2009 H2 Maths Paper 2 :

Date: No.

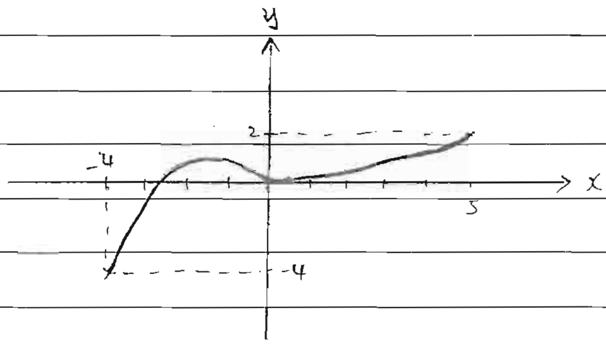
(1) C : $x = t^2 + 4t$ —— (1)

$y = t^3 + t^2$ —— (2)

(i) when $t = -2$, $x = -4$, $y = -4$

when $t = 0$, $x = 0$, $y = 0$

when $t = 1$, $x = 5$, $y = 2$



(ii) $\frac{dx}{dt} = 2t + 4$

$\frac{dy}{dt} = 3t^2 + 2t$

$\therefore \frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} = \frac{3t^2+2t}{2t+4}$

when $t = 2$, we have.

$\frac{dy}{dx} = \frac{3(2)^2+2(2)}{2(2)+4} = 2.$

and $x = 12$, $y = 12$ $\Rightarrow$ $P(12, 12)$

$\therefore$ Equation of l :

$y - 12 = 2(x - 12)$

$y = 2x - 12$ # —— (3)

(iii) Given tangent l meet at Q :

sub (1) & (2) into (3)

$t^3 + t^2 = 2(t^2 + 4t) - 12.$

$t^3 - t^2 - 8t + 12 = 0.$

$(t-2)(t^2 + t - 6) = 0$ {using factor theorem}.

$(t-2)(t+3)(t-2) = 0$

$\therefore t = 2$ or $t = -3$.

(reject as this belong to point P)

$\therefore$ when $t = -3$, $x = -3.$ {from (1)}

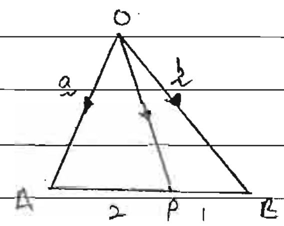
$y = -18.$ {from (2)}.

$\therefore$ Coordinates of Q is $(-3, -18)$

2009 H2 Maths Paper 2:

Date No.

(2) Given $\overrightarrow{OA} = \underset{\sim}{a} = \begin{pmatrix}14\\14\\14\end{pmatrix}$ and $\overrightarrow{OB} = \underset{\sim}{b} = \begin{pmatrix}11\\-13\\2\end{pmatrix}$



(i) Given P divides AB in the ratio 2 : 1

By ratio theorem :

$$\overrightarrow{OP} = \frac{1}{3}\left[1\underset{\sim}{a} + 2\underset{\sim}{b}\right]$$

$$= \frac{1}{3}\left[\begin{pmatrix}14\\14\\14\end{pmatrix} + 2\begin{pmatrix}11\\-13\\2\end{pmatrix}\right]$$

$$= \begin{pmatrix}12\\-4\\6\end{pmatrix}$$

$\therefore$ Coordinates of P is $(12, -4, 6)$

(ii) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}11\\-13\\2\end{pmatrix} - \begin{pmatrix}14\\14\\14\end{pmatrix} = \begin{pmatrix}-3\\-27\\-12\end{pmatrix}$

$$\overrightarrow{AB} \cdot \overrightarrow{OP} = \begin{pmatrix}-3\\-27\\-12\end{pmatrix} \cdot \begin{pmatrix}12\\-4\\6\end{pmatrix}$$

$$= -36 + 108 - 72$$

$$= 0$$

$\therefore \overrightarrow{AB} \cdot \overrightarrow{OP} = 0 \Rightarrow AB \perp OP$ (shown).

