

ELEMENTARY MATHEMATICS
Paper 1 Suggested Solutions

4016/01
October/November 2009

1. **Topic: Algebra**

(a) $20xy - 5y = 5y(4x - 1)$

(b) $4x - 4(x + 3) = 4x - 4x - 12$
 $= -12$

Answer (a) $\frac{5y(4x - 1)}{\quad}$ [1]
(b) $\frac{-12}{\quad}$ [1]

2. **Topic: Arithmetic (Percentages)**

$100\% \rightarrow \$48$

$1\% \rightarrow \frac{\$48}{100}$

$235\% \rightarrow \frac{\$48}{100} \times 235$
 $= \$112.80$

\therefore Selling price = \$112.80

Selling price
= Cost price (100%) + Profit (135%)
= 235% \times Cost price

Answer \$ $\frac{112.80}{\quad}$ [2]

3. **Topic: Statistics (Frequency Table & Pie Chart)**

(a) Modal colour = **Blue**

Modal colour
= Mode of frequency table
= Colour of car that appears most frequently

(b) Angle representing the colour green = $\frac{5}{9+5+1+6+3} \times 360^\circ$
 $= 75^\circ$

\angle of sector in pie chart
 \propto its given data

Answer (a) $\frac{\text{Blue}}{\quad}$ [1]
(b) $\frac{75^\circ}{\quad}$ [1]

4. **Topic: Arithmetic (Percentages & Fractions)**

(a) $\frac{17}{24} \times 100\% = 70\frac{5}{6}\%$

(b) Fraction of candidates who were not awarded an A or B grade

$= 1 - \frac{1}{3} - \frac{1}{4}$
 $= \frac{5}{12}$

Answer (a) $\frac{70\frac{5}{6}}{\quad} \%$ [1]
(b) $\frac{5}{12}$ [1]

5. **Topic: Algebra (Indices)**

(a) $2^p \times 5 = 40$

$2^p = \frac{40}{5}$

$2^p = 8$

$2^p = 2^3$

$\therefore p = 3$

$$\begin{aligned} \text{(b)} \quad 1 \div x^{-4} &= 1 \div \frac{1}{x^4} \\ &= 1 \times \frac{x^4}{1} \\ &= x^4 \end{aligned}$$

$$\frac{1}{a^m} = a^{-m}$$

Answer (a) $p = \underline{\quad 3 \quad}$ [1]
(b) $\underline{\quad x^4 \quad}$ [1]

6. Topic: Angle Properties of Polygon

$$\begin{aligned} \text{(a)} \quad x^\circ + \angle EDC &= 180^\circ && \text{(int. } \angle\text{s, } AE \parallel DC) \\ \therefore x &= 180 - 130 \\ &= \mathbf{50} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \angle EAB &= 180^\circ - x^\circ \\ &= 130^\circ && \text{(int. } \angle\text{s)} \end{aligned}$$

$$\text{Sum of interior } \angle\text{s} = (5 - 2) \times 180^\circ$$

$$x^\circ + \angle EAB + y^\circ + 80^\circ + 130^\circ = 3 \times 180^\circ$$

$$50^\circ + 130^\circ + y^\circ + 80^\circ + 130^\circ = 540^\circ$$

$$y = \mathbf{150}$$

Answer (a) $x = \underline{\quad 50 \quad}$ [1]
(b) $y = \underline{\quad 150 \quad}$ [1]

Sum of int. $\angle\text{s}$ of an n -sided polygon = $(n - 2) \times 180^\circ$

7. Topic: Linear Inequalities

$$\begin{aligned} -2 &< 2x - 5 < 7 \\ -2 + 5 &< 2x < 7 + 5 \\ \therefore \frac{3}{2} &< x < 6 \end{aligned}$$

Answer (a) $\underline{\quad \frac{3}{2} < x < 6 \quad}$ [2]

