2009 H2 Maths Paper 1

Date No.

① (i) Given U_n is a quadratic polynomial in n.

Then Let $U_n = an^2 + bn + c$, where a, b & c are constants

Given $u_1 = 10$, then $u_1 = a + b + c = 10$

Given $u_2 = 6$, then $u_2 = 4a + 2b + c = 6$

Given $u_3 = 5$, then $u_3 = 9a + 3b + c = 5$

Using GC, we have $a = \frac{3}{2}$, $b = -\frac{17}{2}$ and $c = 17$.

$\therefore U_n = \frac{3}{2}n^2 - \frac{17}{2}n + 17$ #.

(ii) For

$\frac{3}{2}n^2 - \frac{17}{2}n + 17 > 100$

$3n^2 - 17n + 34 > 200.$

$3n^2 - 17n - 166 > 0.$ —— ①

Consider $3n^2 - 17n - 166 = 0$

$$n = \frac{17 \pm \sqrt{17^2 + 4(166)(3)}}{2(3)}$$

$$\therefore n = \frac{17 - \sqrt{2281}}{6} \quad \text{or} \quad \frac{17 + \sqrt{2281}}{6}$$

$$= -5.13 \quad \text{or} \quad 10.8$$

From ①, we have

$n < -5.13$ or $n > 10.8$

But $n \in \mathbb{Z}^+$, $\therefore n \geq 11$ is the set of values of n.

② $$\int_0^1 \frac{1}{4-x^2}\,dx = \int_0^1 \left[\frac{1}{4(2-x)} + \frac{1}{4(2+x)}\right] dx = \frac{1}{4}\left[\ln\left|\frac{2+x}{2-x}\right|\right]_0^1 = \frac{1}{4}\ln 3$$

$$\int_0^{1/2p} \frac{1}{\sqrt{1-p^2x^2}}\,dx = \frac{1}{p}\int_0^{1/2p} \frac{p}{\sqrt{1-(px)^2}}\,dx = \frac{1}{p}\left[\sin^{-1}(px)\right]_0^{\frac{1}{2p}} = \frac{1}{p}\left[\sin^{-1}\left(\tfrac{1}{2}\right) - \sin^{-1}(0)\right]$$
$$= \frac{\pi}{6p}$$

Given $\displaystyle\int_0^1 \frac{1}{4-x^2}\,dx = \int_0^{\frac{1}{2p}} \frac{1}{\sqrt{1-p^2x^2}}\,dx$

From above,

$$\frac{1}{4}\ln 3 = \frac{\pi}{6p}$$

$$\therefore p = \frac{4\pi}{6\ln 3}$$

$$= \frac{2\pi}{3\ln 3} \#$$

2009 H2 Maths Paper 1:

Date No.

3(i)

$$\frac{1}{n-1} - \frac{2}{n} + \frac{1}{n+1} = \frac{n(n+1) - 2(n+1)(n-1) + n(n-1)}{(n-1)\ n\ (n+1)}$$

$$= \frac{n^2+n - 2n^2+2 + n^2 - n}{n(n^2-1)}$$

$$= \frac{2}{n^3-n}$$

$\therefore A = 2$ #

(ii)

$$\sum_{r=2}^{n} \frac{1}{r^3-r} = \frac{1}{2}\sum_{r=2}^{n} \frac{2}{r^3-r}$$

$$= \frac{1}{2}\sum_{r=2}^{n}\left(\frac{1}{r-1} - \frac{2}{r} + \frac{1}{r+1}\right) \qquad \{\text{use } 3(i)\}.$$

$$= \frac{1}{2}\Big[\frac{1}{1} - \frac{2}{2} + \frac{1}{3}$$
$$+ \frac{1}{2} - \frac{2}{3} + \frac{1}{4}$$
$$+ \frac{1}{3} - \frac{2}{4} + \frac{1}{5}$$
$$+ \frac{1}{4} - \frac{2}{5} + \frac{1}{6}$$
$$\vdots$$
$$+ \frac{1}{n-3} - \frac{2}{n-2} + \frac{1}{n-1}$$
$$+ \frac{1}{n-2} - \frac{2}{n-1} + \frac{1}{n}$$
$$+ \frac{1}{n-1} - \frac{2}{n} + \frac{1}{n+1}\Big]$$

