.6$$

$$v = \pm \sqrt{429.6}$$

$$= \pm 20.726$$

$$\approx \pm 20.7 \quad (3 \text{ s.f.}) \quad \#$$

$$(ii) \quad v^2 = u^2 - 2gh$$

$$u^2 = v^2 + 2gh$$

$$u = \pm \sqrt{v^2 + 2gh} \quad \#$$

$$3. (a) \sin 20^\circ = \frac{BC}{12}$$

$$BC = 12 \sin 20^\circ \\ = 4.1042$$

$$\therefore BD = 6 + 4.1042 \\ = 10.1042 \\ \approx 10.1 \text{ m (3 s.f.)} \#$$

$$(b) (i) (AG)^2 = (AN)^2 + (GN)^2$$

$$12^2 = 5^2 + (GN)^2 \quad (\because AN = EF)$$

$$(GN)^2 = 12^2 - 5^2$$

$$GN = \sqrt{119} \quad \text{or} \quad -\sqrt{119} \quad (\text{neg.})$$

$$= 10.908$$

$$\approx 10.9 \text{ m (3 s.f.)}$$

$$(ii) \cos \hat{GAN} = \frac{5}{12}$$

$$\hat{GAN} = \cos^{-1}\left(\frac{5}{12}\right) \\ = 65.375^\circ$$

$$\therefore \text{The angle through which the jib has rotated} \\ = 65.375^\circ - 20^\circ \\ = 45.375^\circ \\ \approx 45.4^\circ \text{ (1 d.p.)}$$

$$4. (a) (i) \hat{BEC} = \hat{BAC} \\ = 33^\circ \text{ (angle in the same segment) } \#$$

$$(ii) \hat{BAE} = 90^\circ \text{ (} \Delta \text{ in semicircle)}$$

$$\hat{CAE} = 90^\circ - 33^\circ \\ = 57^\circ$$

$$\therefore \hat{EBC} = \hat{CAE} \\ = 57^\circ \text{ (angle in the same segment) } \#$$

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$$(iii) \hat{CDE} + \hat{EBC} = 180^\circ \text{ (cyclic quad.)}$$

$$\therefore \hat{CDE} = 180^\circ - 57^\circ \\ = 123^\circ \#$$

$$(b) \hat{AFE} = \hat{DFC} \text{ (common } \sphericalangle \text{)}$$

$$\hat{FAE} = \hat{FDC} = 57^\circ$$

$\therefore \Delta FAE$ and ΔFDC are similar. $\#$

(c) Since ΔFAE and ΔFDC are similar,

$$\frac{FA}{FD} = \frac{FE}{FC}$$

$$\frac{AC+3}{4} = \frac{8+4}{3}$$

$$3(AC+3) = 4(12)$$

$$3AC + 9 = 48$$

$$3AC = 39$$

$$AC = 13 \text{ cm } \#$$

$$5(a) \quad E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$$

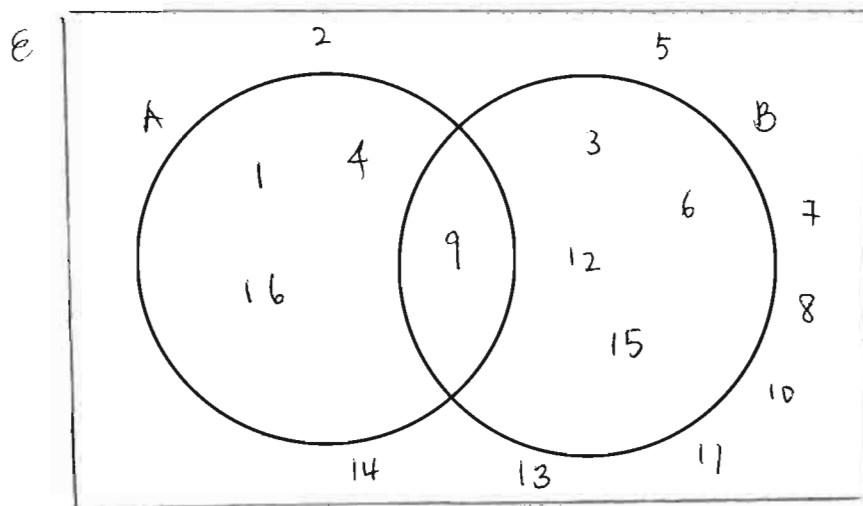
$$A = \{1, 4, 9, 16\}$$

$$B = \{3, 6, 9, 12, 15\}$$

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(i)