(iii) Given $\underset{\sim}{c}$ is a unit vector of $\overrightarrow{OP}$:

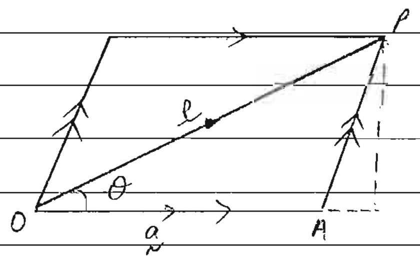
$$|\overrightarrow{OP}| = \sqrt{12^2 + (-4)^2 + 6^2} = 14$$

$$\therefore \underset{\sim}{c} = \frac{\overrightarrow{OP}}{|\overrightarrow{OP}|} = \begin{pmatrix}\frac{12}{14}\\-\frac{4}{14}\\\frac{6}{14}\end{pmatrix} = \begin{pmatrix}6/7\\-2/7\\3/7\end{pmatrix} \#$$

The geometrical meaning of $|\underset{\sim}{a} \cdot \underset{\sim}{c}|$ is length of projection of $\underset{\sim}{a}$ onto $\overrightarrow{OP}$. #

(iv) $$\underset{\sim}{a} \times \underset{\sim}{\ell} = \begin{pmatrix}14\\14\\14\end{pmatrix} \times \begin{pmatrix}12\\-4\\6\end{pmatrix} = \begin{pmatrix}14\times6 + 4\times14\\-(14\times6 - 14\times12)\\14\times(-4) - 14\times12\end{pmatrix} = \begin{pmatrix}140\\84\\-224\end{pmatrix}$$

The geometrical meaning of $|\underset{\sim}{a} \times \underset{\sim}{\ell}|$ is area of parallelogram.



Note area of parallelogram

$= |\underset{\sim}{a}||\underset{\sim}{\ell}|\sin\theta$

$= |\underset{\sim}{a} \times \underset{\sim}{\ell}|$

$\therefore$ Area of $\triangle OAP$

$$= \frac{1}{2}|\underset{\sim}{a} \times \underset{\sim}{\ell}|$$

$$= \frac{1}{2}\left|\begin{pmatrix}140\\84\\-224\end{pmatrix}\right|$$

$$= \frac{1}{2}\sqrt{140^2 + 84^2 + (-224)^2}$$

$$= \frac{1}{2}\sqrt{76832}$$

$$= 98\sqrt{2} \text{ units}^2$$

2009 H2 Maths Paper 2 :

Date: No.

(3) Given $f(x) = \frac{ax}{bx-a}$, for $x \in \mathbb{R}$, $x \neq \frac{a}{b}$, $a, b \neq 0$

(i) Let $y = \frac{ax}{bx-a}$ where $y = f(x)$

$byx - ay = ax$.

$(by-a)x = ay$.

$x = \frac{ay}{by-a}$

$\therefore f^{-1}(x) = \frac{ax}{bx-a}$. # ——— (1)

$\therefore f^{-1}(x) = f(x)$.

$\Rightarrow x = f(f(x))$

$\Rightarrow f^2(x) = x$ #

$\therefore$ Range of $f^2(x)$ is $R_{f^2} \in \mathbb{R} / \{\frac{a}{b}\}$ #

(ii) Given $g(x) = \frac{1}{x}$ for $x \in \mathbb{R}$, $x \neq 0$.

$\therefore R_g = \mathbb{R} / \{0\}$

and $D_f = \mathbb{R} / \{\frac{a}{b}\}$ where $a, b \neq 0$

$\therefore$ fg does not exist because $R_g \not\subseteq D_f$.

(iii) Given $f^{-1}(x) = x$.

From (1), we have $\frac{ax}{bx-a} = x$.

$\left(\frac{a}{bx-a} - 1\right)x = 0$.

$\therefore x = 0$ or $\frac{a}{bx-a} - 1 = 0$.

$a = bx - a$.

$x = \frac{2a}{b}$

$\therefore$ The solution are : $x = 0$ or $x = \frac{2a}{b}$ #

2009 H2 Maths Paper 2:

Date No.

(4) Given by one scientist:

(i) $\frac{d^2n}{dt^2} = 10 - 6t$.

