

'01 Level 2009 AM 4038 Paper 2

$$1. (i) \sin(A-B) = \frac{3}{8}$$

$$\sin A \cos B - \cos A \sin B = \frac{3}{8}$$

$$\frac{5}{8} - \cos A \sin B = \frac{3}{8}$$

$$\cos A \sin B = \frac{5}{8} - \frac{3}{8}$$

$$= \frac{1}{4} \#$$

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$$(ii) \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{5}{8} + \frac{1}{4}$$

$$= \frac{7}{8} \#$$

$$(iii) \frac{\tan A}{\tan B} = \frac{\sin A}{\cos A} \div \frac{\sin B}{\cos B}$$

$$= \frac{\sin A}{\cos A} \times \frac{\cos B}{\sin B}$$

$$= \frac{\sin A \cos B}{\cos A \sin B}$$

$$= \frac{\frac{5}{8}}{\frac{1}{4}}$$

$$= 2\frac{1}{2} \#$$

$$2. (i) \frac{7}{2x^2 - x - 6} = \frac{7}{(2x+3)(x-2)}$$

$$\Rightarrow \frac{7}{(2x+3)(x-2)} = \frac{A}{2x+3} + \frac{B}{x-2}$$

$$7 = A(x-2) + B(2x+3)$$

when $x=2$,

$$7 = 0 + B(7)$$

$$B = 1.$$

when $x = -\frac{3}{2}$,

$$7 = A\left(-\frac{3}{2} - 2\right) + 0$$

$$A = -2$$

$$\therefore \frac{7}{2x^2 - x - 6} = -\frac{2}{2x+3} + \frac{1}{x-2}$$

$$\begin{aligned} (ii) \int_3^9 \frac{7}{2x^2 - x - 6} dx &= \int_3^9 \left(-\frac{2}{2x+3} + \frac{1}{x-2}\right) dx \\ &= \left[-\frac{\ln(2x+3)}{1} + \ln(x-2) \right]_3^9 \\ &= [-\ln(21) + \ln 7] - [-\ln 9 + \ln 1] \\ &= -1.0986 - (-2.1972) \\ &= 1.098 \\ &\approx 1.10 \text{ (3 s.f.)} \\ &\quad \# \end{aligned}$$

3. (i) $u = 2^x$,

$$8^x - 2^{x+2} = 15$$

$$(2^3)^x - 2^x \times 2^2 = 15$$

$$(2^x)^3 - 4(2^x) = 15$$

$$\therefore u^3 - 4u - 15 = 0 \quad \#$$

(ii) let $f(u) = u^3 - 4u - 15$

when $u = 3$,

$$f(3) = 3^3 - 4(3) - 15 = 0$$

\therefore By factor theorem, $(u-3)$ is a factor.

$$f(u) = (u-3)(u^2 + bu + 5)$$

Compare coeff. of u : $-4 = 5 - 3b$

$$3b = 9$$

$$b = 3$$

$$\therefore f(u) = (u-3)(u^2 + 3u + 5) = 0$$

$$\Rightarrow u-3 = 0 \quad \text{or} \quad u^2 + 3u + 5 = 0$$

$$u = 3$$

$$u = \frac{-3 \pm \sqrt{3^2 - 4(1)(5)}}{2}$$

$$= \frac{-3 \pm \sqrt{-11}}{2} \quad (\text{N.A.})$$

$\therefore u = 3$ is the only real solution of this equation.
(shown) $\#$

(ii)

