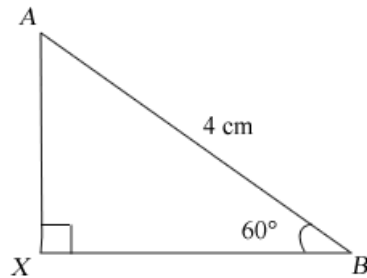


**ADDITIONAL MATHEMATICS**  
 Paper 1 Suggested Solutions

**4038/01**  
**October/November 2008**

1. **Topic: Trigonometry (Trigonometric Ratios)**

(i)  $\angle ABX = 60^\circ$   
 $\sin \angle ABX = \frac{AX}{AB}$   
 $\sin 60^\circ = \frac{AX}{4}$   
 $AX = \frac{\sqrt{3}}{2} \times 4$   
 $= 2\sqrt{3} \text{ cm}$



(ii) Using Pythagoras' theorem in  $\triangle ABX$ ,

$$AB^2 = AX^2 + BX^2$$

$$16 = (2\sqrt{3})^2 + BX^2$$

$$BX^2 = 16 - 12$$

$$= 4$$

$$BX = 2 \text{ cm}$$

$AX = 2\sqrt{3}$  from (i)  
 $CX = BC + BX$   
 $= 2 + 2 = 4 \text{ cm}$

In  $\triangle AXC$ ,  $\tan \angle ACB = \frac{AX}{CX}$   
 $= \frac{2\sqrt{3}}{4}$   
 $\angle ACB = \tan^{-1} \left( \frac{\sqrt{3}}{2} \right)$  (Shown)

2. **Topics: Indices, Simultaneous Equations**

$$9^x (27)^y = 1$$

$$(3^{2x})(3^{3y}) = 3^0$$

$$3^{2x+3y} = 3^0$$

Comparing the indices:  $2x + 3y = 0$   
 $6y = -4x \dots\dots\dots (1)$

$$8^y \div (\sqrt{2})^x = 16\sqrt{2}$$

$$2^{3y} \div 2^{\frac{1}{2}x} = (2^4) \left( 2^{\frac{1}{2}} \right)$$

$$2^{3y-\frac{1}{2}x} = 2^{4+\frac{1}{2}}$$

Comparing the indices:  $3y - \frac{1}{2}x = 4 + \frac{1}{2}$   
 $3y - \frac{1}{2}x = \frac{9}{2}$   
 $6y - x = 9 \dots\dots\dots (2)$

Sub (1) into (2):  $-4x - x = 9$   
 $x = -\frac{9}{5}$

Sub  $x = -\frac{9}{5}$  into (1):  $6y = -4 \left( -\frac{9}{5} \right)$   
 $= \frac{36}{5}$   
 $y = \frac{6}{5}$

$\therefore x = -\frac{9}{5}, y = \frac{6}{5}$

### 3. Topic: Simultaneous Equations (Solution by Inverse Matrix Method)

$$\mathbf{A} = \begin{pmatrix} 7 & -8 \\ 1 & 6 \end{pmatrix}$$

$$\mathbf{A}^{-1} = \frac{1}{(7)(6) - (-8)(1)} \begin{pmatrix} 6 & 8 \\ -1 & 7 \end{pmatrix}$$

$$= \frac{1}{50} \begin{pmatrix} 6 & 8 \\ -1 & 7 \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\mathbf{M}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Given:  $7q - 8p = 11$   
 $q + 6p = -7$

Expressing the above as a matrix equation,

$$\begin{pmatrix} 7 & -8 \\ 1 & 6 \end{pmatrix} \begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} 11 \\ -7 \end{pmatrix} \quad \mathbf{A} \begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} 11 \\ -7 \end{pmatrix}$$

