

ELEMENTARY MATHEMATICS Paper 2 Suggested Solutions

4016/02
October/November 2009

1. Topics: Arithmetic (Time, Speed and Percentages)

(a) (i) $1\frac{1}{2}$ hours = $1\frac{1}{2} \times 60$
= 90 minutes

Time he spends warming up = $\frac{90}{12} \times 3$
= **22.5 minutes**

(ii) % of the $1\frac{1}{2}$ hours at the sports centre he spends running
= $\frac{7}{2+3+7} \times 100\%$
= **$58\frac{1}{3}\%$**

(b) (i) Speed = $\frac{3000 \text{ m}}{9\frac{1}{2} \text{ minutes}}$ Speed = $\frac{\text{Distance travelled}}{\text{Time taken}}$
= $\frac{3}{9\frac{1}{2}}$ km/h
= $\frac{3}{\frac{19}{2}}$ km/h
= **$18\frac{18}{19}$ km/h**

(ii) His best time in 2009 = $90\% \times 9\frac{1}{2}$ minutes 100% → 2008 time
90% → 2009 time
= 8.55 minutes
= **8 minutes 33 seconds**

(iii) His best time in 2007 = $\frac{9.5}{95} \times 100$ 100% → 2007 time
95% → 2008 time
= **10 minutes 0 seconds**

2. Topic: Algebra (Factorisation, Formulae)

(a) (i) $25 - p^2 = 5^2 - p^2$ $a^2 - b^2 = (a + b)(a - b)$
= **$(5 + p)(5 - p)$**

(ii) $\frac{25 - p^2}{15 + 3p} = \frac{(5 + p)(5 - p)}{3(5 + p)}$ Using answer from part (i)
= $\frac{5 - p}{3}$

(b) $\frac{3}{(x+2)^2} - \frac{4}{x+2} = \frac{3 - 4(x+2)}{(x+2)^2}$
= $\frac{3 - 4x - 8}{(x+2)^2}$
= $\frac{-4x - 5}{(x+2)^2}$

(c) $v^2 = u^2 - 2gh$

(i) When $u = 30$, $g = 9.8$ and $h = 24$,
 $v^2 = 30^2 - 2(9.8)(24)$
= $900 - 470.4$
 $v^2 = 429.6$
 $v = \pm\sqrt{429.6}$
= ± 20.726
 $\approx \pm 20.7$ (3 sig. fig.)

(ii) $v^2 = u^2 - 2gh$
 $u^2 = v^2 + 2gh$
 $u = \pm\sqrt{v^2 + 2gh}$

3. Topic: Trigonometry (Trigonometrical Ratios, Pythagoras' Theorem)

(a) $\sin 20^\circ = \frac{BC}{12}$

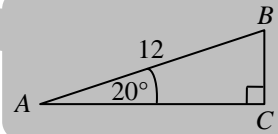
$$BC = 12 \sin 20^\circ$$

$$= 4.1042$$

$$\therefore BD = 6 + 4.1042$$

$$= 10.1042$$

$$\approx \mathbf{10.1 \text{ m (3 sig. fig.)}}$$



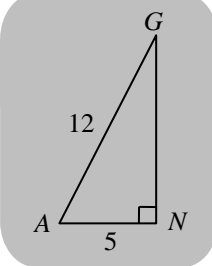
(b) (i) $(AG)^2 = (AM)^2 + (GN)^2$
 $12^2 = 5^2 + (GN)^2$ ($\because AN = EF$)

$$(GN)^2 = 12^2 - 5^2$$

$$GN = \sqrt{119} \text{ or } -\sqrt{119} \text{ (rejected)}$$

$$= 10.908$$

$$\approx \mathbf{10.9 \text{ m (3 sig. fig.)}}$$



(ii) $\cos \angle GAN = \frac{5}{12}$

$$\angle GAN = \cos^{-1} \left(\frac{5}{12} \right)$$

$$= 65.375^\circ$$

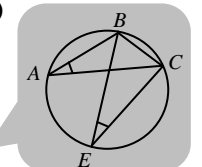
$$\therefore \text{The angle through which the jib has rotated} = 65.375^\circ - 20^\circ$$

$$= 45.375^\circ$$

$$\approx \mathbf{45.4^\circ (1 \text{ d.p.})}$$

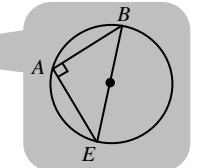
4. Topic: Geometry (Geometrical Properties of Circles & Similarity)