$$u = 3$$

$$\Rightarrow 2^x = 3$$

$$\lg 2^x = \lg 3$$

$$x \lg 2 = \lg 3$$

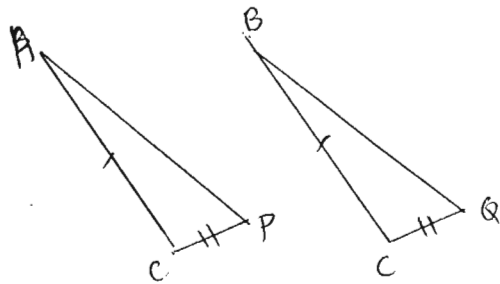
$$x = \frac{\lg 3}{\lg 2}$$

$$= 1.584$$

$$\approx 1.58 \text{ (3.s.f.)}$$

4. GIVEN $\triangle ABC$ IS ISOSCELES \triangle ,

$$(i) \therefore \hat{CAB} = \hat{CBA} = \theta$$



In $\triangle ACP$ and $\triangle BCQ$,
 $AC = BC$ (GIVEN)
 $CP = CQ$ (GIVEN)
 $\hat{ACP} = \hat{BCQ}$ (COMMON)

$\therefore \triangle ACP$ IS CONGRUENT TO $\triangle BCQ$.
 Hence $\hat{CAP} = \hat{CBQ} = \alpha$

$$\therefore \hat{XAB} = \hat{CAB} - \hat{CAP} = \theta - \alpha$$

$$\text{and } \hat{XBA} = \hat{CBA} - \hat{CBQ} = \theta - \alpha$$

$$\therefore \hat{XAB} = \hat{XBA}$$

Hence $\triangle XAB$ IS ISOSCELES \triangle . (SHOWN) \neq

(ii) Since $\triangle XAB$ IS ISOSCELES \triangle ,

$$XA = XB = a \quad \text{--- (1)}$$

Since $\triangle ACP$ IS CONGRUENT TO $\triangle BCQ$ (FROM PART (i))

$$AP = BQ$$

$$AX + XP = BX + XQ$$

$$a + XP = a + XQ \quad (\text{USE RESULT (1)})$$

$$XP = XQ$$

$$PX = QX \quad (\text{SHOWN}) \neq$$

$$\begin{aligned}
 5. (i) \quad \left(2 - \frac{x}{4}\right)^n &= \binom{n}{0} (2)^n + \binom{n}{1} (2)^{n-1} \left(-\frac{x}{4}\right)^1 + \binom{n}{2} (2)^{n-2} \left(-\frac{x}{4}\right)^2 + \dots \\
 &= 2^n + n(2)^{n-1} \left(-\frac{x}{4}\right) + \frac{n(n-1)}{2} (2)^{n-2} \frac{x^2}{2^4} + \dots \\
 &= 2^n - n2^{n-3}x + n(n-1)(2)^{n-7}x^2 + \dots \quad *
 \end{aligned}$$

$$(ii) \quad (1+x) \left(2 - \frac{x}{4}\right)^n = a + bx^2$$

$$\Rightarrow (1+x) \left[2^n - n2^{n-3}x + n(n-1)(2)^{n-7}x^2 + \dots \right] = a + bx^2$$

$$2^n - n2^{n-3}x + n(n-1)(2)^{n-7}x^2 + 2^n x - n2^{n-3}x^2 = a + bx^2$$

$$(ii) \text{ Compare coeff. of } x^0: \quad 2^n = a \quad \text{--- (1)}$$

$$\text{Compare coeff. of } x^1: \quad -n2^{n-3} + 2^n = 0$$

$$2^{n-3} [-n + 2^3] = 0$$

$$2^{n-3} = 0$$

(N.A)

$$\text{or } -n + 8 = 0$$

$$n = 8$$

*

$$(iii) \text{ Compare coeff. of } x^2: \quad n(n-1)2^{n-7} - n2^{n-3} = b \quad \text{--- (2)}$$

$$\text{Sub } n=8 \text{ into (1), } a = 2^8 = 256 \quad *$$

$$\text{Sub } n=8 \text{ into (2), } b = 8(7)2^1 - 8(2)^5 = -144 \quad *$$

$$6. (i) \quad y = 1 + 2\cos x$$

$$\text{when } y=0, \quad 1 + 2\cos x = 0$$

$$\cos x = -\frac{1}{2}$$

$$\text{Basic } \angle = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\therefore x = \pi - \frac{\pi}{3}, \quad \pi + \frac{\pi}{3}$$

$$= \frac{2\pi}{3}, \quad \frac{4\pi}{3}$$

\therefore x-coordinate of A is $\frac{2\pi}{3}$ (shown).