$$= \frac{1}{2}\left[1 - \frac{2}{2} + \frac{1}{2} + \frac{1}{n} - \frac{2}{n} + \frac{1}{n+1}\right] \qquad \{\text{using method of difference}\}$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{1}{n} + \frac{1}{n+1}\right) \#$$

$$\therefore \sum_{r=2}^{n} \frac{1}{r^3-r} = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{n} + \frac{1}{n+1}\right)$$

$$\lim_{n\to\infty} \sum_{r=2}^{n} \frac{1}{r^3-r} = \lim_{n\to\infty} \frac{1}{2}\left(\frac{1}{2} - \frac{1}{n} + \frac{1}{n+1}\right)$$

$$\sum_{r=2}^{\infty} \frac{1}{r^3-r} = \frac{1}{2}\left(\frac{1}{2} - 0 + 0\right)$$

$$= \frac{1}{4}$$

$\therefore$ The series converges to a constant as $n \to \infty$ #

Its value $= \frac{1}{4}$ #

2009 H2 Maths Paper 1:

Date: No.

(4) Given $f(x) = \begin{cases} 7 - x^2 & \text{for } 0 < x \leq 2 \\ 2x - 1 & \text{for } 2 < x \leq 4 \end{cases}$

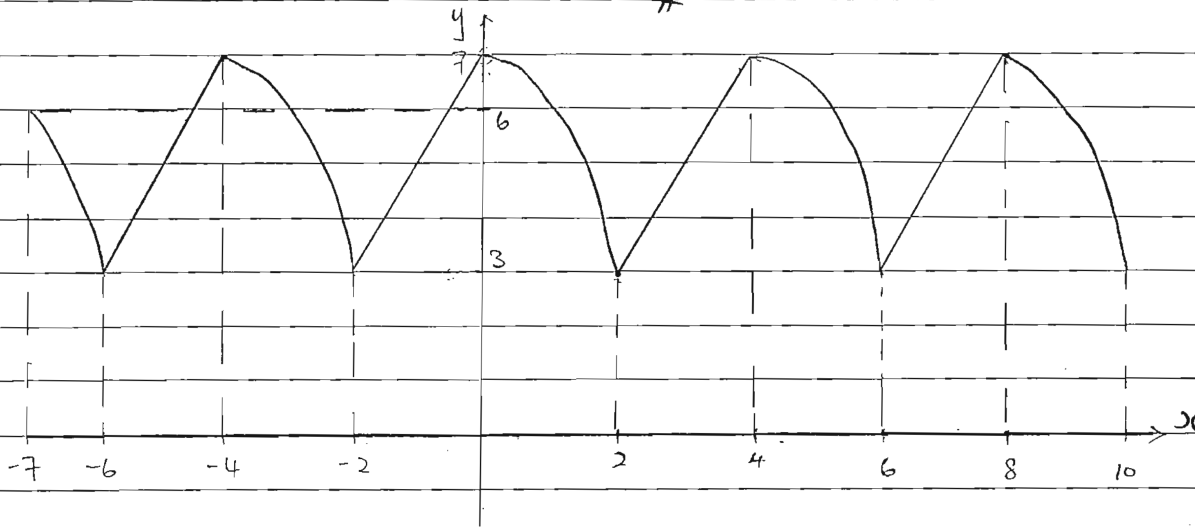
and $f(x) = f(x+4)$

(i) $f(27) = f(23+4) = f(23) = f(19+4) = f(19) = f(15) = f(11)$
$= f(7) = f(3) = 2(3) - 1 = 5.$

$f(45) = f(41+4) = f(41) = f(37) = f(33) = \ldots\ldots = f(1) = 7 - 1 = 6$

$\therefore f(27) + f(45) = 5 + 6 = 11$ #

(ii)



$f(-7) = f(-7+4) = f(-3) = f(-3+4) = f(+1) = 7 - 1^2 = 6$

(iii) $\therefore \int_{-4}^{3} f(x)\,dx = \int_{-4}^{4} f(x)\,dx - \int_{3}^{4} f(x)\,dx.$

$= 2\int_0^4 f(x)\,dx$ − area of trapezium.

