

ELEMENTARY MATHEMATICS
Paper 1 Suggested Solutions

4016/01
October/November 2008

1. **Topic: Standard Form**

(a) $\frac{4.32}{32.8 \times 0.593} = 0.2221$

$\approx 2.22 \times 10^{-1}$ (3 sig. fig.)

(b) $\frac{6.82 \times 10^5}{1.55 \times 10^{-2}} = \left(\frac{6.82}{1.55}\right) \times 10^{5-(-2)}$

$= 4.4 \times 10^7$

Answer (a) $\frac{2.22 \times 10^{-1}}{4.4 \times 10^7}$ [1]

(b) $\frac{4.4 \times 10^7}{4.4 \times 10^7}$ [1]

2. **Topic: Statistics (Bar Chart & Pie Chart)**

(a) Total no. of children = $6 + 30 + 12$
 $= 48$

$\frac{\text{Total no. of children who walked or cycled}}{\text{Total no. of children}} = \frac{30+12}{48}$
 $= \frac{7}{8}$

(b) $\frac{6}{48} \times 360^\circ = 45^\circ$

∠ of sector in pie chart \propto its given data

Answer (a) $\frac{7}{8}$ [1]

(b) 45° [1]

3. **Topic: Indices**

(a) $\left(\frac{2}{x}\right)^{-3} = \left(\frac{x}{2}\right)^3$
 $= \frac{x^3}{8}$

$\left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{a}\right)^c$

(b) $2^{34} \div 2^4 = 2^k$
 $2^{34-4} = 2^k$

$k = 30$

$a^m \div a^n = a^{m-n}$

Answer (a) $\frac{x^3}{8}$ [1]

(b) $k = 30$ [1]

4. **Topic: Arithmetic**

(a) 3 hours 41 min = $3 \frac{41}{60}$ hr

(b) Distance travelled = $3 \frac{41}{60} \times 50$

$= 184.16$

≈ 184 km (nearest km)

Answer (a) $3 \frac{41}{60}$ hours [1]

(b) 184 km [1]

5. **Topic: Arithmetic**

(a) $\frac{2}{5} \rightarrow \4.20

$$\frac{3}{5} \rightarrow \frac{\$4.20}{2} \times 3$$

$$= \$6.30$$

(b) $\$0.99 \rightarrow x$ grams

$$\$1 \rightarrow \frac{100x}{99} \text{ grams}$$

$$\$y \rightarrow \frac{100}{99}xy \text{ grams}$$

Answer (a) \$ 6.30 [1]

(b) $\frac{100}{99}xy$ grams [1]

6. **Topic: Mensuration**

$$\text{Arc } AB = \frac{1}{4}(2)(\pi)(6)$$

$$= 3\pi$$

Circumference of circle = $2\pi r$

$$\text{Arc } OC = \frac{1}{2}(2)(\pi)(1.5)$$

$$= 1.5\pi$$

$$\text{Perimeter of the shaded region} = 3\pi + 1.5\pi + 6 + 3$$

$$= 4.5\pi + 9$$

Answer $4.5\pi + 9$ [2]

7. **Topic: Standard Form**

(a) $1.03 \times 10^{10} = 10.3 \times 10^9$

$$\therefore k = 10.3$$

1 billion = 1.0×10^9

(b) Estimated increase = $1.03 \times 10^{10} - 6.5 \times 10^9$

$$= 10^9[1.03 \times 10 - 6.5]$$

$$= 3.8 \times 10^9$$

Answer (a) $k = \frac{10.3}{1}$ [1]

(b) 3.8×10^9 [1]

8. **Topic: Coordinate Geometry**

(a) At y-axis, $x = 0$,

$$3(0) + 2y = 8$$

$$2y = 8$$

$$y = 4$$

$$\therefore A(0, 4)$$

Coordinates of y-intercept

(b) $3x + 2y = 8$

$$2y = -3x + 8$$

$$y = -\frac{3}{2}x + 4$$

Equation of straight line with gradient m and y-intercept c :

$$y = mx + c$$

Answer (a) (0 , 4) [1]

(b) $-\frac{3}{2}$ [1]

9. **Topics: Pythagoras Theorem, Trigonometry**

(a) If $\angle ABC$ is a right angle, by Pythagoras' Theorem,

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 17^2 = 289 \text{ cm}^2$$

$$AB^2 + BC^2 = 15^2 + 8^2 = 225 + 64 = 289 \text{ cm}^2 = AC^2$$

$\therefore \angle ABC$ is a right angle

(b) (i) Area of $\triangle DAC = \frac{1}{2}(5)(8)$
 $= 20 \text{ cm}^2$

Area of $\triangle = \frac{1}{2} \times \text{base} \times \text{height}$

(ii) $\cos \angle DAC = \cos (180^\circ - \angle BAC)$

$$= -\cos \angle BAC$$

$-\cos \theta = \cos (180^\circ - \theta)$

$$= -\frac{15}{17}$$

Answer (a) $AC^2 = AB^2 + BC^2$ (Pythagoras' theorem)