(a) (i) $\angle BEC = \angle BAC$ (angle in the same segment)
 $= 33^\circ$



(ii) $\angle BAE = 90^\circ$ (Δ in semi circle)
 $\angle CAE = \angle BAE - \angle BAC$
 $= 90^\circ - 33^\circ$
 $= 57^\circ$

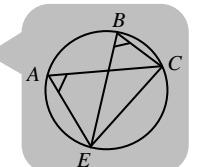
$\angle BAC = 33^\circ$
[from (a)(i)]



$$\therefore \angle EBC = \angle CAE$$

$$= 57^\circ$$

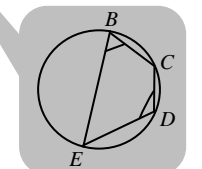
(angle in the same segment)



(iii) $\angle CDE + \angle EBC = 180^\circ$ (opp. \angle s of cyclic quad)
 $\therefore \angle CDE = 180^\circ - 57^\circ$
 $= 123^\circ$

$\angle EBC = \angle CAE = 57^\circ$
[from (a)(ii)]

(b) (i) $\angle AFE = \angle DFC$ (Common \angle)
 $\angle FAE = \angle FDC = 57^\circ$



$$\therefore \Delta FAE \text{ and } \Delta FDC \text{ are similar}$$

(c) Since ΔFAE and ΔFDC are similar,

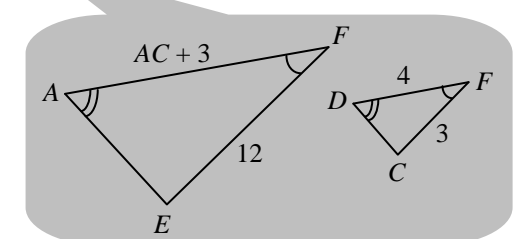
$$\frac{FA}{FD} = \frac{FE}{FC}$$

$$\frac{AC+3}{4} = \frac{8+4}{3}$$

$$3(AC+3) = 4(12)$$

$$3AC = 39$$

$$AC = \mathbf{13 \text{ cm}}$$



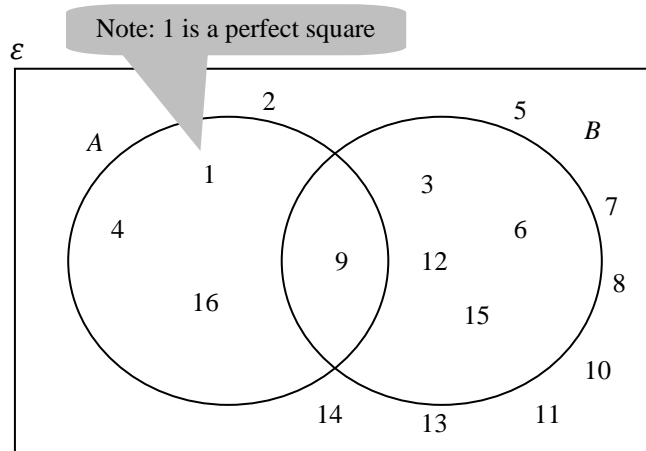
5. Topics: Set Language and Notation, Matrices

(a) $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$

$A = \{1, 4, 9, 16\}$

$B = \{3, 6, 9, 12, 15\}$

(i)



(ii) $A' = \{2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15\}$

$A' \cap B = \{3, 6, 12, 15\}$

(iii) $A \cup B = \{1, 3, 4, 6, 9, 12, 15, 16\}$

$n(A \cup B) = 8$

(b) (i) $D = \begin{pmatrix} 25 & 15 \\ 10 & 30 \end{pmatrix}$

All perfect squares and all integers divisible by 3.