Multiply both sides by  $\mathbf{A}^{-1}$ ,

$$\begin{pmatrix} q \\ p \end{pmatrix} = \frac{1}{50} \begin{pmatrix} 6 & 8 \\ -1 & 7 \end{pmatrix} \begin{pmatrix} 11 \\ -7 \end{pmatrix}$$

$$= \frac{1}{50} \begin{pmatrix} 10 \\ -60 \end{pmatrix}$$

$$= \frac{1}{5} \begin{pmatrix} 1 \\ -6 \end{pmatrix}$$

$$\mathbf{A}^{-1} \mathbf{A} \begin{pmatrix} q \\ p \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} 11 \\ -7 \end{pmatrix}$$

$$\mathbf{I} \begin{pmatrix} q \\ p \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} 11 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} q \\ p \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} 11 \\ -7 \end{pmatrix}$$

$$\therefore q = \frac{1}{5}, p = -\frac{6}{5}$$

### 4. Topic: Differentiation & Integration

$$(i) \frac{d}{dx}(x^3 \ln x) = x^3 \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(x^3)$$

$$= x^3 \times \frac{1}{x} + 3x^2 \ln x$$

$$= x^2 + 3x^2 \ln x$$

Product Rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$(ii) \int x^2 \ln x \, dx = \frac{1}{3} \int 3x^2 \ln x \, dx$$

$$= \frac{1}{3} \left[ \int (3x^2 \ln x + x^2 - x^2) \, dx \right]$$

$$= \frac{1}{3} \left[ \int (3x^2 \ln x + x^2) \, dx - \int x^2 \, dx \right]$$

$$= \frac{1}{3} \int (3x^2 \ln x + x^2) \, dx - \frac{1}{3} \int x^2 \, dx$$

$$= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + c$$

From (i):  $\frac{d}{dx}(x^3 \ln x) = x^3 + 3x^2 \ln x \Rightarrow$   
 $\int (3x^2 \ln x + x^2) \, dx = x^3 \ln x + c$

### 5. Topics: Partial Fractions, Applications of Differentiation (Gradients)

$$(i) \frac{8x-46}{(x-5)(x+1)} = \frac{A}{x-5} + \frac{B}{x+1}$$

Multiply both sides by  $(x-5)(x+1)$ :

$$8x - 46 = A(x+1) + B(x-5)$$

Let  $x = -1$ :  $-8 - 46 = B(-6)$   
 $B = 9$

Let  $x = 5$ :  $40 - 46 = A(6)$   
 $A = -1$

$$\therefore \frac{8x-46}{(x-5)(x+1)} = -\frac{1}{x-5} + \frac{9}{x+1}$$

$$(ii) \text{ From (i), } y = \frac{8x-46}{(x-5)(x+1)}$$

$$= -\frac{1}{x-5} + \frac{9}{x+1}$$

$$\frac{dy}{dx} = (x-5)^{-2} - 9(x+1)^{-2}$$

When  $x = 2$ ,  $\frac{dy}{dx} = (2-5)^{-2} - 9(2+1)^{-2} = \frac{1}{9} - \frac{9}{9}$   
 $= -\frac{8}{9}$

6. **Topic: Applications of Differentiation & Integration (Kinematics)**

(i)  $v = 6t - \frac{1}{2}t^2$

Since the cyclist is at rest,  $v = 0$  at point B.

$$\Rightarrow 6t - \frac{1}{2}t^2 = 0$$

$$\frac{1}{2}t(12 - t) = 0$$

$$t = 0 \text{ (reject) or } 12$$

$\therefore$  Time taken from A to B = **12 s**

(ii) Distance  $AB = \int_0^{12} (6t - \frac{1}{2}t^2) dt$

$$= \left[ 3t^2 - \frac{1}{6}t^3 \right]_0^{12}$$

$$= 3(12)^2 - \frac{1}{6}(12)^3$$

$$= \mathbf{144 \text{ m}}$$

(iii) Acceleration  $a = \frac{dv}{dt} = 6 - t$

When  $t = 8$ ,  $a = 6 - 8$

$$= \mathbf{-2 \text{ ms}^{-2}}$$

7. **Topic: Applications of Differentiation (Gradients, Tangents & Normals)**

Given  $y = \frac{\sin x}{2 - \cos x}$ ,  $0 < x < \frac{\pi}{2}$

$$\frac{dy}{dx} = \frac{(2 - \cos x)\cos x - \sin x \cdot \sin x}{(2 - \cos x)^2}$$