$\therefore \angle ABC$ is a right angle [1]

Answer(b)(i) $\frac{20}{\text{cm}^2}$ [1]

(ii) $\cos \angle DAC = -\frac{15}{17}$ [1]

10. **Topic: Matrices**

(a) $\begin{pmatrix} 14 & 5 & 1 \\ 15 & 0 & 5 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 27 \\ 25 \end{pmatrix}$

$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax + by + cz \\ dx + ey + fz \end{pmatrix}$

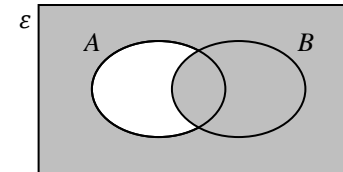
(b) Sandy's score is 27 marks and Roger's score is 25 marks.

Answer (a) $\begin{pmatrix} 27 \\ 25 \end{pmatrix}$ [2]

Answer (b) Sandy's score is 27 marks
 and Roger's score is 25 marks [1]

11. **Topic: Set Language and Notation**

(a)



(b) $\epsilon = \{2, 3, 4, 5, 6, 7 \dots\}$

$P = \{2, 3, 5, 7 \dots\}$

$S = \{4, 9, 16 \dots\}$

$T = \{2, 12, 22 \dots\}$

(i) $P \cap T = \{2\}$

$n(P \cap T) = 1$

(ii) $S \cap T = \{ \}$

$\therefore n(S \cap T) = 0$

$n(A)$ denotes the number of elements in set A

Answer (b)(i) $\frac{1}{\text{[1]}}$

(b) $\frac{0}{\text{[1]}}$

12. Topics: Algebra (Algebraic Representation, Formulae)

(a) $A = \frac{1}{2}(3x + 5x)(h) + (h + 2)(5x)$
 $= 4xh + 5xh + 10x$
 $= 9xh + 10x$ (Shown) Area of trapezium ($\frac{1}{2} \times$ sum of // sides \times height) + Area of rectangle (length \times breadth)

(b) $A = 9xh + 10x$
 $A = x(9h + 10)$
 $x = \frac{A}{(9h+10)}$

Answer (a) $A = \frac{1}{2}(3x + 5x)(h) + (h + 2)(5x)$
 $= 4xh + 5xh + 10x$
 $= 9xh + 10x$ [1]

Answer (b) $x = \frac{A}{(9h+10)}$ [2]

13. Topics: Direct & Inverse Variation, Percentages

Let s be the speed of the car.

$D = ks^2$ Direct variation:
 $y \propto x \Rightarrow y = kx$
 When $s = p$, $D = 6$

$6 = kp^2$
 $k = \frac{6}{p^2}$
 $\therefore D = \frac{6s^2}{p^2}$

(a) When the speed is increased by 300%,
 Initial speed = p
 Increase in speed = $300\% \times p$
 $= 3p$
 New speed = initial speed + increase in speed
 $= p + 3p$
 $= 4p$

(b) When $s = 4p$,
 $D_1 = \frac{6(4p)^2}{p^2}$
 $= 96$

% increase = $\frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$

(c) Percentage increase = $\frac{96-6}{6} \times 100\%$
 $= 1500\%$

Answer	(a) <u>4p</u>	m/s	[1]
	(b) <u>96</u>	m	[1]
	(c) <u>1500</u>	%	[1]

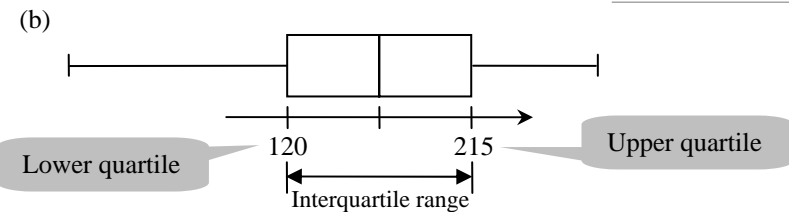
14. Topic: Statistics (Stem-and-Leaf Diagram, Box-and-Whisker Diagram)

(a) (i) **22** Modal score = score that appears most frequently

(ii) Median score = average of 8th & 9th terms
 $= \frac{20+21}{2}$
 $= 20.5$

Total number of terms = 16 (even)
 \Rightarrow median term = average of 2 middle terms