Integers outside of A
 \rightarrow non-perfect squares

Integers in both A' and B
 \rightarrow non-perfect squares and divisible by 3

(ii) $E = 5C + D$

$$= 5 \begin{pmatrix} 10 & 30 \\ 20 & 10 \end{pmatrix} + \begin{pmatrix} 25 & 15 \\ 10 & 30 \end{pmatrix}$$

$$= \begin{pmatrix} 50 & 150 \\ 100 & 50 \end{pmatrix} + \begin{pmatrix} 25 & 15 \\ 10 & 30 \end{pmatrix}$$

$$= \begin{pmatrix} 75 & 165 \\ 110 & 80 \end{pmatrix}$$

(iii) Total number of adults and children carried by bus from Monday to Saturday.

(iv) (a) $F = C \begin{pmatrix} 25 \\ 15 \end{pmatrix}$

$$= \begin{pmatrix} 10 & 30 \\ 20 & 10 \end{pmatrix} \begin{pmatrix} 25 \\ 15 \end{pmatrix}$$

$$= \begin{pmatrix} 700 \\ 650 \end{pmatrix}$$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$

(b) F represents the total fare in cents collected from both adults and children on a weekday morning and weekday afternoon respectively.

(c) $G = \frac{1}{100} (1 \ 1) \begin{pmatrix} 700 \\ 650 \end{pmatrix}$

$$= \frac{1}{100} (1350)$$

$$= (13.50)$$

(d) G represents the total fare amount (in dollars) collected on a weekday.

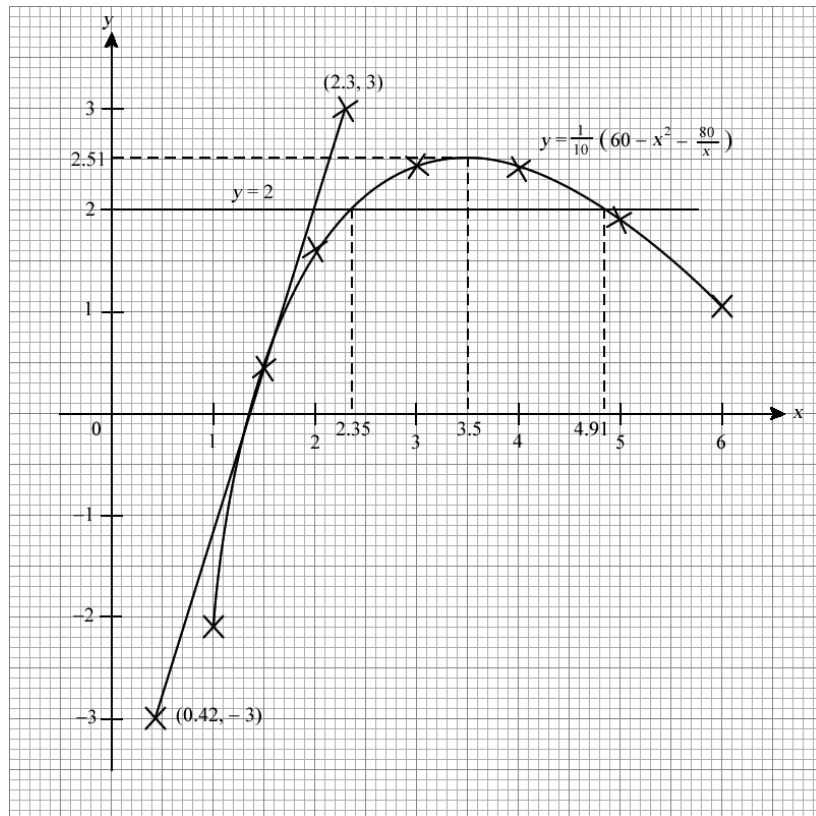
6. **Topic: Graphical Solution of Equations**