8. Topic: Arithmetic (Compound Interest)

$$\begin{aligned} \text{Total amount} &= 5000 \left(1 + \frac{4.8}{100}\right)^6 \\ &= \$6624.265 \end{aligned}$$

$$\begin{aligned} \therefore \text{Interest} &= \$6624.265 - \$5000 \\ &= \$1624.265 \\ &\approx \mathbf{1624.27 \text{ (2 d.p.)}} \end{aligned}$$

Given in formula sheet (compound interest):

$$\text{Total amount} = P \left(1 + \frac{r}{100}\right)^n$$

Total interest

$$= \text{Total amt.} - \text{Principal amt.}$$

Answer \$ $\underline{\quad 1624.27 \quad}$ [2]

9. Topics: Trigonometry and Mensuration

$$\begin{aligned} \text{(a)} \quad \text{Area of } \triangle ABC &= \frac{1}{2} ab \sin c \\ &= \frac{1}{2} (7.43)(7.43) \sin 38^\circ \\ &= 16.993 \\ &\approx \mathbf{17.0 \text{ cm}^2 \text{ (3 sig. fig.)}} \end{aligned}$$

$$\begin{aligned} \text{(b) Volume of the prism} &= \text{Base area} \times \text{height} \\ &= \text{Area of } \triangle ABC \times 20 \\ &= 16.993 \times 20 \\ &= 339.87 \\ &\approx \mathbf{340 \text{ cm}^2} \text{ (3 sig. fig.)} \end{aligned}$$

$$\begin{aligned} \text{Answer (a)} & \underline{\quad 17.0 \quad} \text{ cm}^2 \quad [1] \\ \text{(b)} & \underline{\quad 340 \quad} \text{ cm}^2 \quad [1] \end{aligned}$$

10. Topic: Number Patterns

$$\begin{aligned} \text{(a) 1st term} &= 38 \\ 2^{\text{nd}} \text{ term} &= 38 - 7 \\ &= \mathbf{31} \\ 3^{\text{rd}} \text{ term} &= 31 - 7 \\ &= \mathbf{24} \end{aligned}$$

$$\begin{aligned} \text{(b) } T_1 &= 38 \\ T_2: 31 &= 38 - 7 \\ T_3: 24 &= 38 - 7 - 7 \\ T_n: 38 - 7(n-1) &= 45 - 7n \\ n^{\text{th}} \text{ term} &= \mathbf{45 - 7n} \end{aligned}$$

$$\begin{aligned} \text{Answer (a)} & \underline{\quad 31 \quad}, \underline{\quad 24 \quad} \quad [1] \\ \text{(b)} & \underline{\quad 45 - 7n \quad} \quad [1] \end{aligned}$$

11. Topic: Arithmetic (Application of Mathematics in Practical Situations)

$$\begin{aligned} \text{Small tin: } 415 \text{ g} &\rightarrow \$1.04 \\ 1 \text{ g} &\rightarrow \$\frac{1.04}{415} \\ &= \$0.002506/\text{g} \\ &\approx \$0.00251/\text{g} \text{ (3 sig. fig.)} \end{aligned}$$

$$\begin{aligned} \text{Large tin: } 815 \text{ g} &\rightarrow \$1.98 \\ 1 \text{ g} &\rightarrow \$\frac{1.98}{815} \\ &= \$0.002429/\text{g} \\ &\approx \$0.00243/\text{g} \text{ (3 sig. fig.)} \end{aligned}$$

\therefore the large tin gives better value because it costs less per gram.

$$\text{Answer The } \underline{\text{large}} \text{ tin gives better value} \quad [2]$$

12. Topic: Kinematics

$$\begin{aligned} \text{(a) Acceleration during the 1}^{\text{st}} 40 \text{ seconds} &= \frac{24}{40} \\ &= \mathbf{0.6 \text{ m/s}^2} \end{aligned}$$

$$\text{Acceleration} = \frac{\text{Speed}}{\text{Time}}$$

$$\begin{aligned} \text{(b) Total distance travelled} &= \text{Total area under the graph} \\ &= \frac{1}{2}(60)(24) \\ &= \mathbf{720 \text{ m}} \end{aligned}$$

$$\begin{aligned} \text{Answer (a)} & \underline{\quad 0.6 \quad} \text{ m/s}^2 \quad [1] \\ \text{(b)} & \underline{\quad 720 \quad} \text{ m} \quad [1] \end{aligned}$$

13. Topic: Volumes of Similar Figures

(a) Let w be the width of the prism.