$\frac{dn}{dt} = 10t - 3t^2 + c_1$, where c_1 is a constant

$n = 5t^2 - t^3 + c_1 t + c_2$, where c_2 is a constant.

$\therefore$ Given $n = 100$ when $t = 0$.

$\therefore 100 = 5(0) - 0 + c_1(0) + c_2$

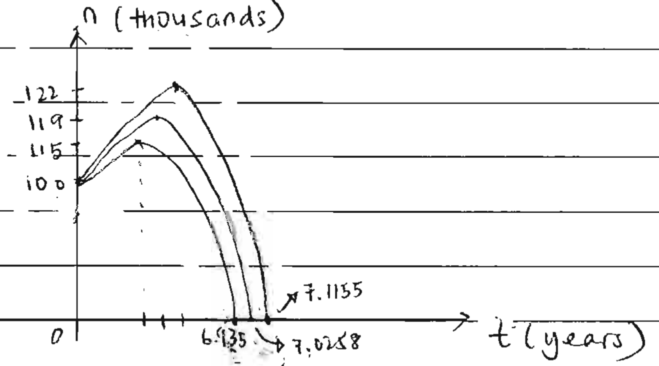
$c_2 = 100$.

$\therefore n = 5t^2 - t^3 + c_1 t + 100$.

when $c_1 = 1$, $n = 5t^2 - t^3 + 100 + t$, turning pt $= (3.43, 122)$

when $c_1 = 0$, $n = 5t^2 - t^3 + 100$, turning pt $= (3.33, 119)$

when $c_1 = -1$, $n = 5t^2 - t^3 + 100 - t$, turning pt $= (3.23, 115)$



(ii) Given by 2nd scientist:

$\frac{dn}{dt} = 3 - 0.02n$.

$\int \frac{1}{3 - 0.02n}\, dn = \int dt$.

$\frac{1}{-0.02} \ln|3 - 0.02n| = t + c_3$ where c_3 is constant.

$\ln|3 - 0.02n| = -0.02t - 0.02c_3$

$3 - 0.02n = e^{-0.02t} \cdot e^{-0.02c_3}$

$0.02n = 3 - e^{-0.02c_3} e^{-0.02t}$

$n = 150 - 50e^{-0.02c_3} e^{-0.02t}$

$\therefore n = 150 - Ae^{-0.02t}$, where A is a constant.

The population will eventually increase to and remain at 150 000 ※

(5) A quota sample of 100 cinema-goers may be obtained by instructing the interviewer to conduct the survey with 50 male and 50 female cinema-goers as they leave the cinema.

A disadvantage of this method is the possibility of bias in the selection process, as interviewers may tend to choose the easiest way to fulfill the survey quota e.g. selecting those who are more open and the easiest to approach; interviewing couples (with 1 male and 1 female) who may tend to give the same opinion. #

(6) (i)

(ii) As far as the data in the scatter diagram is concerned, the linear model is appropriate since its calculated value of $r = -0.986$ indicates a strong negative linear correlation.

In the context of the question, however, it is unlikely that the world record will decrease linearly with time since it's likely to be increasingly difficult to break it as we approach the limits of our human abilities as time goes by. Hence a non-linear model with a negative exponential function may be more appropriate than a linear model. #

(iii) A quadratic model (with a minimum point) would not be appropriate since the world record time can only decrease or remain the same as the years go by. Hence there cannot be a portion where t increases as x increases in the long term. #

NO:............................... DATE:...............................

(iv) By generating another list $\Rightarrow L = \ln t$

and using GC, we have line of regression.

$\therefore \ln t = 34.853 - 0.016127x$

$\approx 34.9 - 0.0161x$

coefficient of correlation, $r = -0.99616$.

Sub $x = 2010$, $\therefore t = 11.447$

≈ 11.4 (3 s.f.)

* Note world record time $= 3$ min 30 s $+ t$ s

$\therefore$ world record time as at 1st January 2010 is 3 minutes 41.4 seconds #

As the 2010 world record time is predicted through extrapolating our data well beyond the year 2000, it is not reliable despite its strong correlation. #

2009 H2 Maths Paper 2

Date No.

(7) (i) Given $p = 25$.