and x-coordinate of B is $\frac{4\pi}{3}$ ~~*~~

(ii) Area of the shaded region

$$= \int_0^{\frac{2\pi}{3}} (1 + 2\cos x) dx - \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (1 + 2\cos x) dx$$

$$= \left[x + 2\sin x \right]_0^{\frac{2\pi}{3}} - \left[x + 2\sin x \right]_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}$$

$$= \left[\frac{2\pi}{3} + 2\sin\frac{2\pi}{3} - 0 \right] - \left[\frac{4\pi}{3} + 2\sin\frac{4\pi}{3} - \frac{2\pi}{3} - 2\sin\frac{2\pi}{3} \right]$$

$$= \frac{2\pi}{3} + 2\left(\frac{\sqrt{3}}{2}\right) - \frac{4\pi}{3} - 2\left(-\frac{\sqrt{3}}{2}\right) + \frac{2\pi}{3} + 2\left(\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{3} + \sqrt{3} + \sqrt{3}$$

$$= 5.196$$

$$\approx 5.20 \text{ units}^2 \text{ (3 s.f.)} \quad \#$$

$$7. (i) \quad y = |3x - 5| - 2$$

$$\text{When } x = 0, \quad y = |0 - 5| - 2 \\ = 3$$

$$\text{When } y = 0, \quad |3x - 5| - 2 = 0$$

$$|3x - 5| = 2$$

$$3x - 5 = 2 \quad \text{or} \quad 3x - 5 = -2$$

$$3x = 7$$

$$3x = 3$$

$$x = \frac{7}{3}$$

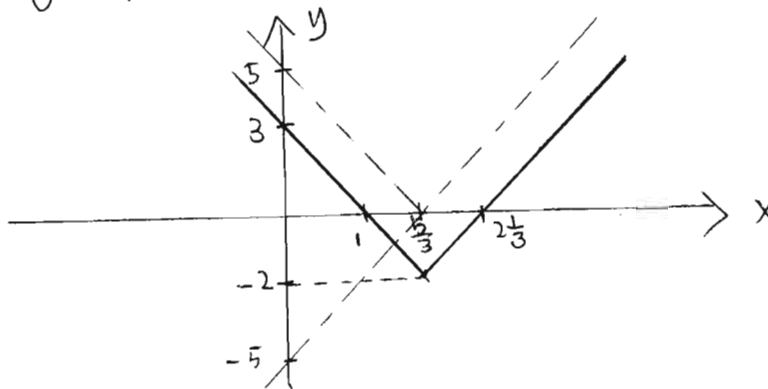
$$x = 1$$

$$= 2\frac{1}{3}$$

\therefore Coordinates of all the points meets the axes are

$(0, 3)$, $(2\frac{1}{3}, 0)$ and $(1, 0)$ #

$$(ii) \quad y = |3x - 5| - 2$$



$$(iii) \quad x = |3x - 5| - 2$$

$$x + 2 = |3x - 5|$$

$$x + 2 = 3x - 5 \quad \text{or} \quad -(x + 2) = 3x - 5$$

$$2x = 7$$

$$-x - 2 = 3x - 5$$

$$x = 3.5$$

$$4x = 3$$

#

$$x = 0.75$$

#

$$8. \text{ (i)} \quad s = 400(1 - e^{-t/10}) - 16t$$

$$\begin{aligned} v &= \frac{ds}{dt} \\ &= 400\left(\frac{1}{10}e^{-t/10}\right) - 16 \\ &= 40e^{-t/10} - 16 \quad \# \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad a &= \frac{dv}{dt} \\ &= 40\left(-\frac{1}{10}\right)e^{-t/10} \\ &= -4e^{-t/10} \quad \# \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \text{At A, } t=0, \\ \therefore v &= 40e^0 - 16 \\ &= 24 \text{ m/s} \quad \# \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad \text{let } v &= 0 \\ 40e^{-t/10} - 16 &= 0 \\ 40e^{-t/10} &= 16 \\ e^{-t/10} &= 0.4 \\ \ln e^{-t/10} &= \ln 0.4 \\ -\frac{t}{10} &= \ln 0.4 \\ t &= 9.1629 \\ &\approx 9.163 \text{ seconds (shown)}. \end{aligned}$$

