

# '0' Level 2009 AM 4038 Paper 1

1. Let  $f(x) = 2x^3 + ax^2 + bx + 3$

$$f(1) = 0$$

$$\Rightarrow 2(1)^3 + a(1)^2 + b(1) + 3 = 0$$

$$2 + a + b + 3 = 0$$

$$a = -5 - b \quad \text{--- (1)}$$

$$f(-2) = 15$$

$$\Rightarrow 2(-2)^3 + a(-2)^2 + b(-2) + 3 = 15$$

$$-16 + 4a - 2b + 3 = 15$$

$$4a - 2b = 28$$

$$2a - b = 14 \quad \text{--- (2)}$$

Sub (1) into (2),

$$2(-5 - b) - b = 14$$

$$-10 - 2b - b = 14$$

$$-3b = 24$$

$$b = -8$$

Sub  $b = -8$  into (1),

$$a = -5 + 8$$

$$= 3$$

$\therefore a = 3$  and  $b = -8$  #

$$2. \quad y = \frac{\ln x}{x}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{x \left(\frac{1}{x}\right) - \ln x}{x^2} \\ &= \frac{1 - \ln x}{x^2} \end{aligned}$$

Since  $y$  is an increasing function of  $x$ ,  $\frac{dy}{dx} > 0$ .

$$\Rightarrow 1 - \ln x > 0 \quad \text{as } x^2 > 0.$$

$$-\ln x > -1$$

$$\ln x < 1$$

$$x < e^1$$

$\therefore$  the set of values of  $x = \{x: 0 < x < e\}$  #

$$3. \quad x\sqrt{24} = x\sqrt{3} + \sqrt{6}$$

$$x\sqrt{24} - x\sqrt{3} = \sqrt{6}$$

$$x(\sqrt{24} - \sqrt{3}) = \sqrt{6}$$

$$x = \frac{\sqrt{6}}{\sqrt{24} - \sqrt{3}} \times \frac{\sqrt{24} + \sqrt{3}}{\sqrt{24} + \sqrt{3}}$$