Let v_1 = volume of water when $d = 12$

and v_2 = volume of water when $d = 24$ cm

$$\frac{v_1}{v_2} = \frac{(\text{Base area when } d=12) \times w}{(\text{Base area when } d=24) \times w}$$

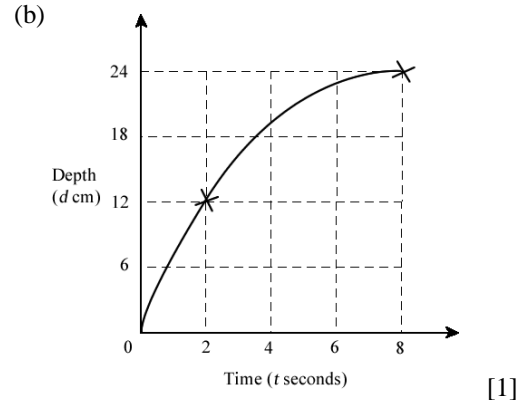
$$\frac{v_1}{v_2} = \left(\frac{12}{24}\right)^2 \times \frac{w}{w}$$

$$v_1 = \frac{1}{4} v_2$$

\therefore since v_2 takes 8 seconds, v_1 takes 2 seconds

$\therefore t = 2$ when $d = 12$

Answer (a) $t =$ 2 [1]



14. Topic: Mensuration (Surface area)

Surface area of hemisphere

$$= \frac{1}{2} \times \text{surface area of sphere (from formula sheet)}$$

$$= \frac{1}{2} (4\pi r^2) = 2\pi r^2$$

Curved surface area of a cone
 $= \pi r l$ (from formula sheet)

$$\begin{aligned} \text{Surface area of the toy} &= 2\pi r^2 + \pi r l \\ &= 2\pi(2.8)^2 + \pi(2.8)(7.2) \\ &\approx 112.59 \\ &\approx \mathbf{113 \text{ cm}^2} \text{ (3 sig. fig.)} \end{aligned}$$

Answer 113 cm² [1]

15. Topic: Areas & Volumes of Similar Figures

(a) (i) $\frac{v_S}{v_L} = \left(\frac{R_S}{R_L}\right)^3$

$$\frac{640}{1250} = \left(\frac{R_S}{R_L}\right)^3$$

$$\therefore \frac{R_S}{R_L} = \sqrt[3]{\frac{640}{1250}}$$

$$= \frac{4}{5}$$

Ratio of the smaller radius to the larger radius = **4 : 5**

(ii) $\frac{A_S}{A_L} = \left(\frac{R_S}{R_L}\right)^2$

$$= \left(\frac{4}{5}\right)^2$$

$$= \frac{16}{25}$$

\therefore Ratio of the surface area to the larger surface area = **16 : 25**

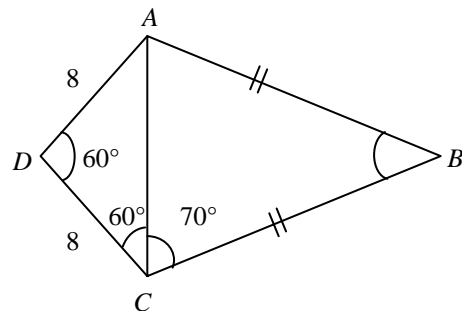
$$\begin{aligned} \text{(b)} \quad \frac{M_S}{M_L} &= \left(\frac{R_S}{R_L}\right)^3 \\ \frac{M_S}{25} &= \left(\frac{4}{5}\right)^3 \\ M_S &= \frac{64}{125} \times 25 \\ &= 12.8 \text{ kg} \end{aligned}$$