$$(v). \text{ Total distance} = 400 \left(1 - e^{-\frac{9.163}{10}} \right) - 16(9.163)$$

$$= 93.393 \text{ m}$$

Average speed of the motorcycle for the journey from A to B

$$= \frac{\text{Total distance}}{\text{total time taken}}$$

$$= \frac{93.393}{9.163}$$

$$= 10.192$$

$$\approx 10.2 \text{ m/s (3 s.f.)} \#$$

9. Equation of the circle:

$$(x-2)^2 + (y+1)^2 = 5^2$$

$$x^2 - 4x + 4 + y^2 + 2y + 1 - 25 = 0$$

$$x^2 - 4x + y^2 + 2y - 20 = 0$$

$$x^2 + y^2 - 4x + 2y - 20 = 0 \quad \text{--- (1)}$$

$$\Rightarrow -4 = 2g$$

$$g = -2 \#$$

$$\Rightarrow 2 = 2f$$

$$f = 1 \#$$

$$\Rightarrow c = -20 \#$$

(ii) Since AC is parallel to x-axis \Rightarrow y-coordinate of A = -1.

Sub $y = -1$ into (1),

$$x^2 + (-1)^2 - 4x + 2(-1) - 20 = 0$$

$$x^2 - 4x - 21 = 0$$

$$(x - 7)(x + 3) = 0$$

$$x - 7 = 0 \quad \text{or} \quad x + 3 = 0$$

$$x = 7 \text{ (rejected)} \quad x = -3$$

\therefore Coordinates of A are $(-3, -1)$ #

(iii) Gradient of AB = Gradient of OA

$$= \frac{-1 - 0}{-3 - 0} = \frac{1}{3}$$

Equation of AB: $y + 1 = \frac{1}{3}(x + 3)$

$$y = \frac{1}{3}x \quad \# \quad \text{--- (2)}$$

(iii) Sub (2) into (1),

$$x^2 + \left(\frac{1}{3}x\right)^2 - 4x + 2\left(\frac{1}{3}x\right) - 20 = 0$$

$$x^2 + \frac{1}{9}x^2 - 4x + \frac{2}{3}x - 20 = 0$$

$$9x^2 + x^2 - 36x + 6x - 180 = 0$$

$$10x^2 - 30x - 180 = 0$$

$$x^2 - 3x - 18 = 0$$

$$(x - 6)(x + 3) = 0$$

$$x - 6 = 0 \quad \text{or} \quad x + 3 = 0$$

$$x = 6$$

$$x = -3 \text{ (rejected)}$$

Sub $x = 6$ into (2),

$$y = \frac{1}{3}(6) = 2$$

\therefore Coordinates of B are $(6, 2)$ #

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10.

$$\frac{dy}{dx} = 6x - 6$$

$$\begin{aligned} \frac{dy}{dx} &= \int (6x - 6) dx \\ &= 3x^2 - 6x + C \end{aligned}$$

when $x=3$, $\frac{dy}{dx} = 12$

$$\Rightarrow 12 = 3(9) - 6(3) + C$$

$$C = 3$$

$$\therefore \frac{dy}{dx} = 3x^2 - 6x + 3$$

$$\begin{aligned} y &= \int (3x^2 - 6x + 3) dx \\ &= x^3 - 3x^2 + 3x + C \end{aligned}$$

Sub (3, 10),

$$\Rightarrow 10 = 27 - 27 + 9 + C$$

$$C = 1$$

$$\therefore y = x^3 - 3x^2 + 3x + 1 \quad \text{--- (1)}$$

For stationary point of the curve: $\frac{dy}{dx} = 0$

$$3x^2 - 6x + 3 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x = 1$$

$$\begin{aligned} \text{Sub } x=1 \text{ into (1), } y &= 1 - 3 + 3 + 1 \\ &= 2 \end{aligned}$$

\therefore coordinates of the stationary point are (1, 2).