$$= \frac{2\cos x - \cos^2 x - \sin^2 x}{(2 - \cos x)^2}$$

$$= \frac{2\cos x - 1}{(2 - \cos x)^2}$$

Tangent to curve is parallel to the x-axis

$$\Rightarrow \frac{dy}{dx} = 0$$

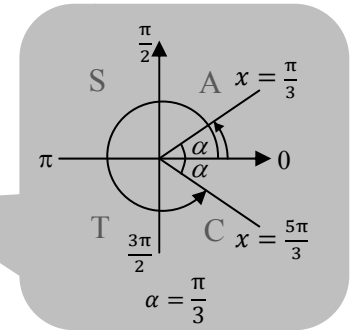
$$\frac{2\cos x - 1}{(2 - \cos x)^2} = 0$$

$$2\cos x - 1 = 0$$

$$\cos x = \frac{1}{2} \Rightarrow \text{Basic angle } \alpha = \frac{\pi}{3}$$

$$x = \frac{\pi}{3} \text{ or } \frac{5\pi}{3} \text{ (reject } \because 0 < x < \frac{\pi}{2})$$

$$\therefore x = \frac{\pi}{3}$$



8. **Topic: Further Trigonometric Identities**

(i) For  $\sin 3x + \sin x = 4 \sin x \cos^2 x$

Factor Formula:

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

L.H.S.:  $\sin 3x + \sin x$

$$= 2 \sin \frac{3x+x}{2} \cos \frac{3x-x}{2}$$

$$= 2 \sin 2x \cos x$$

Double Angle Formula:

$$\sin 2A = 2 \sin A \cos A$$

$$= 2(2 \sin x \cos x) \cos x$$

$$= 4 \sin x \cos^2 x = \text{R. H. S. (Shown)}$$

(ii)  $\sin 3x + \sin x = 2\cos^2 x$

From (i):  $\sin 3x + \sin x = 4\sin x \cos^2 x$

$$4\sin x \cos^2 x = 2\cos^2 x$$

$$4\sin x \cos^2 x - 2\cos^2 x = 0$$

$$2\cos^2 x(2\sin x - 1) = 0$$

$$\cos x = 0$$

$$\sin x = \frac{1}{2}$$

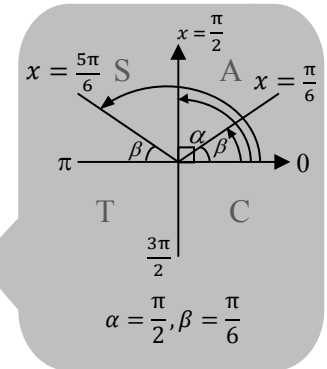
Basic angle  $\alpha = \frac{\pi}{2}$

Basic angle  $\beta = \frac{\pi}{6}$

$$x = \frac{\pi}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\therefore x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$



9. **Topic: Simultaneous Equations**

Let Ann's age be  $x$  and Betty's age be  $y$ .

$$x^2 - 2y^2 = 6(x - y) \dots\dots\dots (1)$$

$$x + y = 5(x - y)$$

$$x + y = 5x - 5y$$

$$6y = 4x$$

$$y = \frac{2}{3}x \dots\dots\dots (2)$$

$x^2 - 2y^2$ : "... twice the square of Betty's age subtracted from the square of Ann's age"  
 $= 6(x - y)$ : "... equal to 6 times the difference of their ages."

