2009 H1 Maths 8863 paper 1

1. $x+2y=3$

$x = 3-2y$ —①

$x^2+xy=2$ —②

Sub ① into ②, $(3-2y)^2 + (3-2y)y = 2$

$9-12y+4y^2+3y-2y^2=2$

$2y^2-9y+7=0$

$(2y-7)(y-1)=0$

$2y-7=0$ or $y-1=0$

$y=3.5$ $\quad$ $y=1$

Sub $y=3.5$ into ①, $x=3-2(3.5)$

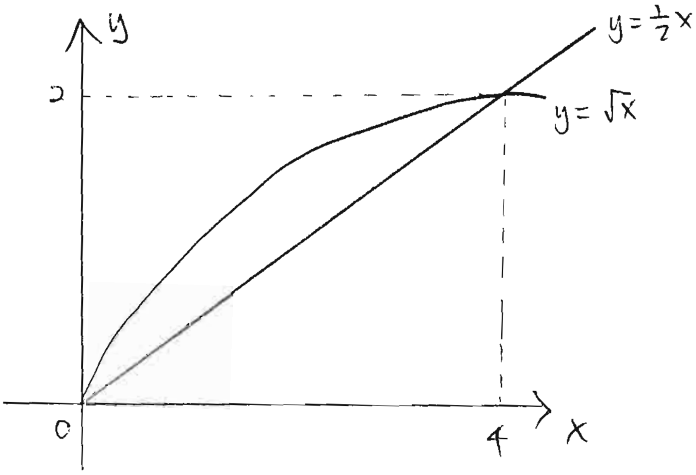
$= -4$

Sub $y=1$ into ①, $x=3-2(1)$

$=1$

$\therefore$ $x=-4$, $y=3.5$ and $x=1, y=1$ #

2. (i)



$y=\sqrt{x}$ —①

$y=\frac{1}{2}x$ —②

① = ②, $\sqrt{x} = \frac{1}{2}x$

$x = \frac{1}{4}x^2$

$x^2 = 4x$

$x^2-4x=0$

$x(x-4)=0$

$x=0$ or $x=4$

$\therefore$ $y=0$ $\quad$ $y=2$

$\therefore$ Coordinates of intersection are $(0,0)$ and $(4,2)$ #

(ii) Area of the region between the two graphs

$= \int_0^4 (\sqrt{x} - \frac{1}{2}x)\,dx$

$= \int_0^4 (x^{\frac{1}{2}} - \frac{1}{2}x)\,dx$

$= \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{4} \right]_0^4$

$= \frac{4^{3/2}}{3/2} - \frac{4^2}{4}$

$= 5\frac{1}{3} - 4$

$= 1\frac{1}{3}$ unit^2 #

3. (i) $f(x) = e^x$

let $y = e^x$

$\ln y = x$

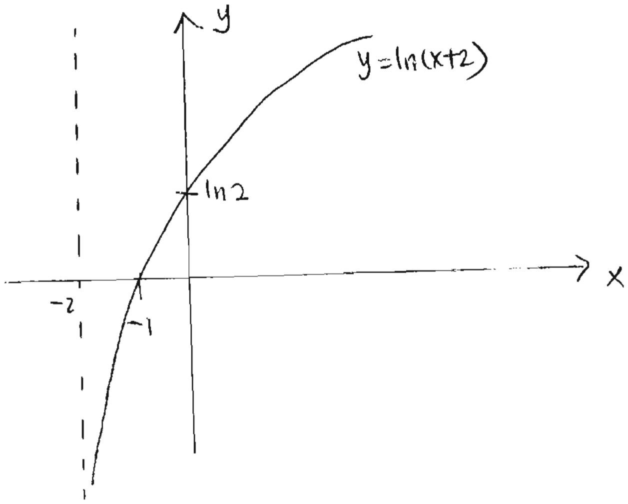
$\therefore f^{-1}(x) = \ln x$, $x > 0$

$\Rightarrow h(x) = f^{-1}g(x)$

$= f^{-1}(x+2)$

$= \ln(x+2)$, $x > -2$. #

(ii)