Probability that a randomly chosen component is faulty

$= P$(component supplied by A is faulty or component supplied by B is faulty)

$= P$(component supplied by A is faulty) + P(component supplied by B is faulty)

$= \frac{25}{100} \times 0.05 + \frac{75}{100} \times 0.03$

$= 0.035$ #

(ii) For a general value of p:

$$f(p) = \frac{\frac{p}{100} \times 0.05}{\frac{p}{100} \times 0.05 + \frac{100-p}{100} \times 0.03} = \frac{\frac{p}{100} \times 0.05}{\frac{1}{100}[0.05p + 3 - 0.03p]} = \frac{0.05p}{0.02p+3} \text{ (shown)} \#$$

$$f'(p) = \frac{(0.02p+3)0.05 - 0.05p(0.02)}{(0.02p+3)^2} = \frac{0.15}{(0.02p+3)^2}$$

for $0 \leq p \leq 100$, $(0.02p+3)^2 > 0$

$\therefore f'(p) = \frac{0.15}{(0.02p+3)^2} > 0$

$\therefore f$ is an increasing function for $0 \leq p \leq 100$.

The increasing function $f(p)$ shows that as the company buys a greater % of its electronic components from supplier A, the probability of a faulty component that is randomly packed from supplier A increases. This translates into a greater likelihood of receiving a greater number of faulty components from supplier A #

(8) ELEVATED

E − 3, L − 1, V − 1, A − 1, T − 1, D − 1

(i) No. of ways to be arrange (without restrictions) $= \frac{8!}{3!} = 6720$ #

(ii) No. of ways for T and D next to one another $= \frac{7!}{3!} \times 2!$ {treat T & D as one group}

$= 1680$.

$\therefore$ No. of ways of T & D must not be next to one another $= 6720 - 1680 = 5040$ #

(iii) No. of ways for consonants (L, V, T, D) and vowels must alternate $= 4! \times \frac{4!}{3!} \times 2$

C V C V C V C V or V C V C V C V C

$= 192$ #

(iv) Case 1, X E X X E X X E X $= 5! \times 2! = 240$

Case 2, E X X X E X X E $= 5! \times 2! = 240$

$\therefore$ No. of ways between any two Es there must be at least 2 other letters

$= 240 + 240$

$= 480$ #

2009 H2 Maths Paper 2:

Date No.

(9) Let M be the random variable of the thickness in cm of a Mechanics textbook.

(i) Then $M \sim N(2.5, 0.1^2)$

$\therefore \quad \bar{M} \sim N(2.5, \frac{0.1^2}{n})$.

Given $P(\bar{M} > 2.53) = 0.0668$

Then $P(Z > \frac{2.53-2.5}{0.1/\sqrt{n}}) = 0.0668$.

$1 - P(Z < \frac{2.53-2.5}{0.1/\sqrt{n}}) = 0.0668$.

$P(Z < 0.3\sqrt{n}) = 0.9332$.

$0.3\sqrt{n} = 1.5$ {using G.C}.

$n \approx 25$ #.

(ii) Let S be the random variable of the thickness in cm of a Statistic textbook.

$S \sim N(2.0, 0.08^2)$.

Let $M_T = M_1 + M_2 + M_3 + \ldots + M_{21}$

$\therefore M_T \sim N(21 \times 2.5, 21 \times 0.1^2) = N(52.5, 0.21)$

Let $S_T = S_1 + S_2 + S_3 + \ldots + S_{24}$

$\therefore S_T \sim N(24 \times 2, 24 \times 0.08^2) = N(48.0, 0.1536)$

Hence $M_T + S_T \sim N(100.5, 0.3636)$.

P(21 mechanic textbooks & 24 statistics text will fit onto a bookshelf of length 1m)

$= P(M_T + S_T \leq 100 \text{ cm}) = \text{normalcdf}(-e99, 100, 100.5, \sqrt{0.3636})$

$= 0.20349$

≈ 0.203 (3 sig fig). #

(iii) Let $D = S_1 + S_2 + S_3 + S_4 - 3M$

Then $E(D) = 4E(S) - 3E(M) = 4(2) - 3(2.5) = 0.5$.

and $Var(D) = 4Var(S) + 9Var(M) = 4(0.08)^2 + 9(0.1)^2 = 0.1156$.