∴ Mass of the smaller sphere = **12.8 kg**

$$\begin{aligned} \text{Answer(a)(i)} \quad &\frac{4}{16} : \frac{5}{25} && [1] \\ \text{(ii)} \quad &\frac{4}{16} : \frac{5}{25} && [1] \\ \text{(b)} \quad &\underline{12.8} \text{ kg} && [1] \end{aligned}$$

16. Topic: Geometry

(a) (i)



$$\angle DAC = \angle ACD = 60^\circ$$

∴ ΔDAC is an equilateral $\Delta \Rightarrow$ Length of $AC = 8 \text{ cm}$

$$\begin{aligned} \text{(ii)} \quad \text{Since } \angle ACD = 60^\circ, \quad \angle ACB &= 130^\circ - 60^\circ \\ &= 70^\circ \\ \therefore \angle ABC &= 180^\circ - 2(70^\circ) \\ &= \mathbf{40^\circ} \text{ (sum of } \angle\text{s in } \Delta) \end{aligned}$$

$$\begin{aligned} \text{Answer(a)(i)} \quad AC &= \underline{8} \text{ cm} && [1] \\ \text{(ii)} \quad \angle ABC &= \underline{40^\circ} && [1] \end{aligned}$$

$$\begin{aligned} \text{(b) (i)} \quad \angle POT &= 2 \times \angle PQO \text{ (}\angle \text{ at centre} = 2 \times \angle\text{s at circumference)} \\ &= 2(32^\circ) \\ &= \mathbf{64^\circ} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \angle OPT &= 90^\circ && \text{(tan } \perp \text{ radius)} \\ \therefore \angle OTP &= 180^\circ - 90^\circ - 64^\circ \\ &= \mathbf{26^\circ} && \text{(sum of } \angle\text{s in } \Delta) \end{aligned}$$

$$\begin{aligned} \text{Answer(b)(i)} \quad \angle POT &= \underline{64^\circ} && [1] \\ \text{(ii)} \quad \angle OTP &= \underline{26^\circ} && [1] \end{aligned}$$

17. Topic: Algebra

$$\text{(a) (i)} \quad 2x^2 + kx - 15 = 0 \dots\dots\dots (1)$$

Sub $x = 3$,

$$2(3)^2 + 3k - 15 = 0$$

$$18 + 3k - 15 = 0$$

$$3k = -3$$

$$k = \mathbf{-1}$$

(ii) Sub $k = -1$ into (1),

$$2x^2 - x - 15 = 0$$

$$(2x + 5)(x - 3) = 0$$

$$2x + 5 = 0 \quad \text{or} \quad x - 3 = 0$$

$$x = -\frac{5}{2} \quad x = 3 \text{ (given)}$$

$$= -2.5$$

$$\begin{aligned} \text{(b)} \quad 6p^2 - 3pq - 10ap + 5a &= 3p(2p - q) - 5a(2p - q) \\ &= (2p - q)(3p - 5a) \end{aligned}$$

Answer (a)(i) $k = \frac{-1}{\quad}$ [1]

(ii) $x = \frac{-2.5}{\quad}$ [1]

(b) $\frac{(2p - q)(3p - 5a)}{\quad}$ [2]

18. Topic: Factors and Multiples

$$\begin{aligned} \text{(a)} \quad 150 &= 2 \times 75 \\ &= 2 \times 3 \times 25 \\ &= 2 \times 3 \times 5^2 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 150 &= 2 \times 3 \times 5^2 \\ 48 &= 2^4 \times 3 \\ \text{HCF} &= 2 \times 3 \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \text{LCM of 48 and 150} &= 2^4 \times 3 \times 5^2 \\ &= 1200 \end{aligned}$$

$$\begin{aligned} \text{Least number of chocolate bars he could have bought} &= \frac{1200}{150} \\ &= 8 \end{aligned}$$

Answer (a) $150 = \frac{2 \times 3 \times 5^2}{\quad}$ [1]

(b) $\frac{6}{\quad}$ [1]

(c) $\frac{8}{\quad}$ [2]