$y = \ln(x+2)$

when $x=0$, $y = \ln 2$

when $y=0$, $\ln(x+2) = 0$

$x+2 = e^0$

$x = -1$

Equation of asymptote : $x = -2$

Coordinates of points that crosses x-axis = $(-1, 0)$

Coordinates of points that crosses y-axis = $(0, \ln 2)$

#

(iii) $g(x) = x+2$

$g(-x) = -x+2$

Given $h(x) = g(-x)$,

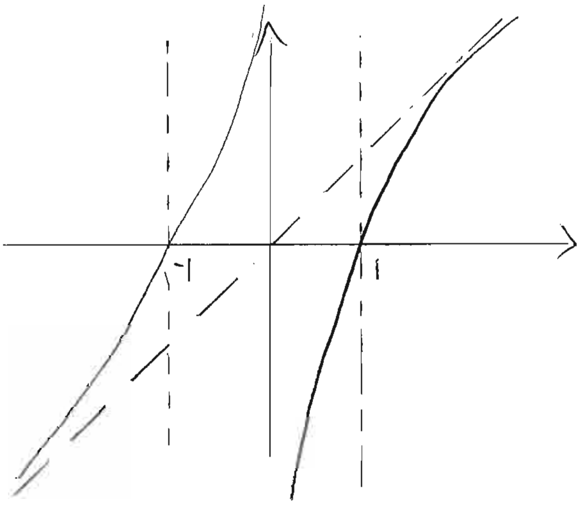
$\Rightarrow \ln(x+2) = -x+2$

By GC, draw $y = \ln(x+2)$ and $y = -x+2$,

$x = 0.9262$

≈ 0.926 (3 d.p) #

4. (i) $y = x - \frac{1}{x}$



when $y=0$, $x - \frac{1}{x} = 0$

$x = \frac{1}{x}$

$x^2 = 1$

$x = 1$ or -1

(ii) $y = x - \frac{1}{x}$

$\frac{dy}{dx} = 1 + \frac{1}{x^2}$

when $x=2$, $\frac{dy}{dx} = \frac{5}{4}$ and $y = \frac{3}{2}$

$\therefore$ Gradient of the normal $= -\frac{4}{5}$

(iii) Equation of the normal at P : $y - \frac{3}{2} = -\frac{4}{5}(x-2)$

$y = -\frac{4}{5}x + \frac{31}{10}$

$5y = -4x + 15.5$

$5y + 4x - 15.5 = 0$ # ——(1)

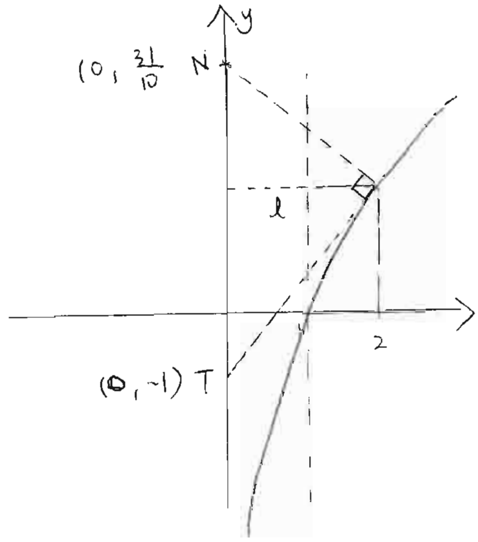
(iv) Sub $x=0$ into (1), $5y = 15.5$

$y = \frac{31}{10}$

$\therefore$ Coordinates of N $= \left(0, \frac{31}{10}\right)$

Equation of tangent at P: $y - \frac{3}{2} = \frac{5}{4}(x-2)$

$$y = \frac{5}{4}x - 1 \quad \text{—— (2)}$$



Sub $x=0$ into (2), $y=-1$

$\therefore$ coordinates of $T = (0, -1)$

Area of $\Delta PTN = \frac{1}{2} \times l \times TN$

$= \frac{1}{2} \times 2 \times \left(\frac{31}{10} + 1\right)$

$= 4.1$ unit^2 #

5. $y = 2x^3 - 5x^2 - 4x + 3$ —— (1)