(a) $y = \frac{1}{10} \left(60 - x^2 - \frac{80}{x} \right)$

When $x = 6$,

$$p = \frac{1}{10} \left(60 - 6^2 - \frac{80}{6} \right) = \mathbf{1.07 \text{ (2 d.p.)}}$$

(b)



(c) Values of x where $\frac{1}{10} \left(60 - x^2 - \frac{80}{x} \right) = 2$ is obtained from x -coordinate of intersection points of the curve $y = \frac{1}{10} \left(60 - x^2 - \frac{80}{x} \right)$ and the line $y = 2$.

From the graph, $x = \mathbf{2.35, 4.91}$

(d) From the graph, gradient of tangent $= \frac{3 - (-3)}{2.3 - 0.42}$

$= \mathbf{3.19 \text{ (3 sig. fig.)}}$

Check:

$$\frac{1}{10} \left(60 - x^2 - \frac{80}{x} \right) = 2$$

$$x^3 - 40x + 80 = 0$$

$$x = -7.15, \mathbf{2.31, 4.85}$$

AMaths students:

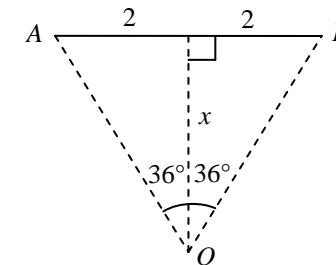
Check: $\frac{dy}{dx} = \frac{1}{10} \left(-2x + \frac{80}{x^2} \right)$
Sub $x = 1.5 \Rightarrow \frac{dy}{dx} = 3.26$

(e) Largest value of $y = \mathbf{2.51}$, when $x = 3.5$

7. **Topics: Trigonometry, Mensuration**

(a) $\angle AOB = \frac{360^\circ}{5}$
 $= 72^\circ$

Let the length of the perpendicular from O to AB be x .



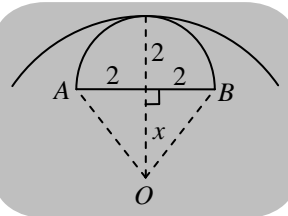
$$\tan 36^\circ = \frac{2}{x}$$

$$x = \frac{2}{\tan 36^\circ}$$

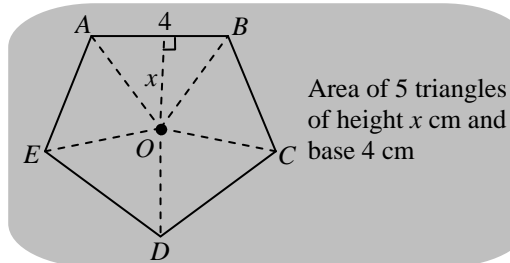
$$= 2.7527$$

$\approx \mathbf{2.75 \text{ cm (3 sig. fig.)}}$

- (b) Radius of outer circle = $x + 2$
 = $2.7527 + 2$
 = 4.7527
 ≈ 4.75 cm (3 sig. fig.) (Shown)



- (c) Total length of wire needed to make the ornament
 = $5(4) + 5\left(\frac{1}{2}\right)(2\pi)(2) + 2\pi(4.7527)$ (5 × sides of pentagon)
 = 81.278 + (5 × semicircles)
 + Circumference of outer circle
 ≈ 81.3 cm (3 sig. fig.)



- (d) Area enclosed by the wire pentagon = $5\left(\frac{1}{2}\right)(2.7527)(4)$
 = 27.527
 ≈ 27.5 cm² (3 sig. fig.)