$= 2\left[\int_0^2 (7 - x^2)\,dx + \int_2^4 (2x-1)\,dx\right] - \frac{1}{2}(5+7)(1)$

$= 2\left\{\left[7x - \frac{1}{3}x^3\right]_0^2 + \frac{1}{2}(3+7)(2)\right\} - 6$

$= 2\left\{\left(14 - \frac{8}{3}\right) + 10\right\} - 6$

$= 36\frac{2}{3}$ unit^2.

2009 H2 Maths Paper 1:

Date: No.

(5) Let $P(n)$ be the statement s.t.

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1) \quad \text{where } n \geq 1$$

when $n=1$

$$L.H.S = \sum_{r=1}^{1} r^2 = 1^2 = 1$$

$$R.H.S = \frac{1}{6}(1)(1+1)(2+1) = \frac{1}{6}(2)(3) = 1$$

$\therefore L.H.S = R.H.S$

$\therefore P(1)$ is true.

Assume $P(k)$ is true for some $k \in \mathbb{Z}^+$

i.e. $\sum_{r=1}^{k} r^2 = \frac{1}{6}k(k+1)(2k+1)$

Need to prove that $P(k+1)$ is true

i.e. $\sum_{r=1}^{k+1} r^2 = \frac{1}{6}(k+1)(k+2)[2(k+1)+1]$

Now, $LHS = \sum_{r=1}^{k+1} r^2 = \sum_{r=1}^{k} r^2 + (k+1)^2$

$$= \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$$

$$= \frac{1}{6}(k+1)[k(2k+1) + 6(k+1)]$$

$$= \frac{1}{6}(k+1)[2k^2 + 7k + 6].$$

$$= \frac{1}{6}(k+1)(k+2)(2k+3)$$

$$= \frac{1}{6}(k+1)(k+2)[2(k+1)+1]$$

$$= R.H.S$$

i.e. $P(k+1)$ is true whenever $P(k)$ is true.

Since $P(1)$ is true, by Mathematical Induction, $P(n)$ is true for all $n \in \mathbb{Z}^+$

$$\sum_{r=n+1}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n} r^2.$$

$$= \frac{1}{6}(2n)(2n+1)(4n+1) - \frac{1}{6}n(n+1)(2n+1)$$

$$= \frac{1}{6}n(2n+1)[8n+2 - n-1]$$

$$= \frac{1}{6}n(2n+1)(7n+1)$$

2009 H2 Maths Paper 1:

(6) Given C_1 : $y = \frac{x-2}{x+2}$ and C_2 : $\frac{x^2}{6} + \frac{y^2}{3} = 1$

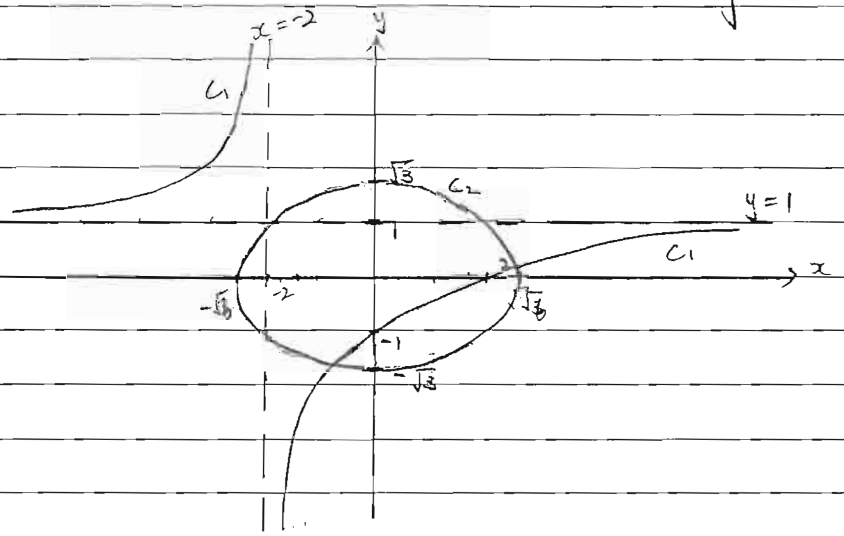
$y = 1 - \frac{4}{x+2}$ $\qquad$ $\frac{x^2}{(\sqrt{6})^2} + \frac{y^2}{(\sqrt{3})^2} = 1.$

(i) Vertical asymptotes : $x = -2$. $\qquad$ C_2 is an Ecllipse.