$$(ii) \quad A' = \{2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15\}$$

$$A' \cap B = \{3, 6, 12, 15\} \quad \#$$

$$(iii) \quad A \cup B = \{1, 3, 4, 6, 9, 12, 15, 16\}$$

$$n(A \cup B) = 8 \quad \#$$

$$5 \text{ (b) (i) } D = \begin{pmatrix} 25 & 15 \\ 10 & 30 \end{pmatrix}$$

$$\begin{aligned} \text{(ii) } E &= 5C + D \\ &= 5 \begin{pmatrix} 10 & 30 \\ 20 & 10 \end{pmatrix} + \begin{pmatrix} 25 & 15 \\ 10 & 30 \end{pmatrix} \\ &= \begin{pmatrix} 50 & 150 \\ 100 & 50 \end{pmatrix} + \begin{pmatrix} 25 & 15 \\ 10 & 30 \end{pmatrix} \\ &= \begin{pmatrix} 75 & 165 \\ 110 & 80 \end{pmatrix} \# \end{aligned}$$

(iii) Total number of adults and children carried by bus from Monday to Saturday.

$$\begin{aligned} \text{(iv) (a) } F &= C \begin{pmatrix} 25 \\ 15 \end{pmatrix} \\ &= \begin{pmatrix} 10 & 30 \\ 20 & 10 \end{pmatrix} \begin{pmatrix} 25 \\ 15 \end{pmatrix} \\ &= \begin{pmatrix} 700 \\ 650 \end{pmatrix} \end{aligned}$$

(b) F represents the total fare (in cents) collected from both adults and children on a weekday morning and weekday afternoon respectively.

$$\begin{aligned} \text{(c) } G &= \frac{1}{100} (1 \ 1) \begin{pmatrix} 700 \\ 650 \end{pmatrix} \\ &= \frac{1}{100} (1350) \\ &= (13.50) \end{aligned}$$

(d) G represents the total fare amount (in dollars) collected on a weekday.