$$= \frac{\sqrt{6}\sqrt{24} + \sqrt{6}\sqrt{3}}{24 - 3}$$

$$= \frac{\sqrt{144} + \sqrt{18}}{21}$$

$$= \frac{12 + \sqrt{9 \times 2}}{21}$$

$$= \frac{12 + 3\sqrt{2}}{21} = \frac{4 + \sqrt{2}}{7}$$

$\therefore a=4, b=2$  #

$$4. (i) \lg(x+14) - \lg(x-2) = 2\lg 5$$

$$\lg\left(\frac{x+14}{x-2}\right) = \lg 5^2$$

$$\frac{x+14}{x-2} = 25$$

$$x+14 = 25(x-2)$$

$$x+14 = 25x - 50$$

$$24x = 64$$

$$x = 2\frac{2}{3} \#$$

$$(ii) \log_2 y + \log_4 y = 6$$

$$\log_2 y + \frac{\log_2 y}{\log_2 4} = 6$$

$$\log_2 y + \frac{\log_2 y}{2} = 6$$

$$2\log_2 y + \log_2 y = 12$$

$$3\log_2 y = 12$$

$$\log_2 y = 4$$

$$y = 2^4$$

$$= 16 \#$$

5.  $y = 1 - 3 \tan x$

when  $x = 0$ ,  $y = 1 - 3 \tan 0$   
 $= 1$

$$\frac{dy}{dx} = -3 \sec^2 x$$

when  $x = 0$ ,  $\frac{dy}{dx} = -3 \sec^2 0$   
 $= -3$

$\therefore$  Gradient of tangent  $= -3$

Gradient of normal  $= \frac{1}{3}$

Equation of normal:  $y - 1 = \frac{1}{3}(x - 0)$

$$y = \frac{1}{3}x + 1 \quad \text{--- (1)}$$

Sub  $(k, 3)$  into (1),

$$3 = \frac{1}{3}k + 1$$

$$\frac{1}{3}k = 2$$

$$k = 6 \quad \#$$

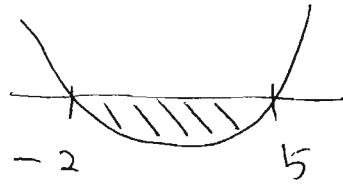
6.  $y = 2x^2 - 6x + c$

(i) when  $c = -20$ ,

$$2x^2 - 6x - 20 \leq 0$$

$$x^2 - 3x - 10 \leq 0$$

$$(x - 5)(x + 2) \leq 0$$



The sets of values of  $x = \{x: -2 \leq x \leq 5\}$  #

(ii)  $y = 2x^2 - 6x + c$  — (1)

$y + 2x = 8$  — (2)

Sub (1) into (2),

$$2x^2 - 6x + c + 2x = 8$$

$$2x^2 - 4x + c - 8 = 0$$

$a = 2$  ,  $b = -4$  ,  $c = c - 8$

Since the line is a tangent to the curve,

$$b^2 - 4ac = 0$$

$$(-4)^2 - 4(2)(c - 8) = 0$$

$$16 - 8c + 64 = 0$$

$$8c = 80$$

$$c = 10$$
 #

$$7. \quad x^2 + 2y^2 + 5x = 68 \quad \text{--- (1)}$$

$$2y + 3x = 9$$

$$2y = 9 - 3x$$

$$y = \frac{9 - 3x}{2} \quad \text{--- (2)}$$

Sub (2) into (1),

$$x^2 + 2\left(\frac{9 - 3x}{2}\right)^2 + 5x = 68$$

$$x^2 + \frac{2(9 - 3x)^2}{2^2} + 5x = 68$$

$$x^2 + \frac{81 - 54x + 9x^2}{2} + 5x = 68.$$

$$2x^2 + 81 - 54x + 9x^2 + 10x = 136$$

$$11x^2 - 44x - 55 = 0$$

$$x^2 - 4x - 5 = 0$$

$$(x + ( ))(x - 5) = 0$$

$$x = -1 \quad \text{or} \quad x = 5$$

$$\text{Sub } x = -1 \text{ into (2), } y = \frac{9 - 3(-1)}{2} = 6$$

$$\text{Sub } x = 5 \text{ into (2), } y = \frac{9 - 3(5)}{2} = -3$$

The coordinates of intersection points  $\Rightarrow (-1, 6)$  and  $(5, -3)$   
 $\therefore$  mid-point =  $\left(\frac{-1+5}{2}, \frac{6-3}{2}\right) = (2, 1.5) \#$

$$8. (i) \cos 3x - \cos x = -4 \sin^2 x \cos x$$

$$\text{LHS} = -2 \sin\left(\frac{3x+x}{2}\right) \sin\left(\frac{3x-x}{2}\right)$$

$$= -2 \sin 2x \sin x$$

$$= -2 (2 \sin x \cos x) \sin x$$

$$= -4 \sin^2 x \cos x$$

$\checkmark$  RHS (shown)  $\#$

**Joss Sticks**

www.exampaper.com.sg

(ii)

$$\cos 3x + 2 \cos x = 0$$

$$\cos 3x - \cos x = -3 \cos x$$

$$-4 \sin^2 x \cos x = -3 \cos x$$

$$4 \sin^2 x \cos x = 3 \cos x$$

$$4 \sin^2 x \cos x - 3 \cos x = 0$$

$$\cos x [4 \sin^2 x - 3] = 0$$

$$\cos x = 0 \quad \text{or} \quad 4 \sin^2 x - 3 = 0$$

$$\text{Basic } \theta = \frac{\pi}{2}$$

$$x = \frac{\pi}{2}$$

$$= 1.5707$$

$$\sin^2 x = \frac{3}{4}$$

$$\sin x = \pm \sqrt{\frac{3}{4}}$$

$$\text{Basic } \theta = \frac{\pi}{3}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$= 1.047, 2.094$$