19. Topics: Approximation & Estimation, Trigonometry

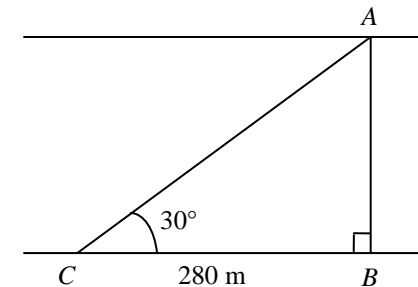
$$\begin{aligned} \text{(a)} \quad \text{(i)} \quad \frac{494.6}{56.33 \times 98.12} &= \frac{494.6}{5527.0996} \\ &= 0.08948635556 \end{aligned}$$

(ii) $0.08948635556 \approx 0.1$ (1 d.p.)

Answer (a)(i) $\frac{0.08948635556}{\quad}$ [1]

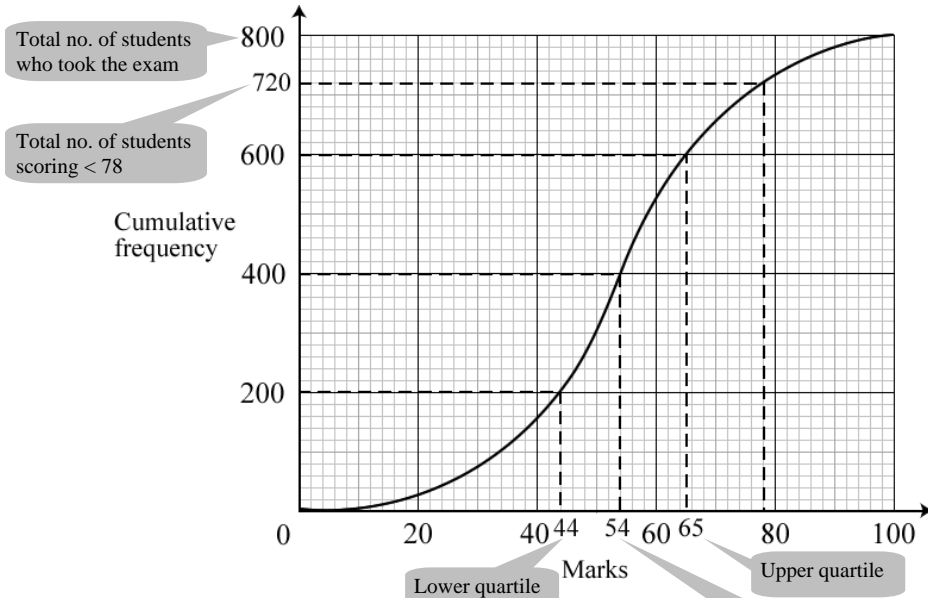
(ii) $\frac{0.1}{\quad}$ [1]

$$\begin{aligned} \text{(b)} \quad \tan 30^\circ &= \frac{AB}{280} \\ AB &= 280 \tan 30^\circ \\ &= 161.65 \\ &\approx 162 \text{ m (3 sig. fig.)} \end{aligned}$$



Answer (b) $\frac{162}{\quad}$ m [2]

20. Topic: Statistics (Cumulative Frequency)



- (a) Median mark = 50th percentile of 800 students
= **54**
- (b) Interquartile range = Upper quartile – Lower quartile
= 65 – 44
= **21**
- (c) Number of students who are awarded a grade A = 800 – 720
= **80**

<i>Answer</i>	(a)	54	[1]
	(b)	21	[2]
	(c)	80	[1]

21. Topic: Standard Form

(a) $1.32 \times 10^9 - 832 \times 10^6 = 10^6[1.32 \times 10^3 - 832]$
 $= 10^6[1320 - 832]$
 $= 488 \times 10^6$
 $= \mathbf{4.88 \times 10^8}$

1 million = 1×10^6
1 billion = 1×10^9

- (b) Average number of per square kilometer living in Africa

$$= \frac{832 \times 10^6}{26.6 \times 10^6}$$

= **31.3 people per sq. km**

(c) $\frac{\text{Number of people living in Singapore}}{\text{Number of people living in China}} = \frac{4.48 \times 10^6}{1.32 \times 10^9}$
 $= \frac{4.48}{1.32 \times 10^3}$
 $= \frac{4.48}{1320}$
 $= \frac{14}{4125}$