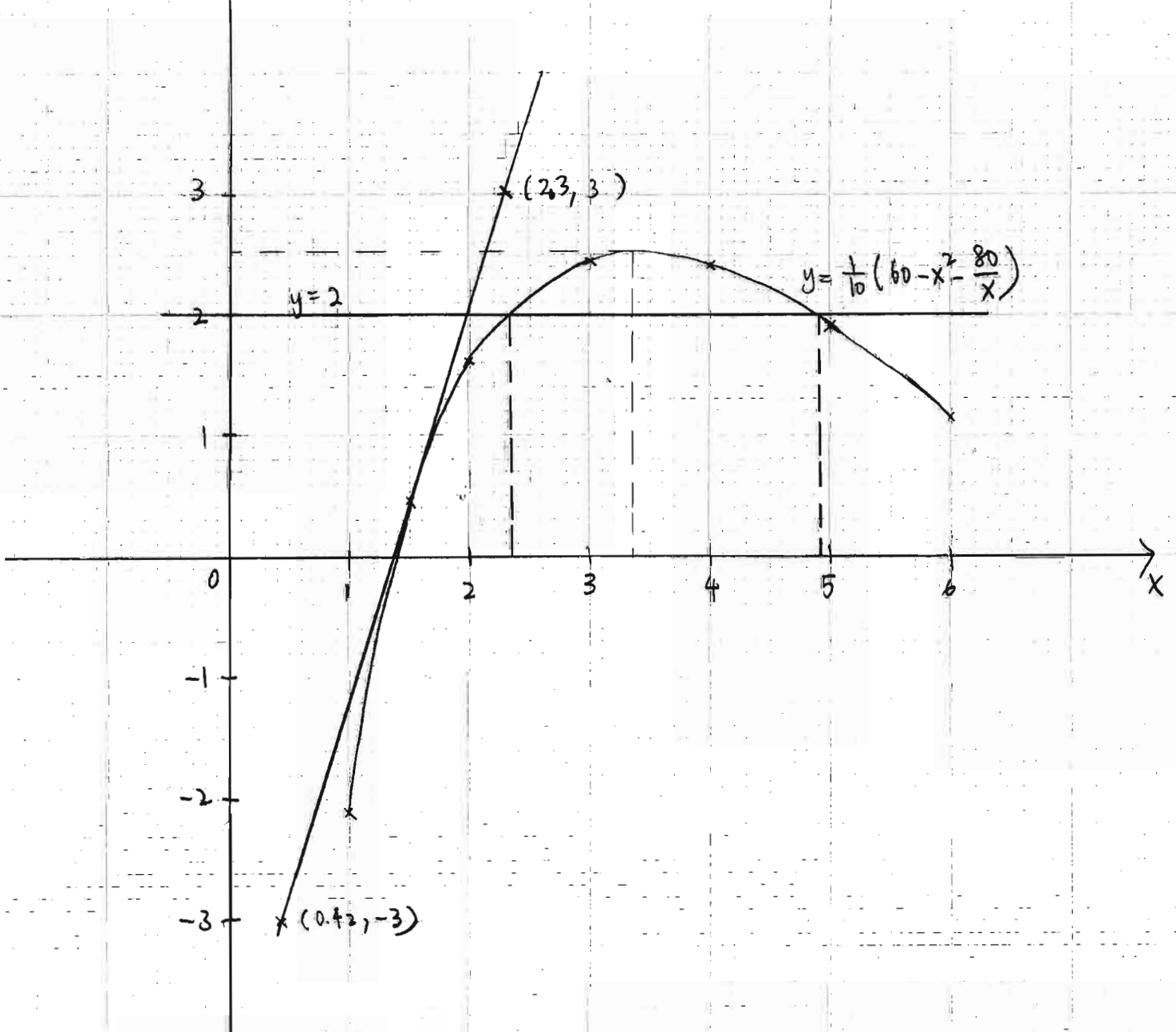
6.

(a) $y = \frac{1}{10} (60 - x^2 - \frac{80}{x})$
 when $x = 6$, $p = \frac{1}{10} [60 - 6^2 - \frac{80}{6}]$
 $= 1.07 (2 \text{ d.p.})$

(c) $\frac{1}{10} (60 - x^2 - \frac{80}{x}) = 2$
 $y = 2$
 $\therefore x = 2.35 \text{ or } 4.91$

(d) Gradient of tangent
 $= \frac{3 - (-3)}{2.3 - 0.42}$
 $\approx 3.19 (3 \text{ s.f.})$

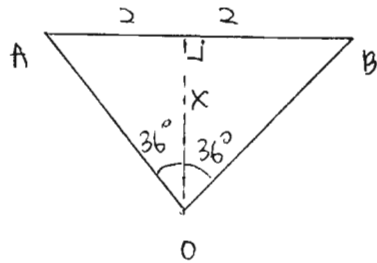
(e) Largest value of $y = 2.51$,
 when $x = 3.5$



$$7. (a) \hat{AOB} = \frac{360^\circ}{5}$$

$$= 72^\circ$$

Let the length of the perpendicular from O to AB be X.