For (1,2), using 1st Derivative Test:

x	0.9	1	1.1
$\frac{dy}{dx}$	+	0	+
	/	-	/

$$\text{when } x=0.9, \frac{dy}{dx} = 3(0.9)^2 - 6(0.9) + 3 \\ = 0.03 > 0$$

$$\text{when } x=1.1, \frac{dy}{dx} = 3(1.1)^2 - 6(1.1) + 3 \\ = 0.03 > 0$$

$\therefore (1,2)$ is a point of inflexion. #

11. (i) $\hat{BAO} = \theta$

$$\cos \theta = \frac{AB}{17}$$

$$AB = 17 \cos \theta$$

$$\sin \theta = \frac{OB}{17}$$

$$OB = 17 \sin \theta$$

$$\cos \theta = \frac{OC}{31}$$

$$OC = 31 \cos \theta$$

$$\therefore BC = OC - OB \\ = 31 \cos \theta - 17 \sin \theta.$$

$$\sin \theta = \frac{CD}{31}$$

$$CD = 31 \sin \theta$$

$$\therefore AB + BC + CD = 17 \cos \theta + 31 \cos \theta - 17 \sin \theta + 31 \sin \theta \\ = (48 \cos \theta + 14 \sin \theta) \text{ cm (shown)} \#$$

(ii) Given $AB + BC + CD = 49$

$$\Rightarrow 48 \cos \theta + 14 \sin \theta = 49 \quad \text{--- (1)}$$

Using R-formula,

$$\begin{aligned} 48 \cos \theta + 14 \sin \theta &= R \cos(\theta - \alpha) \\ &= R [\cos \theta \cos \alpha + \sin \theta \sin \alpha] \\ &= R \cos \alpha \cos \theta + R \sin \alpha \sin \theta \end{aligned}$$

Comparing coefficient,

$$48 = R \cos \alpha \quad \text{--- (1)}$$

$$14 = R \sin \alpha \quad \text{--- (2)}$$

$$\frac{(2)}{(1)} : \quad \frac{R \sin \alpha}{R \cos \alpha} = \frac{14}{48}$$

$$\tan \alpha = 0.29166$$

$$\alpha = 16.26^\circ$$

$$(1)^2 + (2)^2 : \quad R^2 \cos^2 \alpha + R^2 \sin^2 \alpha = 48^2 + 14^2$$

$$R^2 (\cos^2 \alpha + \sin^2 \alpha) = 2500$$

$$R^2 = 2500$$

$$R = 50 \quad \text{or} \quad -50 \quad (\text{rejected})$$

$$\therefore 48 \cos \theta + 14 \sin \theta = 50 \cos(\theta - 16.26^\circ) \quad \text{--- (2)}$$

Sub (2) into (1),

$$50 \cos(\theta - 16.26^\circ) = 49$$

$$\cos(\theta - 16.26^\circ) = 0.98$$

$$\text{Basic } \Delta = 11.478^\circ$$

$$\theta - 16.26^\circ = 11.478^\circ, 360^\circ - 11.478^\circ$$

$$\theta = 27.73^\circ, 364.78^\circ$$

$$= 4.78^\circ, 27.73^\circ$$

$$\approx 4.8^\circ, 27.7^\circ \quad (\text{1 d.p.})$$

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(ii) Maximum value of $AB + BC + CD = 50$ *

when $\cos(\theta - 16.26^\circ) = 1$

$$\Rightarrow \theta - 16.26^\circ = 0^\circ$$

$$\theta = 16.26^\circ$$

$$\approx 16.3^\circ \text{ (1 d.p.)} *$$

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