Ratio of no. of people living in Singapore : no. of people living in China
 $= 14 : 4125$
 $= \mathbf{1 : 294 \frac{9}{14}}$

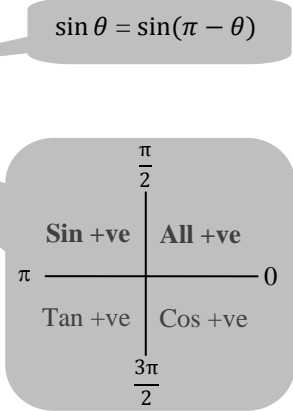
<i>Answer</i>	(a)	4.88×10^8	[2]
	(b)	31.3	[1]
	(c)	$1 : 294 \frac{9}{14}$	[1]

22. Topics: Mensuration and Trigonometry

θ must be in radians

(a) Length of the major arc $AB = r\theta$
 $= 10(2\pi - 2.3)$
 $= 39.83$
 \approx **39.8 cm (3 sig. fig.)**

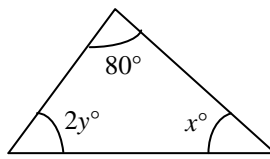
(b) $\sin x = \frac{1}{2}$
 $\therefore \sin \frac{\pi}{6} = \frac{1}{2}$ and $\sin \left(\pi - \frac{\pi}{6}\right) = \sin \frac{\pi}{6} = \frac{1}{2}$
 $\therefore x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$
 $= 0.5235$ or 2.617
 \approx **0.524 or 2.62 (3 sig. fig.)**



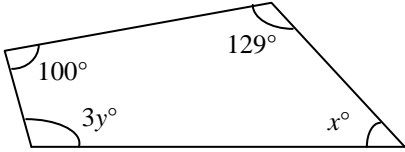
Answer (a) 39.8 cm [2]
 (b) $x =$ 0.524 or 2.62 [2]

23. Topics: Angle Properties of Polygons and Simultaneous Equations

(a)



$2y^\circ + 80^\circ + x^\circ = 180^\circ$
 $x = 100 - 2y \dots\dots\dots (1)$



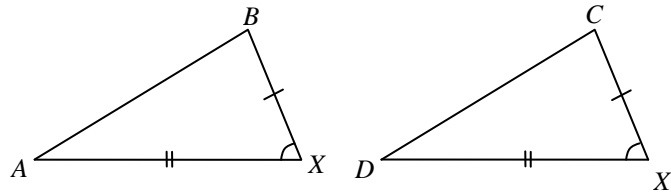
$100^\circ + 129^\circ + 3y^\circ + x^\circ = 360^\circ$ (int. \angle s of quadrilaterals)
 $3y + x = 131 \dots\dots\dots (2)$

(b) Sub (1) into (2):
 $3y + 100 - 2y = 131$
 $y = 31$
 Sub $y = 31$ into (1), $x = 100 - 2(31)$
 $= 38$
 $\therefore x = 38$ and $y = 31$

Answer (a) $x = 100 - 2y$
 $3y + x = 131$ [2]
 (b) $x =$ 38
 $y =$ 31 [2]

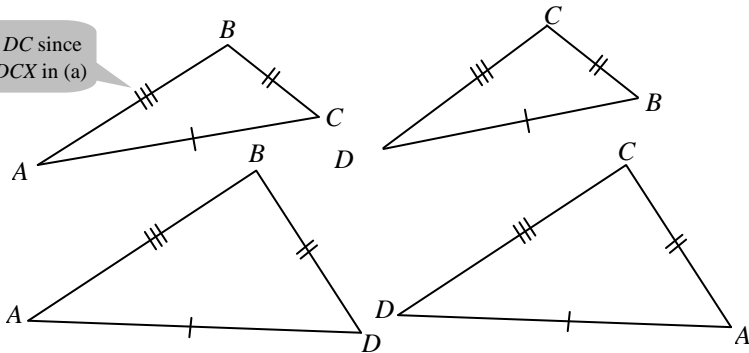
24. Topic: Congruency and Similarity

(a)