Horizontal asymptotes : $y = 1$

when $x = 0$, $y = -1$ $\qquad$ when $x = 0$, $y = \pm\sqrt{3}$

when $y = 0$ $x = 2$ $\qquad$ when $y = 0$, $x = \pm\sqrt{6}$



(ii) Given C_1 : $y = \frac{x-2}{x+2}$ —— ①

C_2 : $\frac{x^2}{6} + \frac{y^2}{3} = 1.$

$x^2 + 2y^2 = 6.$ —— ②.

sub ① into ②,

$$x^2 + 2\frac{(x-2)^2}{(x+2)^2} = 6.$$

$$x^2(x+2)^2 + 2(x-2)^2 = 6(x+2)^2$$

$$2(x-2)^2 = 6(x+2)^2 - x^2(x+2)^2.$$

$$2(x-2)^2 = (6-x^2)(x+2)^2 \text{ (shown).}$$

(iii) Using GC : $x = -0.515$ or $x = 2.45$ {correct to 3 sig fig}.

2009 H2 Maths Paper 1:

Date No.

7 (i) Given $f(x) = e^{\cos x}$ $\Rightarrow$ $f(0) = e$ #

Let $y = e^{\cos x}$

$\frac{dy}{dx} = -\sin x \, e^{\cos x}$ $\Rightarrow$ $f'(0) = 0$ #

$\frac{dy}{dx} = -y \sin x$.

$\frac{d^2y}{dx^2} = -\sin x \cdot \frac{dy}{dx} - y\cos x$ $\Rightarrow$ $f''(0) = -e$

By Maclaurin theorem:

$$f(x) = f(0) + \frac{f'(0)}{1!}x^1 + \frac{f''(0)}{2!}x^2$$
$$= e + 0x^1 + \frac{-e}{2}x^2$$
$$= e - \frac{1}{2}ex^2 \text{ \# } \quad \text{———— (1)}$$

(ii) $$\frac{1}{a+bx^2} = (a+bx^2)^{-1}$$
$$= a^{-1}\left(1+\frac{b}{a}x^2\right)^{-1}$$
$$= \frac{1}{a}\left[1 + \frac{(-1)}{1!}\left(\frac{b}{a}x^2\right)^1 + \dots\right]$$
$$= \frac{1}{a}\left[1 - \frac{b}{a}x^2 + \dots\right]$$
$$= \frac{1}{a} - \frac{b}{a^2}x^2 + \dots \quad \text{———— (2)}$$

Given 1st two non-zero terms of (1) = 1st two non-zero terms of (2)

$\therefore$ $e - \frac{1}{2}ex^2 = \frac{1}{a} - \frac{b}{a^2}x^2$.

Compare constant terms:

$$e = \frac{1}{a}$$
$$a = \frac{1}{e}$$

Compare coeff of x^2:

$$-\frac{1}{2}e = -\frac{b}{a^2}$$
$$b = \frac{1}{2}ea^2$$
$$= \frac{1}{2}e\left(\frac{1}{e^2}\right)$$
$$= \frac{1}{2e} \text{ \#}$$

Date No

(8) (i) Given Instrument A, the bars form G. P.

Given $T_1 = a = 20$ cm.

Given $T_{25} = ar^{24} = 5$ cm.

$$20r^{24} = 5.$$

$$r^{24} = \frac{1}{4}$$

$$r = [2^{-2}]^{1/24}$$

$$= 2^{-\frac{1}{12}}$$

$\therefore$ Total length of n bars $= \frac{a(1-r^n)}{1-r} = \frac{20[1-(2^{-\frac{1}{12}})^n]}{1-2^{-1/12}}$

Since $r = 2^{-\frac{1}{12}} < 1$, $(2^{-\frac{1}{12}})^n \to 0$ as $n \to \infty$.

$\therefore$ Total length $= 20/(1-2^{-\frac{1}{12}}) \approx 356.34$ cm < 357 cm (shown)

(ii) Given Instrument B consists of only 25 bars which are identical to the 1st 25 bars of instrument A.