$x + y = 5(x - y)$ :  
 "... sum of their ages is equal to 5 times the difference of their ages"

Sub (2) into (1):  $x^2 - 2\left(\frac{4}{9}x^2\right) = 6\left(x - \frac{2}{3}x\right)$

$$\frac{1}{9}x^2 = 2x$$

$$x^2 - 18x = 0$$

$$x(x - 18) = 0$$

$x = 0$  (reject as Ann is older than Betty) or  $x = 18$

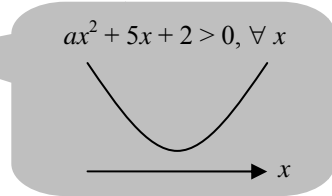
When  $x = 18$ ,

$$y = \frac{2}{3}(18) = 12$$

**∴ Ann is 18 years old and Betty is 12 years old.**

10. **Topic: Quadratic Equations & Inequalities**

(a)  $ax^2 + 5x + 2 > 0$   
 $\Rightarrow b^2 - 4ac < 0$  (no real roots)  
 $25 - 4a(2) < 0$   
 $25 < 8a$   
 $a > \frac{25}{8}$   
 $> 3\frac{1}{8}$



**∴ smallest integer  $a = 4$**

(b)  $-5x^2 + bx - 2 < 0$

$$b^2 - 4ac < 0 \text{ (no real roots)}$$

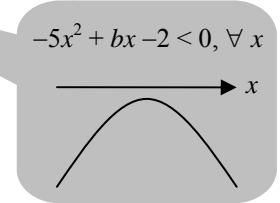
$$b^2 - 4(-5)(-2) < 0$$

$$b^2 - 40 < 0$$

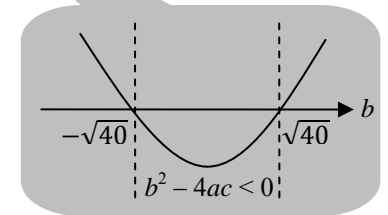
$$(b - \sqrt{40})(b + \sqrt{40}) < 0$$

$$-\sqrt{40} < b < \sqrt{40}$$

$$\Rightarrow b > -\sqrt{40} \approx -6.32$$



**∴ smallest integer  $b = -6$**



## 11. Topic: Binomial Expansions

(i)  $(r+1)^{\text{th}}$  term of  $(x + \frac{k}{x})^7 = \binom{7}{r} x^{7-r} (kx^{-1})^r$   
 $= \binom{7}{r} k^r x^{7-2r}$

In expanding  $(a+b)^n$ ,  
 $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

Coefficient of  $x^3 = \binom{7}{2} k^2$   
 $= 21k^2$

$x^3 = x^{7-2r} \Rightarrow r = 2 \rightarrow T_2$

Coefficient of  $x = \binom{7}{3} k^3$   
 $= 35k^3$

$x^1 = x^{7-2r} \Rightarrow r = 3 \rightarrow T_3$

Equating coefficients of  $x^3$  and  $x$ ,

$$21k^2 = 35k^3$$

$$3k^2 = 5k^3$$

$$5k^3 - 3k^2 = 0$$

$$k^2(5k - 3) = 0$$

$$k = 0 \text{ (reject) or } k = \frac{3}{5}$$

$$\therefore k = \frac{3}{5}$$

(ii)  $(1 - 5x^2)(x + \frac{k}{x})^7 = (1 - 5x^2) \left[ \binom{7}{0} x^7 + \binom{7}{1} kx^5 + \dots \right]$   
 $= x^7 - 35kx^7 + \dots$

$\therefore$  coefficient of  $x^7 = 1 - 35k$   
 $= 1 - 35\left(\frac{3}{5}\right)$   
 $= 1 - 7(3)$   
 $= -20$

Terms beyond the 1<sup>st</sup> 2 terms of  $(x + \frac{k}{x})^7$  are ignored as they do not form  $x^7$  terms when multiplied by  $(1 - 5x^2)$ .