$$\therefore x \approx 1.05, 1.57, 2.09 \quad (3 \text{ s.f.}) \quad \#$$

9.  $f(x) = 3 \sin\left(\frac{x}{3}\right) - 1$

(i)  $-1 \leq \sin\left(\frac{x}{3}\right) \leq 1$

$-3 \leq 3 \sin\left(\frac{x}{3}\right) \leq 3$

$-3 - 1 \leq 3 \sin\left(\frac{x}{3}\right) - 1 \leq 3 - 1$

$-4 \leq 3 \sin\left(\frac{x}{3}\right) - 1 \leq 2$

Maximum value of  $f(x) = 2$

Minimum value of  $f(x) = -4$

(ii) amplitude of  $f = 3$  #

(iii) period of  $f = \frac{360^\circ}{\frac{1}{3}}$

$= 1080^\circ$  #

(iv)  $3 \sin\left(\frac{x}{3}\right) - 1 = 0$

$\sin\left(\frac{x}{3}\right) = \frac{1}{3}$

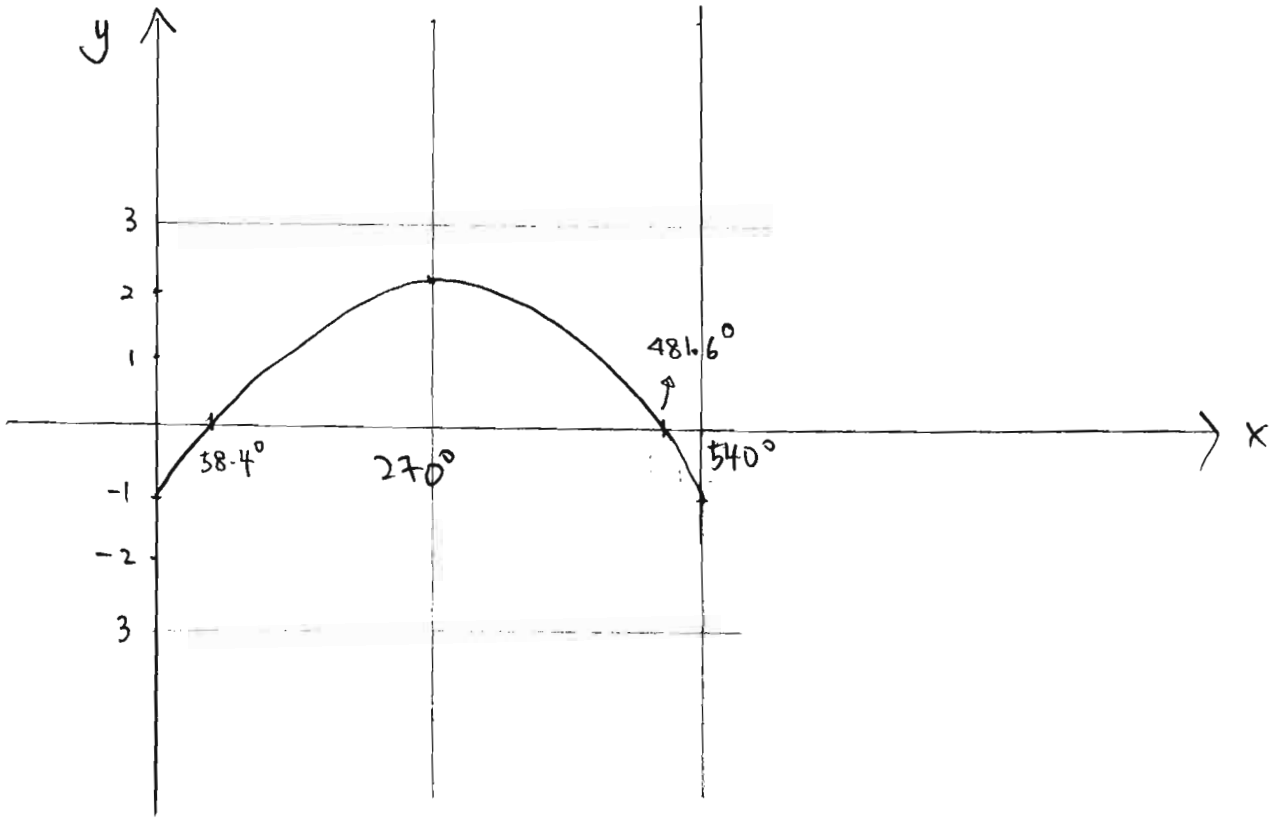
Basic  $\angle = 19.47^\circ$

$\frac{x}{3} = 19.47^\circ$

$x = 58.41$

$\approx 58.4^\circ$  (1 d.p.) #

9. (v)  $y = 3\sin\left(\frac{x}{3}\right) - 1$



10.

t	2	4	6	8	10
ln m	3.875	3.725	3.575	3.424	3.277

(ii)  $m = m_0 e^{-kt}$

$$\begin{aligned} \ln m &= \ln m_0 e^{-kt} \\ &= \ln m_0 + \ln e^{-kt} \\ &= \ln m_0 - kt \ln e \\ \ln m &= \ln m_0 - kt \end{aligned}$$

⇒ gradient = -k

$$\frac{3.277 - 3.875}{10 - 2} = -k$$

$$k = 0.07475$$

$$\approx 0.0748 \quad \#$$

⇒  $\ln m_0 = 4.04$  (intercept on ln m-axis)