(i) $\frac{dy}{dx} = 6x^2 - 10x - 4$

For stationary points, $\frac{dy}{dx} = 0$

$6x^2 - 10x - 4 = 0$

$3x^2 - 5x - 2 = 0$

$(3x+1)(x-2) = 0$

$3x+1 = 0$ or $x-2=0$

$x = -\frac{1}{3}$ $\quad$ $x = 2$

Sub $x = -\frac{1}{3}$ into (1), $y = 2(-\frac{1}{3})^3 - 5(-\frac{1}{3})^2 - 4(-\frac{1}{3}) + 3$

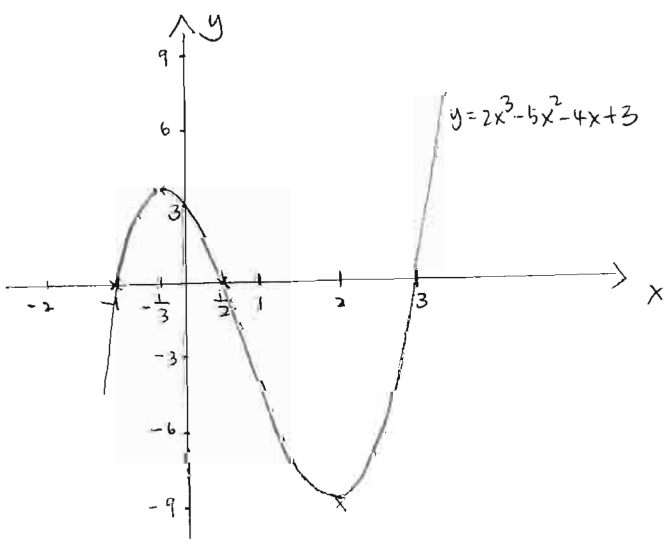
$= 3\frac{19}{27}$

Sub $x = 2$ into (1), $y = 2(2)^3 - 5(2)^2 - 4(2) + 3$

$= -9$

$\therefore$ Coordinates of the stationary points on the curve are $\left(-\frac{1}{3}, 3\frac{19}{27}\right)$ & $(2, -9)$ #

(ii)



$y = 2x^3 - 5x^2 - 4x + 3$

$= (x+1)(2x^2 - 7x + 3)$

$= (x+1)(2x-1)(x-3)$

when $y = 0$,

$x = -1, \frac{1}{2}, 3.$

(iii) $2x^3 - 5x^2 - 4x + 3 > 0$

From the graph in (ii), $-1 < x < \frac{1}{2}$, $x > 3$

$$2e^{3x} - 5e^{2x} - 4e^{x} + 3 > 0$$
$$2(e^{x})^3 - 5(e^{x})^2 - 4(e^{x}) + 3 > 0$$

$\Rightarrow -1 < e^{x} < \frac{1}{2}$, $e^{x} > 3$

$\Rightarrow 0 < e^{x} < \frac{1}{2}$ (e^{x} is always positive) $\therefore x > \ln 3$ #

$\therefore x < \ln\frac{1}{2}$ #

6. (i) P (the call is for A and A is in the office) $= 0.2 \times 0.7$
$= 0.14$ #

(ii) P (the researcher being called is in the office)
= P (A is in office or B is in office or C is in office)
$= 0.2(0.7) + 0.3(0.6) + 0.5(0.8)$
$= 0.72$ #