$\therefore D \sim N(0.5, 0.1156)$.

P(the total thickness of 4 statistic textbooks < 3 times the thickness of 1 mechanics textbooks)

$= P(S_1 + S_2 + S_3 + S_4 < 3M) = P(S_1 + S_2 + S_3 + S_4 - 3M < 0) = P(D < 0)$

$= \text{normalcdf}(-E99, 0, 0.5, \sqrt{0.1156})$

$= 0.07070 \approx 0.0707$ (3 s.f.) #

(iv) The thickness of a mechanics textbook is independent of the thickness of a statistics textbook. #

NO: 2009 H2 Maths Paper 2

10. (i) Unbiased estimates of the mean, $\bar{x} = \frac{\Sigma x}{n}$

$= \frac{86.4}{9}$

$= 9.6$ #

Unbiased estimates of the variance of X $= \frac{n}{n-1}\left[\frac{\Sigma x^2}{n} - (\bar{x})^2\right]$

$= \frac{9}{8}\left[\frac{835.02}{9} - (9.6)^2\right]$

$= 0.81$ #

(ii) Assumption: Mass of sugar follows a normal distribution

$H_0: \mu = 10$ grams

$H_1: \mu \neq 10$ grams

$T = \frac{\bar{x}-\mu}{s/\sqrt{n}} \sim t_{n-1}$

$= \frac{9.6-10}{\sqrt{\frac{0.81}{9}}}$

$= -1.333$ when $\mu=10$, $s=\sqrt{0.81}$ and $n=9$.

By GC, $p = 0.2191$ (> 0.05),

Hence we do not reject H_0 and conclude that at the 5% level of significance, there is insufficient evidence to conclude the mass of the packet is not 10 grams. #

In this case, the sample size is small (< 30 say) its not large enough to assume a normal distribution according to central limit theorem.

(iii) As the population variance of X is known, the z-test is carried out in-stead of the t-test. i.e. $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$\therefore z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

#

2009 H2 Maths Paper 2 :

Date: No.

11(i) Two assumptions needed for R to be well modelled by a binomial distribution:

(a) The color of the car is either red or not red.

(b) The trials are independent i.e. the color of the car in each observation is independent of the color of the car in every other observation.

Given $R \sim B(n, p)$.

(ii) Given also $n = 20$, $p = 0.15$

$P(4 \leq R < 8)$

$= P(4 \leq R \leq 7)$

$= P(R \leq 7) - P(R \leq 3)$

$=$ binomcdf $(20, 0.15, 7)$ − binomcdf $(20, 0.15, 3)$

$= 0.99407 - 0.64772$

≈ 0.346 (corr to 3 sig fig).

(iii) Given that $n = 240$, $p = 0.3$

Since $n = 240$ (> 50) is large, $np = 72 > 5$ and $n(1-p) = 168 > 5$ the binomial distribution can be approximated using the normal distribution with mean $np (= 72)$ and variance $np(1-p) = 50.4$

$\therefore$ $R \sim N(72, 50.4)$ approximately.

$P(R < 60) \overset{c.c}{\approx} P(R < 59.5)$

$=$ normalcdf $(-E99, 59.5, 72, \sqrt{50.4})$

$= 0.0391$ (corr to 3 sig fig).

(iv) Given that $n = 240$ and $p = 0.02$.

Since $n > 50$, $p < 0.1$ and $np = 4.8 < 5$, the binomial distribution can be approximated using the poisson distribution with mean $\lambda = np = 4.8$

i.e. $R \sim Po(4.8)$ approximately.

$P(R = 3)$ = poissonpdf $(4.8, 3) \approx 0.1517$ (corr to 4 dec pl)

(v) Given that $n = 20$ and $P(R = 0 \text{ or } 1) = 0.2$

Then $P(R=0) + P(R=1) = 0.2$.

$(1-p)^{20} + \binom{20}{1} p^1 (1-p)^{19} = 0.2$.

$(1-p)^{19} (1 + 19p) = 0.2$.

Using G.C : $p = 0.142$ (corr to 3 sig fig.)