$$m_0 = e^{4.04}$$

$$= 56.82$$

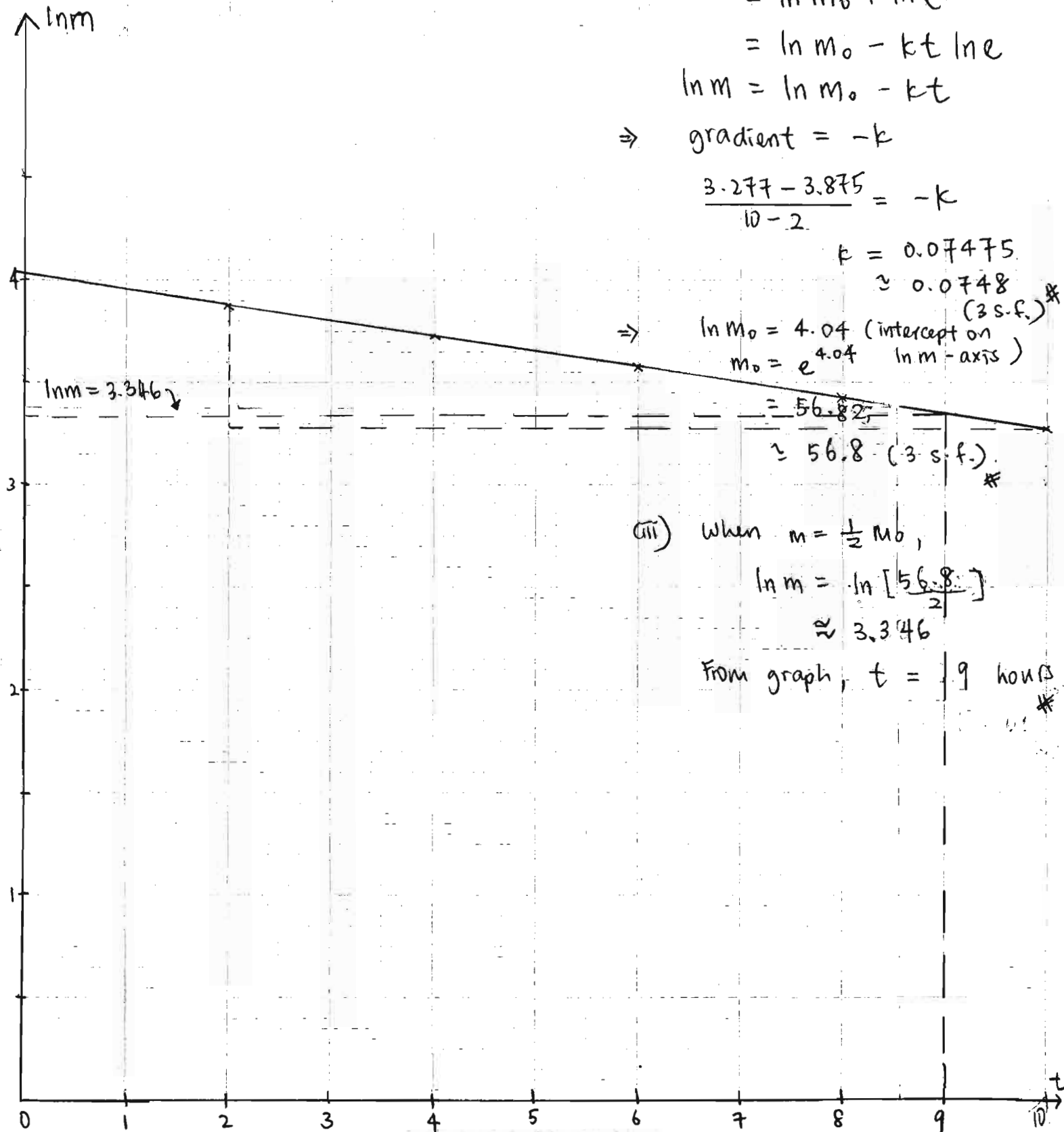
$$\approx 56.8 \quad \#$$

(iii) When  $m = \frac{1}{2} m_0$ ,

$$\ln m = \ln \left[ \frac{56.8}{2} \right]$$

$$\approx 3.346$$

From graph, t = 9 hours #



$$11. (i) \text{ Gradient of AD} = \frac{6+2}{0-2} = -4$$

$$\text{Gradient of AB} = \frac{1}{4}$$

$$\text{Equation of AB: } y - 6 = \frac{1}{4}(x - 0)$$

$$y = \frac{1}{4}x + 6 \quad \text{--- (1)}$$

$$(ii) \quad y = x \quad \text{--- (2)}$$

Sub (1) into (2),

$$x = \frac{1}{4}x + 6$$

$$4x = x + 24$$

$$3x = 24$$

$$x = 8$$

$$\therefore y = 8$$

Coordinates of B are (8, 8) ~~✗~~

(iii) let C be (x, y)

length of DC = 2 × length of AB

$$\sqrt{(2-x)^2 + (-2-y)^2} = 2\sqrt{(0-8)^2 + (6-8)^2}$$

$$(2-x)^2 + (-2-y)^2 = 4(68)$$

$$(2-x)^2 + (-2-y)^2 = 272 \quad \text{--- (3)}$$

$$\text{Equation of CD: } y + 2 = -\frac{1}{4}(x - 2)$$