$$\tan 36^\circ = \frac{2}{x}$$

$$x = \frac{2}{\tan 36^\circ}$$

$$= 2.7527$$

$$\approx 2.75 \text{ cm (3 s.f.)}$$

(b) radius of outer circle = $x + 2$

$$= 2.7527$$

$$= 4.7527$$

$$\approx 4.75 \text{ cm (3 s.f.) (shown) \#}$$

(c) Total length of wire needed to make the ornament

$$= 4(5) + 5\left(\frac{1}{2}\right)(2\pi)(2) + 2\pi(4.7527)$$

$$= 81.278$$

$$\approx 81.3 \text{ cm (3 s.f.) \#}$$

(d) Area enclosed by the wire pentagon = $5\left(\frac{1}{2}\right)(2.7527)(4)$

$$= 27.527$$

$$\approx 27.5 \text{ cm}^2 \text{ (3 s.f.) \#}$$

(e) Area of the region shaded on the diagram

$$= [\pi(4.7527)^2 - 27.527 - 5\left(\frac{1}{2}\right)(\pi)(2)^2] \div 5$$

$$= (12.019) \div 5$$

$$\approx 2.40 \text{ cm}^2 \text{ (3 s.f.) \#}$$

$$8. \text{ (a) (i)} \quad (BC)^2 = (AB)^2 + (AC)^2 - 2(AB)(AC) \cos \hat{BAC}$$

$$= 550^2 + 645^2 - 2(550)(645) \cos 38^\circ$$

$$BC = \sqrt{159431.3703}$$

$$= 399.28$$

$$\approx 399 \text{ m (3 s.f.)}$$

$$\text{(ii)} \quad \frac{\sin \hat{ACB}}{AB} = \frac{\sin \hat{BAC}}{BC}$$

$$\frac{\sin \hat{ACB}}{550} = \frac{\sin 38^\circ}{399.28}$$

$$\sin \hat{ACB} = 0.84806$$

$$\hat{ACB} = 58.00^\circ$$

$$\approx 58.0^\circ \text{ (1 d.p.)}$$

$$\text{(iii)} \quad \hat{ABC} = 180^\circ - 38^\circ - 58.0$$

$$= 84^\circ$$

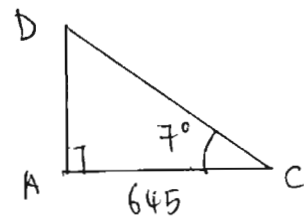
$$\text{Bearing of C from B} = 360^\circ - (180^\circ - 62^\circ) - 84^\circ$$

$$= 158^\circ \#$$

$$\text{(b) (i)} \quad \tan 7^\circ = \frac{AD}{645}$$

$$AD = 79.196$$

$$\approx 79.2 \text{ m (3 s.f.)}$$



(ii) Let x be the shortest distance from A to BC .

$$\frac{1}{2}(x)(BC) = \text{Area of } \triangle ABC$$

$$\frac{1}{2}(x)(399.28) = \frac{1}{2}(550)(645) \sin 38^\circ$$

$$x = 546.999$$

Let θ be the greatest possible angle of elevation of D from a point on BC .

$$\tan \theta = \frac{79.196}{546.999}$$

$$\theta = 8.238 \approx 8.2^\circ \text{ (1 d.p.)}$$

9. (a) $AT = TP = x$

$SB = SQ = 7$

$\therefore ST = 17 - AT - SB$

$= 17 - x - 7$

$= (10 - x) \text{ cm (shown)} \#$

(b) (i) $PQ = (x + 7) \text{ cm} \#$

(ii) $QR = (7 - x) \text{ cm} \#$

By Pythagoras' theorem,

(c) $(PQ)^2 = (PR)^2 + (QR)^2$

$(x + 7)^2 = (ST)^2 + (7 - x)^2$

$x^2 + 14x + 49 = (10 - x)^2 + 49 - 14x + x^2$

$x^2 + 14x + 49 = 100 - 20x + x^2 + 49 - 14x + x^2$

$x^2 - 48x + 100 = 0 \text{ (shown).}$

(d) $a = 1, b = -48, c = 100$

$$x = \frac{48 \pm \sqrt{(-48)^2 - 4(1)(100)}}{2(1)}$$

$$= \frac{48 \pm \sqrt{1904}}{2}$$

$= 45.817 \quad \text{or} \quad 2.182$

$\approx 45.82 \quad 2.18 \text{ (2 d.p.)} \#$

$x = 2.182$

(e) \wedge Diameter of the smaller cylinder

$= 2(2.182) \quad (x = 45.817 \text{ rejected})$

$= 4.364 \text{ cm}$

$= 43.64 \text{ mm}$

$\approx 44 \text{ mm (nearest millimetre)} \#$

10 (a) (i)