Total length of 25 bars of Instrument B, $L = 20[1-(2^{-\frac{1}{12}})^{25}]/(1-2^{-\frac{1}{12}})$

$\therefore$ $L \approx 272$ cm # (corr to 3 sig fig)

Length of 13th bar $= 20(2^{-\frac{1}{12}})^{13-1} = 20(2^{-\frac{1}{12}})^{12} = 10$ cm #

(iii) Using results of 8(ii), $L = 272$ cm.

We have $\frac{25}{2}(a+l) = 272$ —— (1).

Given $T_{25} = 5$ cm

$l = 5$ —— (2).

Sub (2) into (1),

$$\frac{25}{2}(a+5) = 272$$

$$a = 16.76 \text{ cm}$$

From (2), we also have

$$a + 24d = 5$$

$$16.76 + 24d = 5 \quad (\text{use } a = 16.76)$$

$$d = -0.49 \text{ cm}.$$

$\therefore$ Value of $d = -0.49$ cm #

Longest bar $= a = 16.76$ cm # (exact)

2009 H2 Maths Paper 1:

(9) (i) Given $z^7 - (1+i) = 0$

$$z^7 = (1+i)$$
$$z^7 = 2^{\frac{1}{2}} e^{(2k+\frac{1}{4})\pi i}$$
$$z = 2^{\frac{1}{14}} e^{\frac{1}{7}(2k+\frac{1}{4})\pi i}$$

Hence for $k=-3$, $z = 2^{\frac{1}{14}} e^{-\frac{23}{28}\pi i} = z_7$

for $k=-2$, $z = 2^{\frac{1}{14}} e^{-\frac{15}{28}\pi i} = z_6$

for $k=-1$, $z = 2^{\frac{1}{14}} e^{-\frac{7}{28}\pi i} = z_5$

for $k=0$, $z = 2^{\frac{1}{14}} e^{\frac{1}{28}\pi i} = z_1$

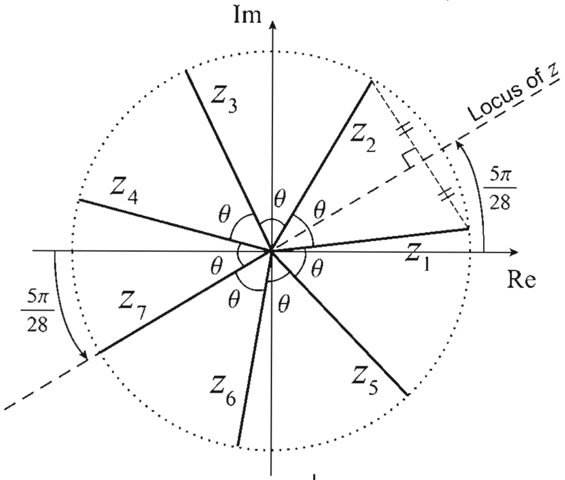
for $k=1$, $z = 2^{\frac{1}{14}} e^{\frac{9}{28}\pi i} = z_2$

for $k=2$, $z = 2^{\frac{1}{14}} e^{\frac{17}{28}\pi i} = z_3$

for $k=3$, $z = 2^{\frac{1}{14}} e^{\frac{25}{28}\pi i} = z_4$

$\therefore$ The roots are $2^{\frac{1}{14}} e^{-\frac{23}{28}\pi i}$, $2^{\frac{1}{14}} e^{-\frac{15}{28}\pi i}$, $2^{\frac{1}{14}} e^{-\frac{7}{28}\pi i}$, $2^{\frac{1}{14}} e^{\frac{1}{28}\pi i}$, $2^{\frac{1}{14}} e^{\frac{9}{28}\pi i}$, $2^{\frac{1}{14}} e^{\frac{17}{28}\pi i}$ and $2^{\frac{1}{14}} e^{\frac{25}{28}\pi i}$ #

(ii)



Note $|z_1| = |z_2| = |z_3| = |z_4| = |z_5| = |z_6| = |z_7| = 2^{1/14}$

and $\theta = \frac{8}{28}\pi = \frac{2}{7}\pi$

(iii) Given $|z - z_1| = |z - z_2|$ —— (1)

for $z = 0 + 0i$, we have $|0 - z_1| = |z_1| = 2^{1/14}$

for $z = 0 + 0i$, we have $|0 - z_2| = |z_2| = 2^{1/14}$

$\therefore$ (0,0) is one of the locus pts for (1)

Hence, the locus of all points z s.t. $|z - z_1| = |z - z_2|$ passes thru' origin.