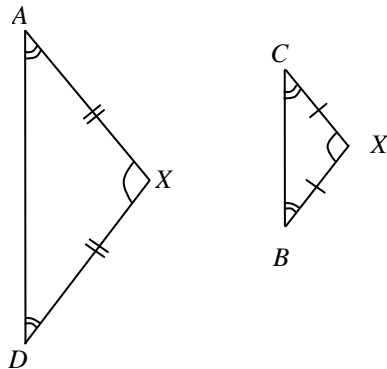


(b)

SSS: $AB = DC$ since $\triangle ABX \cong \triangle DCX$ in (a)



(c)



Answer (a) In triangles ABX and DCX ,
 $AX = DX$ (given)
 $BX = CX$ (given)
 $\angle AXB = \angle CXD$ (vertically opp. \angle s)
 \therefore By SAS property, $\triangle ABX$ and $\triangle DCX$ are congruent. [2]

(b) Triangles ABC and DCB [1]
 or Triangles ABD and DCA [1]

(c) Triangles ADX and CBX [1]

25. Topic: Coordinate Geometry

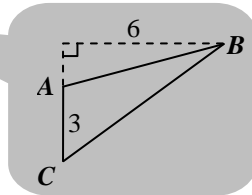
(a) Gradient of $AB = \frac{3-1}{6-0}$
 $= \frac{2}{6}$
 $= \frac{1}{3}$

Gradient of straight line passing through $A(x_1, y_1)$ and $B(x_2, y_2)$
 $= \frac{y_2 - y_1}{x_2 - x_1}$

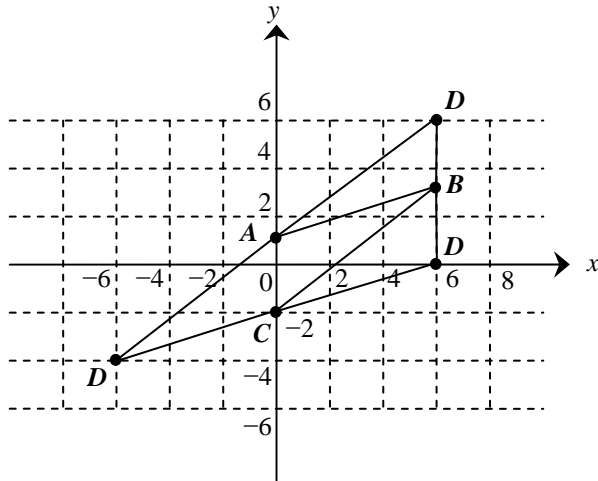
(b) Equation of AB : $y = \frac{1}{3}x + c$
 Sub $(0, 1)$, $\therefore y = \frac{1}{3}x + 1$

Equation of straight line with gradient m & y-intercept c :
 $y = mx + c$

(c) Area of $\triangle ABC = \frac{1}{2} \times 3 \times 6$
 $= 9 \text{ units}^2$



(d) Two possible points of D are $(-6, -4)$, $(6, 0)$ or $(6, 6)$ (Any two)

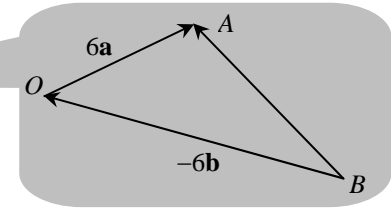


Expressing a vector in terms of its position vectors
 $\vec{LP} = \vec{OP} - \vec{OL}$

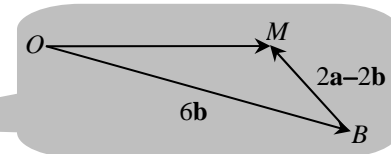
- Answer
- (a) $\frac{1}{3}$ cm [2]
- (b) $y = \frac{1}{3}x + 1$ [1]
- (c) 9 unit² [1]
- (d) $(-6, -4)$
 $(6, 0)$ [2]
 $(6, 6)$