Scores	4	5	6	7	8	10
Frequency	2	5	11	8	3	1

(ii) (a) Mean score = $\frac{4(2) + 5(5) + 6(11) + 7(8) + 8(3) + 10(1)}{30}$

$$= \frac{189}{30}$$

$$= 6.3 \#$$



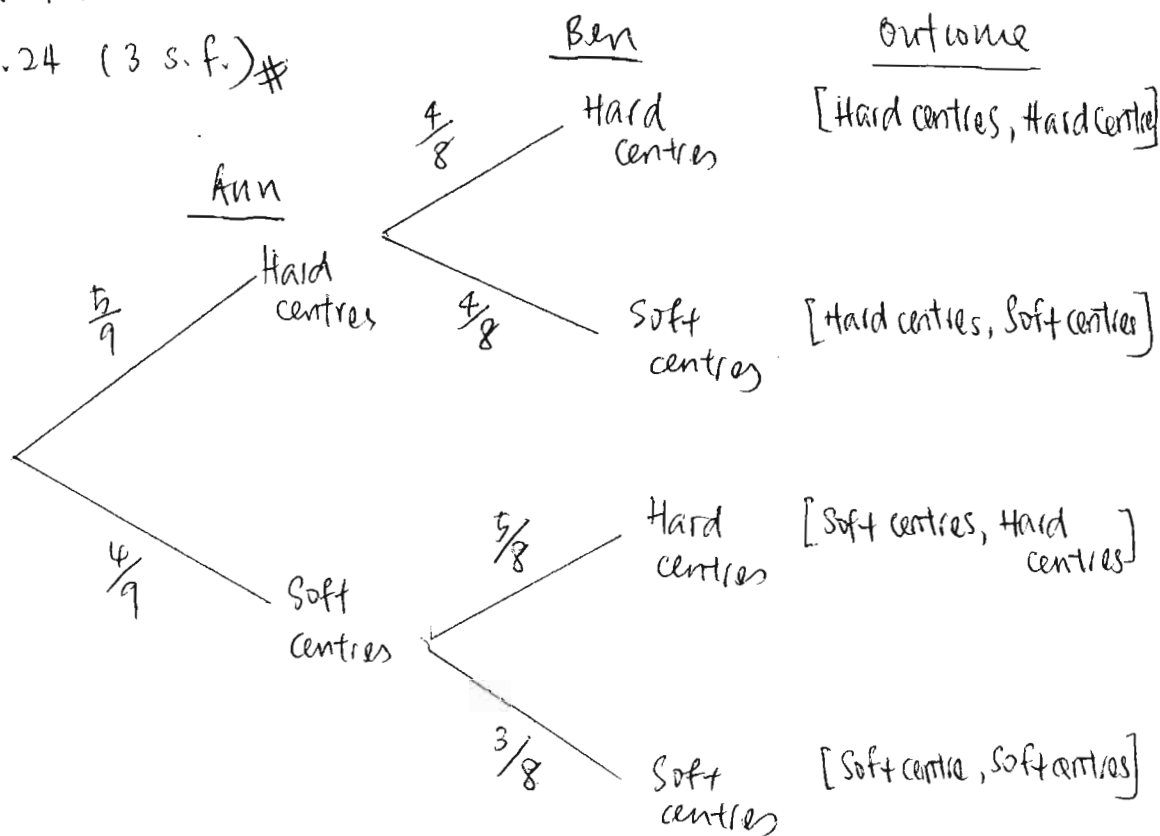
(b) Standard deviation

$$= \sqrt{\frac{2(4)^2 + 5(5)^2 + 11(6)^2 + 8(7)^2 + 3(8)^2 + 1(10)^2}{30} - (6.3)^2}$$

$$= 1.2423$$

$$\approx 1.24 \text{ (3 s.f.)} \#$$

(b) (i)



(ii) (a) $P(\text{Ann and Ben both choose a chocolate with a hard centre})$
 $= \frac{5}{9} \times \frac{4}{8}$
 $= \frac{5}{18} \#$

(b) $P(\text{Ben chooses a chocolate with a soft centre})$
 $= \frac{5}{9} \left(\frac{4}{8}\right) + \frac{4}{9} \times \frac{3}{8} = \frac{4}{9} \#$

(c) $P(\text{one of them choose a chocolate with a hard centre and the other chooses one with a soft centre})$
 $= \frac{5}{9} \left(\frac{4}{8}\right) + \left(\frac{4}{9}\right) \left(\frac{5}{8}\right) = \frac{5}{9} \#$