## 12. Topic: Linear Law

(i) Given  $y = kb^x$ ,  
 $\lg y = \lg(kb^x)$   
 $\lg y = \lg k + x \lg b$   
 $\lg y = (\lg b)x + \lg k$

Letting  $Y$  be  $\lg y$  and  $X$  be  $x$ , the graph of  $\lg y$  against  $\lg x$  is a straight line

$$Y = (\lg b)X + \lg k$$

$$= mX + c$$

From the graph,

Gradient  $m$ :

$$\Rightarrow \lg b = \frac{1.3 - 0.8}{0 - 11}$$

$$= -\frac{1}{22}$$

$$b = 10^{-\frac{1}{22}}$$

$$\approx \mathbf{0.90 \text{ (2 sig. fig.)}}$$

When  $X = 0$ ,  $Y$ -intercept = 1.3

$$\Rightarrow \lg k = 1.3$$

$$k = 10^{1.3}$$

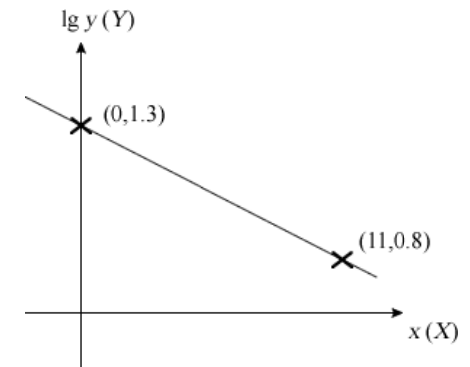
$$\approx \mathbf{20 \text{ (2 sig. fig.)}}$$

(ii) From (i),  $y = 10^{1.3} \left(10^{-\frac{1}{22}}\right)^x$

When  $x = 8$ ,

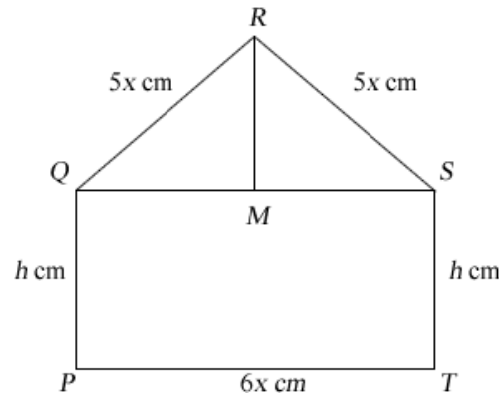
$$y = 10^{1.3} \left(10^{-\frac{1}{22}}\right)^8$$

$$\approx \mathbf{8.64}$$



13. Topic: Applications of Differentiation (Maxima & Minima)

(i)



Perimeter of the window = 360cm (given)

$$\begin{aligned} 5x \times 2 + h \times 2 + 6x &= 360 \\ 5x + h + 3x &= 180 \\ h &= 180 - 8x \end{aligned}$$

Let RM be the height of  $\triangle QRS$ .

$\Rightarrow M$  is the mid-point of QS. ( $\triangle QRS$  is isosceles)

Using Pythagoras' theorem,

$$\begin{aligned} RM &= \sqrt{(5x)^2 - (3x)^2} \\ &= 4x \text{ cm} \end{aligned}$$

$$\begin{aligned} \therefore A &= \frac{1}{2}(6x)(4x) + (h)(6x) \\ &= 12x^2 + (180 - 8x)6x \\ &= \mathbf{1080x - 36x^2} \text{ (Shown)} \end{aligned}$$

Area  $A$  = Area of  $\triangle QRS$  +  
Area of rectangle  $PQST$

To show that  $A = 1080x - 36x^2$ ,  
express all unknowns i.e.  $h$  and  $RM$   
(height of  $\triangle QRS$ ) in terms of  $x$ .

$$(ii) \frac{dA}{dx} = 1080 - 72x$$

$$\text{When } \frac{dA}{dx} = 0,$$

$$\begin{aligned} 72x &= 1080 \\ x &= 15 \text{ cm} \end{aligned}$$

When  $x = 15$  cm,

$$\begin{aligned} A &= 1080(15) - 36(15)^2 \\ &= \mathbf{8100\text{cm}^2} \end{aligned}$$

$$(iii) \frac{d^2A}{dx^2} = -72$$

$$\Rightarrow \frac{d^2y}{dx^2} < 0$$

$\Rightarrow$  The stationary value of  $A$  is a maximum.