$$y = -\frac{1}{4}x - \frac{5}{2} \quad \text{--- (4)}$$

Sub (4) into (3),

$$4 - 4x + x^2 + 4 + 4\left(\frac{1}{4}x - \frac{5}{2}\right) + \left(\frac{1}{4}x - \frac{5}{2}\right)^2 = 272$$

$$4 - 4x + x^2 + 4 + x - 10 + \frac{1}{16}x^2 - \frac{5}{4}x + \frac{25}{4} - 272 = 0$$

$$\frac{17}{16}x^2 - \frac{17}{4}x - \frac{1071}{4} = 0$$

$$17x^2 - 68x - 4284 = 0$$

$$x^2 - 4x - 252 = 0$$

$$(x - 18)(x + 14) = 0$$

$$x - 18 = 0$$

$$x = 18$$

$$\text{or } x + 14 = 0$$

$$x = -14 \text{ (rej.)}$$

$$\text{Sub } x = 18 \text{ into (4), } y = \frac{1}{4}(18) - \frac{5}{2}$$

$$= 2$$

$\therefore$  Coordinates of C are  $(18, 2)$  #

(iv) Area of trapezium ABCD

$$= \frac{1}{2} \begin{vmatrix} 0 & 2 & 18 & 8 & 0 \\ 6 & -2 & 2 & 8 & 6 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 0 + 4 + 144 + 48 - 12 + 36 - 16 \end{vmatrix}$$