- (e) Area of the shaded region on the diagram
 = $[\pi(4.7527)^2 - 27.527 - 5\left(\frac{1}{2}\right)(\pi)(2)^2] \div 5$ [Area of outer circle
 - Area of pentagon
 - Area of 5 semicircles]
 = (12.019) $\div 5$
 ≈ 2.40 cm² (3 sig. fig.) $\div 5$

8. Topic: Trigonometry (Cosine Rule, Sine Rule, Bearings, Angle of Elevation)

- (a) (i) $(BC)^2 = (AB)^2 + (AC)^2 - 2(AB)(AC)\cos \angle BAC$
 = $550^2 + 645^2 - 2(550)(645)\cos 38^\circ$
 $BC = \sqrt{159431.3703}$
 = 399.28
 ≈ 399 m (3 sig. fig.)

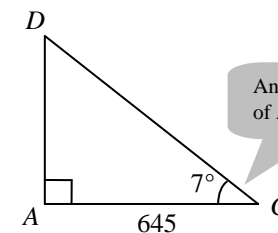
Cosine Rule:
 $c^2 = a^2 + b^2 - 2ab\cos c$

- (ii) $\frac{\sin \angle ACB}{AB} = \frac{\sin \angle BAC}{BC}$
 $\frac{\sin \angle ACB}{550} = \frac{\sin 38^\circ}{399.28}$
 $\sin \angle ACB = 0.84806$
 $\angle ACB = 58.00^\circ$
 $\approx 58.0^\circ$ (1 d.p.)

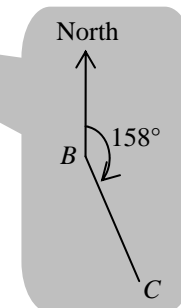
Sine Rule:
 $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

- (iii) $\angle ABC = 180^\circ - 38^\circ - 58.0^\circ$
 = 84°
 Bearing of C from B = $360^\circ - (180^\circ - 62^\circ) - 84^\circ$
 = 158°

- (b) (i) $\tan 7^\circ = \frac{AD}{645}$
 $AD = 79.196$
 ≈ 79.2 m (3 sig. fig.)



Angle of elevation
 of D from C



- (ii) Let x be the shortest distance from A to BC .

$$\frac{1}{2}(x)(BC) = \text{Area of } \triangle ABC$$

$$\frac{1}{2}(x)(399.28) = \frac{1}{2}(550)(645)\sin 38^\circ$$

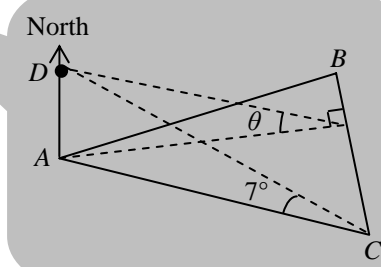
$$x = 546.999$$

Let θ be the greatest possible angle of elevation of D from a point on BC .

$$\tan \theta = \frac{79.196}{546.999}$$

$$\theta = 8.238$$

$$\approx 8.2^\circ \text{ (1 d.p.)}$$



Max \angle of elevation occurs at min distance from A to BC .

9. Topics: Pythagoras' Theorem, Solution to Quadratic Equation

(a) $AT = TP = x$

$$SB = SQ = 7$$

$$\therefore ST = 17 - AT - SB$$

$$= 17 - x - 7$$

$$= (10 - x) \text{ cm (Shown)}$$

(b) (i) $PQ = (x + 7) \text{ cm}$

(ii) $QR = (7 - x) \text{ cm}$

(c) By Pythagoras' Theorem,

$$(PQ)^2 = (PR)^2 + (QR)^2$$

$$(x + 7)^2 = (ST)^2 + (7 - x)^2$$

$$x^2 + 14x + 49 = (10 - x)^2 + 49 - 14x + x^2$$

$$x^2 + 14x + 49 = 100 - 20x + x^2 + 49 - 14x + x^2$$

$$x^2 - 48x + 100 = 0 \text{ (Shown)}$$

(d) $a = 1, b = -48, c = 100$

$$x = \frac{48 \pm \sqrt{(-48)^2 - 4(1)(100)}}{2(1)}$$

$$= \frac{48 \pm \sqrt{1904}}{2}$$

$$= 45.817 \quad \text{or} \quad 2.182$$

$$\approx 45.82 \quad \quad 2.18 \text{ (2 d.p.)}$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- (e) When $x = 2.182$, (45.82 rejected)

$$\text{Diameter of smaller cylinder} = 2(2.182)$$

$$= 4.364 \text{ cm}$$

$$= 43.64 \text{ mm}$$

$$\approx 44 \text{ mm (nearest millimeter)}$$

$\therefore x < 14$ in the diagram

$$1 \text{ cm} = 10 \text{ mm}$$

10. Topics: Statistics, Simple Probability

(a) (i)