26. Topic: Vectors in Two Dimensions

(a) $\vec{BA} = \vec{BO} + \vec{OA}$
 $= -6\mathbf{b} + 6\mathbf{a}$

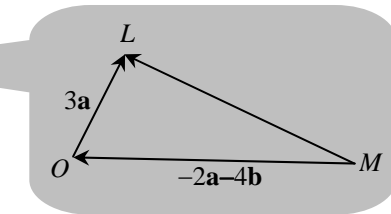


(i) $\vec{BM} = \frac{1}{3}\vec{BA}$
 $= \frac{1}{3}[-6\mathbf{b} + 6\mathbf{a}]$
 $= -2\mathbf{b} + 2\mathbf{a}$
 $= 2\mathbf{a} - 2\mathbf{b}$



(ii) $\vec{OM} = \vec{OB} + \vec{BM}$
 $= 6\mathbf{b} + 2\mathbf{a} - 2\mathbf{b}$
 $= 2\mathbf{a} + 4\mathbf{b}$

(iii) $\vec{ML} = \vec{MO} + \vec{OL}$
 $= -2\mathbf{a} - 4\mathbf{b} + 3\mathbf{a}$
 $= \mathbf{a} - 4\mathbf{b}$



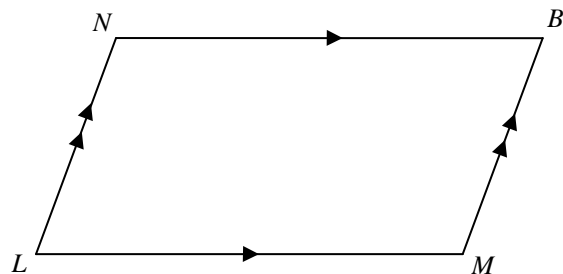
(b) $\vec{LP} = 3\vec{LM}$
 $\vec{OP} - \vec{OL} = 3(4\mathbf{b} - \mathbf{a})$
 $\vec{OP} = 12\mathbf{b} - 3\mathbf{a} + 3\mathbf{a}$
 $= 12\mathbf{b}$

(c) $\vec{OP} = 12\mathbf{b}$
 $= 2(6\mathbf{b})$
 $= 2\vec{OB}$

$\vec{AB} = k\vec{BC}$
 $\Rightarrow A, B$ are collinear
 (straight line)

$\therefore O, B$ and P are collinear and OP is twice of OB .

(d)



Since $LMBN$ is a parallelogram,

$$\begin{aligned} \overrightarrow{NB} &= \overrightarrow{LM} \\ \overrightarrow{OB} - \overrightarrow{ON} &= 4\mathbf{b} - \mathbf{a} \\ \overrightarrow{ON} &= \overrightarrow{OB} - 4\mathbf{b} + \mathbf{a} \\ &= 6\mathbf{b} - 4\mathbf{b} + \mathbf{a} \\ &= 2\mathbf{b} + \mathbf{a} \\ &= \mathbf{a} + 2\mathbf{b} \end{aligned}$$

\overrightarrow{NB} and \overrightarrow{LM} are equal vectors
 \Rightarrow same direction and magnitude

- Answer (a)(i) $\overrightarrow{BM} = \underline{\underline{2\mathbf{a} - 2\mathbf{b}}}$ [2]
 (ii) $\overrightarrow{OM} = \underline{\underline{2\mathbf{a} + 4\mathbf{b}}}$ [1]
 (iii) $\overrightarrow{ML} = \underline{\underline{\mathbf{a} - 4\mathbf{b}}}$ [1]
 (b) $\overrightarrow{OP} = \underline{\underline{12\mathbf{b}}}$ [1]
 (c) $\underline{\underline{O, B \text{ and } P \text{ are collinear,}}}$
 $\underline{\underline{OP \text{ is twice of } OB.}}$ [2]
 (d) $\overrightarrow{ON} = \underline{\underline{\mathbf{a} + 2\mathbf{b}}}$ [1]