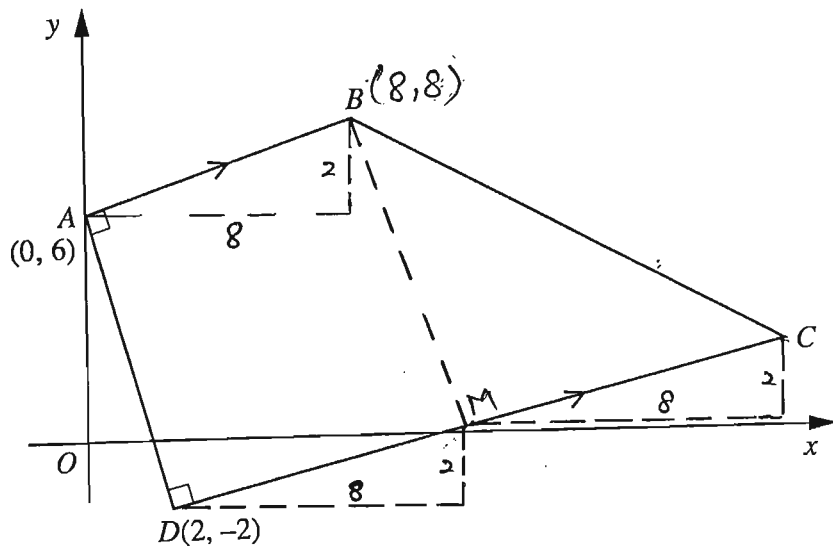
$$= \frac{1}{2} \begin{vmatrix} 204 \end{vmatrix}$$

$$= 102 \text{ units}^2 \text{ #}$$

**Joss Sticks**

www.exampaper.com.sg

1(iii) Alternate Method



Given DC is twice the length of AB.

Let M be the mid-point of DC.

Then  $DM = AB$  and  $DM \parallel AB$ .

Let M be  $(a, b)$  and  $C(x, y)$ .

$$\text{Then } b = -2 + 2 = 0$$

$$a = 2 + 8 = 10$$

$$\therefore M(10, 0).$$

Since M is the mid-point of DC.

$$\frac{2+x}{2} = 10 \quad \text{and} \quad \frac{y+(-2)}{2} = 0$$

$$y = 2$$

$\therefore$  Coordinates of C are  $(18, 2)$  #

12.

$$(i) \quad y = (2x-1)\sqrt{4x+1}$$

$$\frac{dy}{dx} = (2x-1) \frac{1}{2} (4x+1)^{-\frac{1}{2}} (4) + \sqrt{4x+1} (2)$$

$$= \frac{2(2x-1)}{\sqrt{4x+1}} + \frac{2\sqrt{4x+1}}{1}$$

$$= \frac{4x-2 + 2(4x+1)}{\sqrt{4x+1}}$$

$$= \frac{4x-2+8x+2}{\sqrt{4x+1}}$$

$$= \frac{12x}{\sqrt{4x+1}} \quad \#$$

**Joss Sticks**  
www.exampaper.com.sg

$$(ii) \quad \frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$2 = \frac{12(2)}{\sqrt{8+1}} \times \frac{dx}{dt}$$

$$2 = \frac{24}{3} \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{4} \text{ units per second} \quad \#$$

$$(iii) \quad \int_0^2 \frac{3x}{\sqrt{4x+1}} dx = \frac{1}{4} \int_0^2 \frac{12x}{\sqrt{4x+1}} dx$$

$$= \frac{1}{4} \left[ (2x-1)\sqrt{4x+1} \right]_0^2$$

$$= \frac{1}{4} \left[ 3\sqrt{9} - (-1)(1) \right]$$

$$= \frac{1}{4} [9+1]$$

$$= 2.5 \quad \#$$