Scores	4	5	6	7	8	10
Frequency	2	5	11	8	3	1

(ii) (a) Mean score = $\frac{\sum fx}{\sum f}$

$$= \frac{4(2)+5(5)+6(11)+7(8)+8(3)+10(1)}{30}$$

$$= \frac{189}{30}$$

$$= \mathbf{6.3}$$

(b) Standard deviation = $\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$

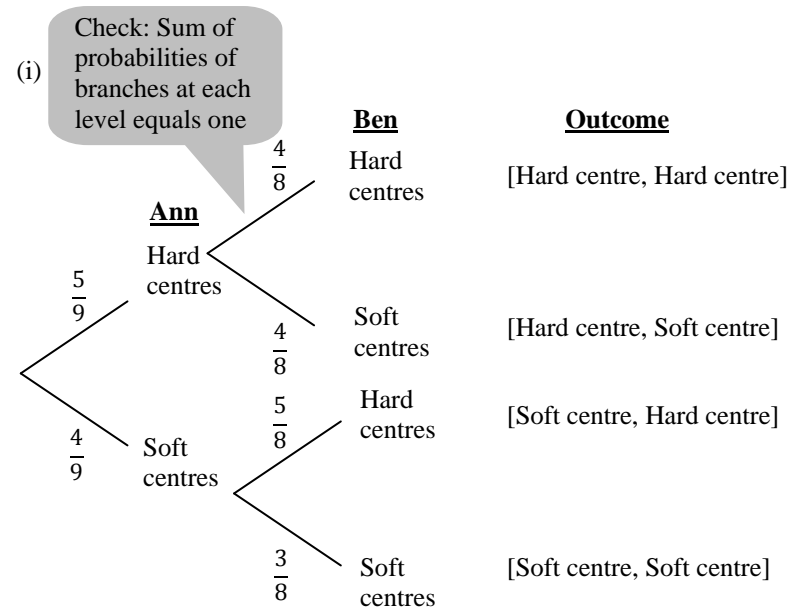
$$= \sqrt{\frac{2(4)^2+5(5)^2+11(6)^2+8(7)^2+3(8)^2+1(10)^2}{30} - (6.3)^2}$$

$$= 1.2423$$

$\frac{\sum fx}{\sum f} = 6.3$ from (ii)(a)

$$\approx \mathbf{1.24 \text{ (3 sig. fig.)}}$$

(b) (i)



(ii) (a) P(Ann and Ben both choose a chocolate with hard centre)

$$= \frac{5}{9} \times \frac{4}{8}$$

P(Ann gets hard centre) AND P(Ben gets hard centre)

$$= \frac{5}{18}$$

(b) P(Ben chooses a chocolate with soft centre)

$$= \frac{5}{9} \left(\frac{4}{8}\right) + \frac{4}{9} \left(\frac{3}{8}\right)$$

P(Ann gets hard centre AND Ben gets soft centre) OR P(Ann gets soft centre AND Ben gets soft centre)

$$= \frac{4}{9}$$

(c) P(One of them chooses a chocolate with hard centre and the other chooses one with soft centre)

$$= \frac{5}{9} \left(\frac{4}{8}\right) + \frac{4}{9} \left(\frac{5}{8}\right)$$

P(Ann gets hard centre AND Ben gets soft centre) OR P(Ann gets soft centre AND Ben gets hard centre)

$$= \frac{5}{9}$$