Answer	(a)(i) <u>22</u>	[1]
	(ii) <u>20.5</u>	[1]



Interquartile range = upper quartile - lower quartile
 $= 215 - 120$
 $= 95$

Answer	(b) <u>95</u>	g	[1]
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15. Topic: Algebra (Factorisation, Expansion)

$$(a) \quad 2x^3 - 13x^2 + 6x = x(2x^2 - 13x + 6) \\ = x(2x - 1)(x - 6)$$

Answer (a) $\underline{x(2x - 1)(x - 6)}$ [2]

$$(b) \quad 9a^2 + 1 - [9a^2 - 6a + 1] = 9a^2 + 1 - 9a^2 + 6a - 1 \\ = 6a$$

Answer (b) $\underline{6a}$ [2]

16. Topic: Angle Properties of Polygons

$$(a) \quad \frac{(n-2)180^\circ}{n} = 165^\circ \\ 180n - 360 = 165n \\ 180n - 165n = 360 \\ 15n = 360 \\ n = 24$$

Each interior \angle of a regular n-sided polygon

$$= \frac{(n-2) \times 180^\circ}{n}$$

$$(b) \quad \text{Let } x \text{ be the remaining interior } \angle. \\ x + 6(125) = (7 - 2)180 \\ x = 150^\circ$$

Answer (a) $\underline{24}$ [2]

(b) $\underline{150^\circ}$ [2]

17. Topic: Factors and Multiples

$$(a) \quad (i) \quad 1800 = 2^3 \times 3^2 \times 5^2$$

$$(ii) \quad k = 3 \times 5 \\ = 15$$

$$(iii) \quad 42 = 2 \times 3 \times 7 \\ \text{HCF} = 2 \times 3 \\ = 6$$

$$(b) \quad 12 = 2^2 \times 3 \\ 27 = 3^3 \\ 90 = 2 \times 3^2 \times 5 \\ \text{LCM} = 2^2 \times 3^3 \times 5 \\ = 540$$

540 seconds = 9 min
They next flash together at **09 09**.

Question asked for the time,
NOT time elapsed!

Answer(a)(i) $\underline{2^3 \times 3^2 \times 5^2}$ [1]

(ii) $k = \underline{15}$ [1]

(iii) $\underline{6}$ [1]

(b) $\underline{09 09}$ [2]

18. Topics: Linear Inequalities and Simultaneous Equations

(a) (i) $2 - 2x > 9$
 $-2x > 9 - 2$
 $-2x > 7$
 $x < -3.5$

Inequality sign reversed when multiplying both sides by -1

(ii) The greatest integer is -4 .

Answer(a)(ii) $x < -3.5$ [1]
 (ii) -4 [1]

(b) $x - 2y = 8$
 $x = 2y + 8$ (1)
 $3x = 19 + 4y$ (2)

Sub (1) into (2),

$3(2y + 8) = 19 + 4y$
 $6y + 24 = 19 + 4y$
 $2y = -5$
 $y = -2.5$

Sub $y = -2.5$ into (1),

$x = 2(-2.5) + 8$
 $= 3$

Answer (b) $x = 3$
 $y = -2.5$ [3]

19. Topic: Areas & Volumes of Similar Figures

Let h_1, A_1, V_1 be the height, surface area and volume, respectively, of the smaller jug.

Let h_2, A_2, V_2 be the height, surface area and volume, respectively, of the larger jug.

(a) $\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2$
 $\frac{h_1}{h_2} = \sqrt{\frac{45}{125}}$
 $= \frac{3}{5}$

(b) $\frac{A_1}{A_2} = \frac{45}{125}$
 $\frac{63}{A_2} = \frac{45}{125}$
 $A_2 = 175 \text{ cm}^2$

(c) $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$
 $\frac{V_1}{2.5} = \left(\frac{3}{5}\right)^3$
 $V_1 = 0.54 \text{ l}$
 $= 540 \text{ cm}^3$

Using ratio of $\frac{h_1}{h_2}$ obtained in (a)

1 litre = 1000 cm^3

Answer (a) $3 : 5$ [1]
 (b) 175 cm^2 [2]
 (c) 540 cm^3 [2]