(iii) let X : call is for C.
y : the researcher being called is not in the office

$$\therefore P(X|y) = \frac{P(X \cap y)}{P(y)}$$
$$= \frac{0.5(0.2)}{1 - P(y')}$$
$$= \frac{0.5(0.2)}{1 - 0.72}$$
$$= 0.3571$$
$$\approx 0.357 \text{ (3 s.f)}$$ #

7. (i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{17}{30} = \frac{1}{3} + \frac{2}{5} - P(A \cap B)$

$P(A \cap B) = \frac{1}{6}$ #

(ii) If A and B are not independent $\Rightarrow P(A) \cdot P(B) = P(A \cap B)$

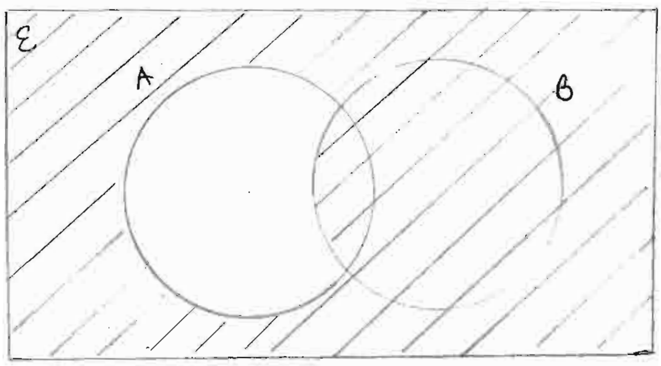
We have $P(A)P(B) = \frac{1}{3} \times \frac{2}{5}$ and $P(A \cap B) = \frac{1}{6}$

$= \frac{2}{15}$

$\therefore P(A)P(B) \neq P(A \cap B)$,

So A and B are not independent. (shown) #

(iii)



$P(A' \cup B) = 1 - P(A) + P(A \cap B)$

$= 1 - \frac{1}{3} + \frac{1}{6}$

$= \frac{5}{6}$ #

8. Let the random variable X be the lifetime of a component.

$X \sim N(120, 18^2)$

(i) $P(X > 144) = \text{normal cdf}(144, e^{99}, 120, 18)$

$= 0.091211$

≈ 0.0912 (3 s.f.) #

(ii) $P(X_1 > 144 \text{ and } X_2 < 144) + P(X_1 < 144 \text{ and } X_2 > 144)$

$= 0.091211(1 - 0.091211) + (1 - 0.091211)(0.091211)$

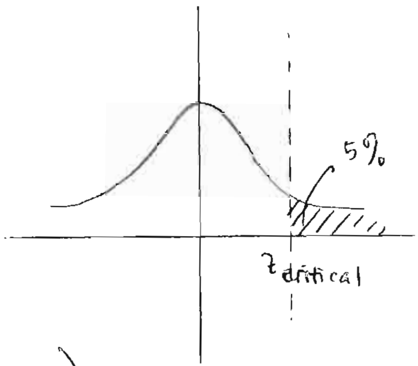
$= 0.16578$

≈ 0.166 (3 s.f.) #

$H_0: \mu = 120$ days

$H_1: \mu > 120$ days

$\bar{X} \sim N\left(120, \frac{18^2}{50}\right)$



$$z_{test} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{124 - 120}{18/\sqrt{50}}$$