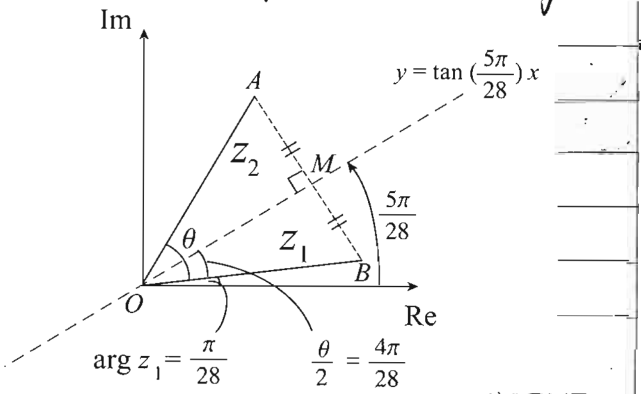
Since $OA = OB$, OM bisects $\angle AOB$.

$\therefore$ Gradient $OM = \tan\left(\frac{\pi}{28} + \frac{4}{28}\pi\right)$

$$= \tan\left(\frac{5}{28}\pi\right)$$

$\therefore$ Cartesian Equation of OM:

$$y = \tan\left(\frac{5\pi}{28}\right) \cdot x$$



2009 H2 Maths Paper 1:

(10) $p_1 : \underset{\sim}{r} \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 1$ Let $n_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ $|\underset{\sim}{n_1}| = \sqrt{14}$

$p_2 : \underset{\sim}{r} \cdot \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = 2$ Let $n_2 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$. $|\underset{\sim}{n_2}| = \sqrt{6}$

(i) Let θ be the acute angle between p_1 and p_2

Then $\cos\theta = \left| \dfrac{\underset{\sim}{n_1} \cdot \underset{\sim}{n_2}}{|\underset{\sim}{n_1}|\,|\underset{\sim}{n_2}|} \right| = \left| \dfrac{\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}}{\sqrt{14}\ \sqrt{6}} \right|$

$\therefore \theta = \cos^{-1}\left(\frac{3}{\sqrt{84}}\right)$

$= 70.89^\circ$

$\approx 70.9^\circ$ (corr to 1 dec pl).

(ii) $\underset{\sim}{n_1} \times \underset{\sim}{n_2} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \times \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ -5 \\ 5 \end{pmatrix} = -5\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

Let $\underset{\sim}{d}$ be the direction vector of l.

$\underset{\sim}{d} \,//\, \underset{\sim}{n_1} \times \underset{\sim}{n_2}$ $\therefore$ Take $\underset{\sim}{d} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

Given $p_1 : \underset{\sim}{r} \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 1 \Rightarrow 2x + y + 3z = 1$

and $p_2 : \underset{\sim}{r} \cdot \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = 2 \Rightarrow -x + 2y + z = 2$

Using G.C, the point of intersection of P_1 and P_2 is $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\therefore$ Vector eqn of line l:

$\underset{\sim}{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \lambda \in \mathbb{R}$ —— ①

(iii) Given p_3 : $2x + y + 3z - 1 + k(-x + 2y + z - 2) = 0$.

$(2-k)x + (2k+1)y + (k+3)z = 2k+1$

Let $\underset{\sim}{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, then

$p_3 : \underset{\sim}{r} \cdot \begin{pmatrix} 2-k \\ 2k+1 \\ k+3 \end{pmatrix} = 2k+1$. ② , Let $\underset{\sim}{n_3} = \begin{pmatrix} 2-k \\ 2k+1 \\ k+3 \end{pmatrix}$

From ①, we know $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ lies on line l.

and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2-k \\ 2k+1 \\ k+3 \end{pmatrix} = 2k+1$

$\therefore \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ also lies on P_3.