20. Topic: Graphs in Practical Situations

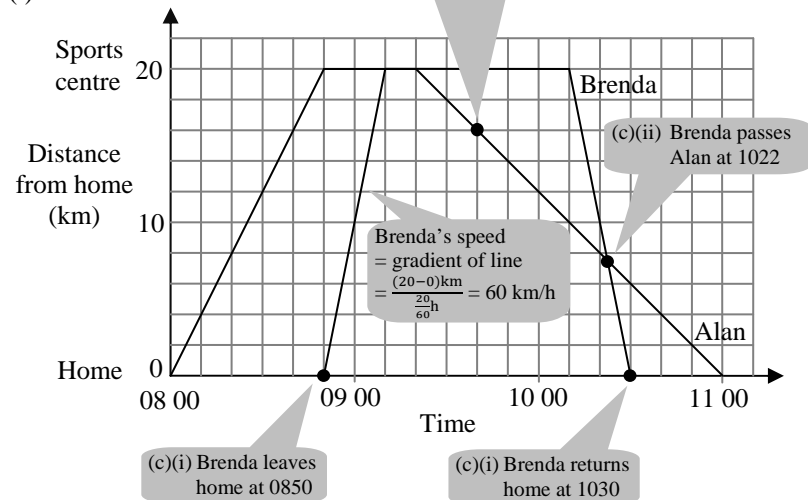
(a) Speed = $20 \div \frac{50}{60}$
= **24 km/h**

(b) He was 4 km from the sports centre.

Answer (a) 24 km/h [1]

(b) 4 km [1]

(c) (i)



(ii) Brenda passed Alan at 1022.

Answer(c)(ii) 1022 [1]

21. Topic: Vectors in Two Dimensions

(a) (i) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
= $-\overrightarrow{OA} + \overrightarrow{OB}$
= $-3\mathbf{a} + 4\mathbf{b}$

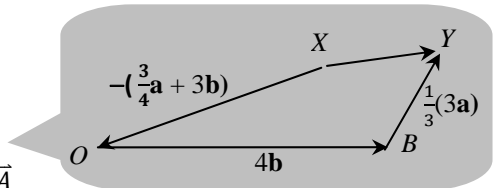
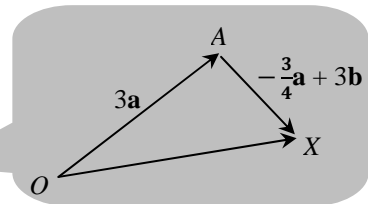
(ii) $\overrightarrow{AX} = \frac{3}{4}\overrightarrow{AB}$
= $\frac{3}{4}(-3\mathbf{a} + 4\mathbf{b})$
= $-\frac{9}{4}\mathbf{a} + 3\mathbf{b}$

(iii) $\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$
= $3\mathbf{a} - \frac{9}{4}\mathbf{a} + 3\mathbf{b}$
= $\frac{3}{4}\mathbf{a} + 3\mathbf{b}$

(iv) $\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OY}$
= $-\overrightarrow{OX} + \overrightarrow{OB} + \overrightarrow{BY}$
= $-\overrightarrow{OX} + \overrightarrow{OB} + \frac{1}{3}\overrightarrow{OA}$
= $-\left(\frac{3}{4}\mathbf{a} + 3\mathbf{b}\right) + 4\mathbf{b} + \frac{1}{3}(3\mathbf{a})$
= $\frac{1}{4}\mathbf{a} + \mathbf{b}$

(b) $\overrightarrow{OX} = 3\left(\frac{1}{4}\mathbf{a} + \mathbf{b}\right)$
= $3\overrightarrow{XY}$

$\therefore O, X$ and Y lie in a straight line



$\overrightarrow{AB} = k\overrightarrow{BC}$
 $\Rightarrow A, B, C$ are collinear
(straight line)

Answer(a)(i) $\overrightarrow{AB} = -3\mathbf{a} + 4\mathbf{b}$ [1]

(ii) $\overrightarrow{AX} = -\frac{9}{4}\mathbf{a} + 3\mathbf{b}$ [1]

(iii) $\overrightarrow{OX} = \frac{3}{4}\mathbf{a} + 3\mathbf{b}$ [1]

(iv) $\overrightarrow{XY} = \frac{1}{4}\mathbf{a} + \mathbf{b}$ [1]

22. Topic: Graphs

(a) (i) x-intercepts: When $y = 0$, $x(3-x) = 0$
 $x = 0$ or $3-x = 0$
 $x = 3$

y-intercept: When $x = 0$, $y = 0$

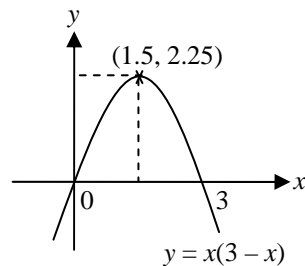
$y = x(3-x)$: coefficient of x^2 is -1
 \Rightarrow graph takes the shape of \cap (max point).