$$= 1.571 \quad (< z_{critical})$$

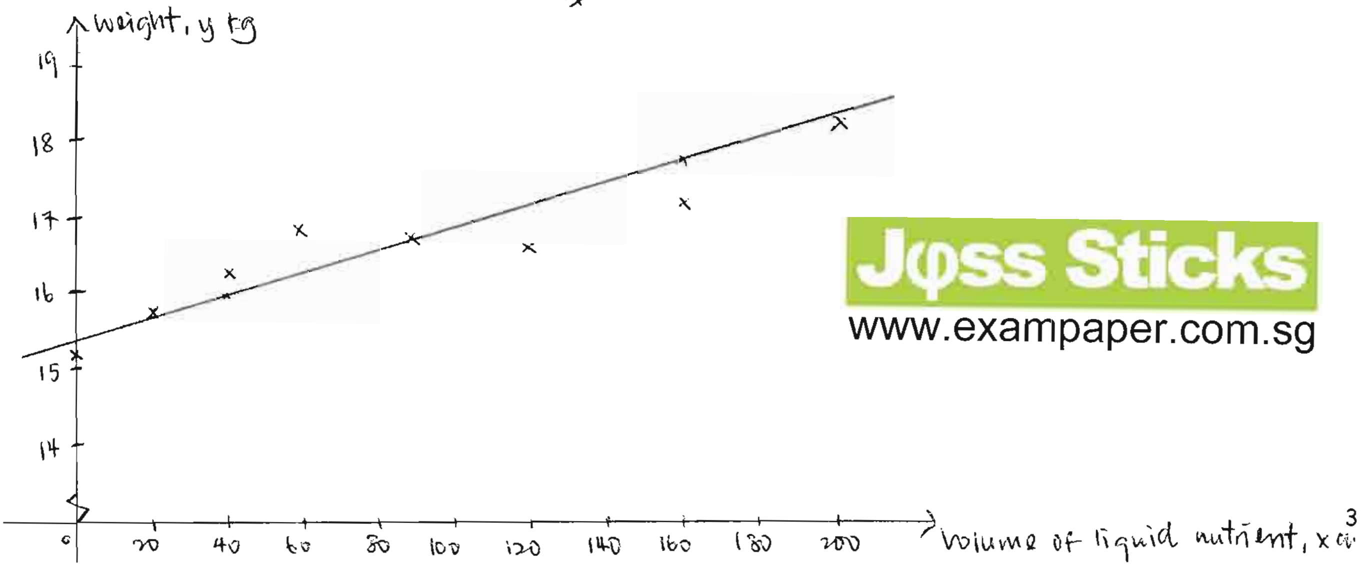
$z_{critical} = \text{InvNorm}(0.95)$

$= 1.645$

Since $z_{test} < z_{critical}$, do not reject H_0.

There is insufficient evidence, at the 5% level of significance, to support the company's claim that the mean lifetime is longer than for the old components. #

9. (i)



(ii) Product moment correlation coefficient, $r = 0.931$

The r-value of 0.931 indicates a reasonably strong positive linear correlation between the volume of liquid nutrient added and the total weight of fruits per tree. #

(iii) From GC: $y = 15.486 + 0.01232x$

$\approx 15.5 + 0.0123x$ #

(iv) When $x = 135$, $y = 15.5 + 0.0123(135)$

$= 17.1605$

≈ 17.2 kg #

(v) Since the volume of liquid nutrient needed for 20 kg of fruit is estimated through extrapolating the data beyond 18.1 kg, it might be unsuitable to use the equation. #

10. (i) Let the random variable X be the number of candidates who fail the piano exam.

$X \sim B(10, 0.2)$

$\therefore P(X=2) = 0.302$ #

(ii) Probability of candidates who pass the piano examination and awarded a distinction $= 0.15(1-0.2)$

$= 0.12$

Let the random variable y be the number of candidates who pass the piano examination and awarded a distinction.

$y \sim B(10, 0.12)$

$P(y<2) = P(y \leq 1)$

$= \text{Binomcdf}(10, 0.12, 1)$

$= 0.65827$

≈ 0.658 (3 s.f.) #

(iii) Let the random variable W be the number of candidates who fail the piano examination.

$W \sim B(50, 0.2)$

Since $np = 10\ (>5)$, $nq = 40\ (>5)$

we can approximate $W \sim N(10, 8)$ using normal distribution.

$\therefore P(W \leq 12) = P(W < 12.5)$

$= 0.812$ #

11 (a) (i) A systematic random sample of 8 may be obtained by first arranging the claims in order of the time in which they are received, and then selecting every 9th claim from the stack, from a random starting point in the stack.