$\underset{\sim}{d} \cdot \underset{\sim}{n_3} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2-k \\ 2k+1 \\ k+3 \end{pmatrix} = 2 - k + 2k + 1 - (k+3) = 0$

$\underset{\sim}{d} \perp n_3 \Rightarrow \underset{\sim}{d} \,//\,$ plane P_3.

Since $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ lies in P_3 and $\underset{\sim}{d} \,//\, P_3$. The l lies in P_3 for any k.

Given $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ lie in P_3. Then $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2-k \\ 2k+1 \\ k+3 \end{pmatrix} = 2k+1$.

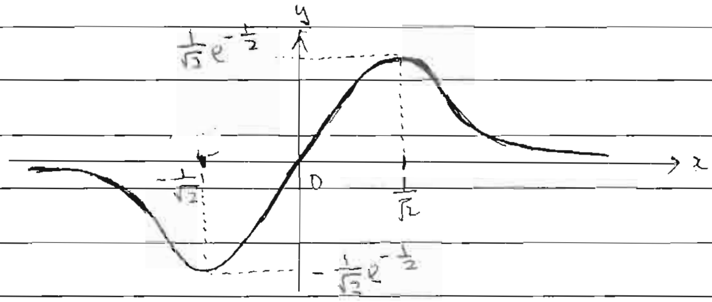
$2(2-k) + 3(2k+1) + 4(k+3) = 2k+1 \Rightarrow k = -3$.

From ② $\therefore P_3 : \underset{\sim}{r} \cdot \begin{pmatrix} 5 \\ -5 \\ 0 \end{pmatrix} = -5 \Rightarrow 5x - 5y = -5$

$z \in \mathbb{R}, \quad x - y = -1$ #

(11) Given $y = f(x)$ where $f(x) = x\,e^{-x^2}$.

(i)



(ii) $y = x e^{-x^2}$

$\frac{dy}{dx} = e^{-x^2} + x\,e^{-x^2}(-2x)$

$= e^{-x^2}[1 - 2x^2]$

When $\frac{dy}{dx} = 0$, $e^{-x^2}[1-2x^2] = 0$.

Since $e^{-x^2} > 0$, $1 - 2x^2 = 0$

$x^2 = \frac{1}{2}$

$x = \pm\frac{1}{\sqrt{2}}$

when $x = -\frac{1}{\sqrt{2}}$, $y = -\frac{1}{\sqrt{2}}e^{-\frac{1}{2}}$

when $x = \frac{1}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2}}e^{-\frac{1}{2}}$

$\therefore$ Coordinates of the turning points are $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}e^{-\frac{1}{2}}\right)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}e^{-\frac{1}{2}}\right)$

(iii) Use $u = x^2$, $du = 2x\,dx$

When $x = 0$, $u = 0$ and when $x = n$, $u = n^2$.

$\int_0^n f(x)\,dx = \int_0^n x\,e^{-x^2}\,dx$.

$= \frac{1}{2}\int_0^n e^{-x^2}\,2x\,dx$

$= \frac{1}{2}\int_0^{n^2} e^{-u}\,du$.

$= \frac{1}{2}\left[-e^{-u}\right]_0^{n^2}$

$= \frac{1}{2}\left[-e^{-n^2} + 1\right]$

$= \frac{1}{2}\left(1 - e^{-n^2}\right)$ ——— (1)

$\therefore$ The area of the region between the curve and the positive x-axis {when $n \to \infty$} $= \lim_{n\to\infty}\int_0^n f(x)\,dx$

$\approx \frac{1}{2}(1 - 0)$ $\{e^{-n^2} \to 0 \text{ as } n \to \infty\}$.

$\approx \frac{1}{2}$ unit^2 *

2009 H2 Maths Paper1

Date No

(11)(iv) $\int_{-2}^{2} |f(x)|\, dx = 2\int_0^2 f(x)\, dx$

$= 2\left[\frac{1}{2}(1 - e^{-2^2})\right]$ {using ①}.

$= 1 - e^{-4}$ #

(v) Volume about x-axis $= \pi \int_0^1 y^2\, dx$

$= \pi \int_0^1 x^2 e^{-2x^2}\, dx$

$= \pi\ (0.11570218)$ {using G.C}.

$= 0.36348$

≈ 0.363 unit^3 (correct to 3 sig fig) ※