At max point:

$x = \frac{3-0}{2} = 1.5$

When $x = 1.5$, $y = 1.5(3-1.5)$
 $= 2.25$

Line of symmetry of quadratic graph passes through max/min point



(ii) Line of symmetry:

$x = \frac{3-0}{2}$
 $= 1.5$

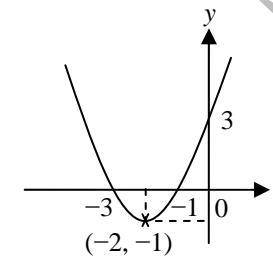
Answer (ii) $x = 1.5$ [1]

(b) (i) x-intercepts: When $y = 0$,
 $(x+2)^2 - 1 = 0$
 $(x+2)^2 = 1$
 $x+2 = 1$ or $x+2 = -1$
 $x = -1$ or $x = -3$

y-intercept: When $x = 0$, $y = 2^2 - 1$
 $= 3$

$y = (x+2)^2 - 1$: coefficient of x^2 is 1
 \Rightarrow graph takes the shape of \cup (min point).

Min point = $(-2, -1)$



$y = (x-h)^2 + k$
 \Rightarrow Line of symmetry $x = h$
 \Rightarrow min/max point = (h, k)

(ii) Coordinates of min pt = $(-2, -1)$

Answer (ii) $(-2, -1)$ [1]

23. Topics: Bearings and Geometrical Constructions

(a) Bearing of C from $A = 116^\circ$

Answer (a) 116° [1]

(d) Given $AB = 60$ km

Scale: 6 cm : 60 km

1 cm : 10 km

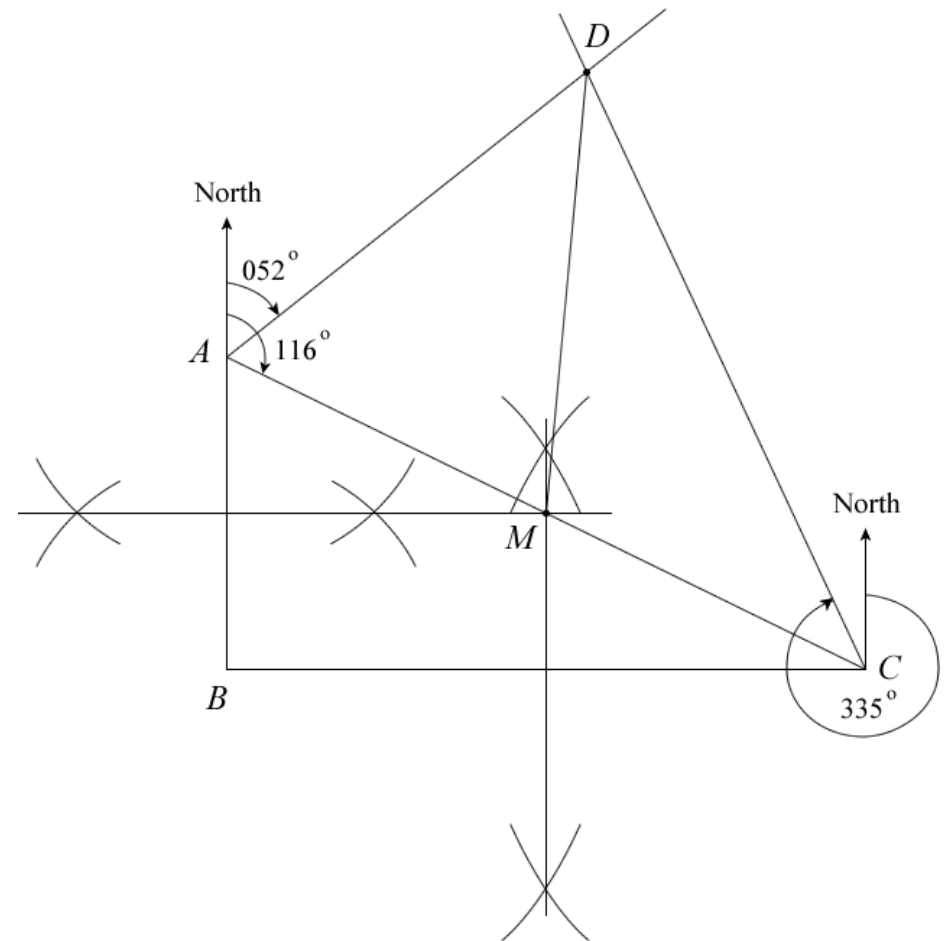
$DM = 8.35$ cm

$\Rightarrow 83.5$ km

Measured on actual question paper.

Answer (b) and (c)

[4]



Answer (d) 83.5 km [1]