(ii) Choosing the first of claims received would not give a good indication as the claims that arrive the earliest will tend to be of relatively low-value, since less time is needed to access the claim amounts when less items have been damaged by the flood. Hence a systematic random sample will give a better indication of the value of the 72 claims. #

(b)(i) $\bar{x} = \frac{\Sigma x}{n}$

$= \frac{\Sigma (x-1000)}{n} + 1000$

$= \frac{5320}{120} + 1000$

$= 1044\frac{1}{3}$ #

Unbiased estimate of population variance

$= \frac{n}{n-1}\left\{\frac{\Sigma (x-1000)^2}{n} - \left[\frac{\Sigma (x-1000)}{n}\right]^2\right\}$

$= \frac{120}{119}\left\{\frac{8282000}{120} - \left(\frac{5320}{120}\right)^2\right\}$

$= 67614.67$

≈ 67600 (3 s.f) #

(ii) An unbiased estimate is an estimate for a parameter of a distribution whose expected value is equal to the true value of the parameter being estimated. #

(iii) $H_0: \mu = 1000$

$H_1: \mu \neq 1000$

$\bar{X} \sim N\left(1000, \frac{67614.67}{120}\right)$

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \frac{1044\frac{1}{3} - 1000}{\sqrt{\frac{67614.67}{120}}}$

$= 1.86753$

From GC, $\alpha\% = 2 \times \text{normcdf}(1.86753, E99)$

$= 0.06182 \times 100\%$

$\approx 6.18\%$

$\therefore \alpha = 6.18$

set of value of α: $|\alpha| > 6.18$ #

12 (a) Let $X \sim N(\mu, \sigma^2)$

$$z = \frac{x-\mu}{\sigma}$$

$$\Rightarrow \frac{22-\mu}{\sigma} = \text{invNorm}(0.3)$$

$$\frac{22-\mu}{\sigma} = -0.52440 \quad \text{—— (1)}$$

$$\Rightarrow \frac{29-\mu}{\sigma} = \text{invNorm}(0.8)$$

$$\frac{29-\mu}{\sigma} = 0.84162 \quad \text{—— (2)}$$

(2) − (1), $\frac{7}{\sigma} = 0.84162 + 0.52440$

$$\sigma = 5.1243$$
$$\approx 5.12 \ (3 \text{ s.f.})$$

Sub $\sigma = 5.1243$ into (1), $22 - \mu = -0.52440(5.1243)$

$$\mu = 24.687$$
$$\approx 24.7 \ (3 \text{ s.f.})$$

$\therefore \sigma = 5.12$ and $\mu = 24.7$ ※

(b)(i) let the random variable X be the mass of Apple and the random variable y be the mass of Nectarines.

$X \sim N(0.15, 0.03^2)$ $\quad y \sim N(0.07, 0.02^2)$

$X_1 + X_2 - (y_1 + \dots + y_4) \sim N[0.15(2) - 0.07(4), (0.03)^2(2) + (0.02)^2(4)]$

$\sim N[0.02, 3.4 \times 10^{-3}]$

$P(X_1 + X_2 > Y_1 + Y_2 + Y_3 + Y_4) = P[X_1 + X_2 - (y_1 + \dots + y_4) > 0]$

$= 0.63419$

$\approx 0.634 \ (3 \text{ s.f.})$ #

(ii) Let $C = 9(X_1 + X_2) + 12(y_1 + y_2 + y_3 + y_4)$

$C \sim N[9(0.15 \times 2) + 12(0.07 \times 4), 9^2(2)(0.03)^2 + 12^2(4)(0.02)^2]$

$\sim N(6.06, 0.3762)$

$P(5 < c < 6) = \text{normalcdf}(5, 6, 6.06, \sqrt{0.3762})$

$= 0.41906$

$\approx 0.419 \ (3 \text{ s.f.})$ #